$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \\ 5 & 6 \end{bmatrix}, A^T = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 3 & 6 \end{bmatrix}, A^{T^T} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \\ 5 & 6 \end{bmatrix}$$

2. For any diagonal matrix **D,** (including the identity matrix **I**), $\mathbf{D}^\mathbf{T} = \mathbf{D}$.

$$\boldsymbol{D} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & -9 \end{bmatrix}^T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & -9 \end{bmatrix}$$

3. $(A+B)^T = A^T + B^T$
4. $(A-B)^T = A^T - B^T$

Matrix Algebra

- **Equality of two matrices**
  Two matrices **A** and **B** are said to be equal if
  (i) They are of same dimensions.
  (ii) Their corresponding elements are equal.

- **Addition and Subtraction of Matrices**

It is possible to add two matrices together, but only if they have the same dimensions. If **A** and **B** are both $(m \times n)$, then the sum

$$\mathbf{C} = \mathbf{A} + \mathbf{B}$$

Where **C** is also $(m \times n)$ and is defined to have each element the sum of the corresponding elements of **A** and **B**, thus

$$c_{ij} = a_{ij} + b_{ij}$$

Matrix addition is both associative, that is

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$$

and commutative

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

Example:

If **A** and **B** are matrices given by

$$A = \begin{bmatrix} 3 & 2 \\ 4 & -9 \\ -2 & -3 \end{bmatrix}, B = \begin{bmatrix} -3 & 5 \\ 8 & -1 \\ 2 & 8 \end{bmatrix}$$

Find **A** + **B**

$$A+B=\begin{bmatrix}3+(-3) & 2+5\\ 4+8 & -9+(-1)\\ (-2)+2 & -3+8\end{bmatrix}=\begin{bmatrix}0 & 7\\ 12 & -10\\ 0 & 5\end{bmatrix}$$

The **subtraction** of two matrices is similarly defined; if **A** and **B** have the same dimensions, then the difference

C = **A** - **B**

Where **C** is also $(m \times n)$ and is defined to have each element the difference of the corresponding elements of **A** and **B**, thus

$$c_{ij} = a_{ij} - b_{ij}$$

Example:

If **A** and **B** are matrices given by

$$A=\begin{bmatrix}3 & 2\\ 4 & -9\\ -2 & -3\end{bmatrix}, B=\begin{bmatrix}-3 & 5\\ 8 & -1\\ 2 & 8\end{bmatrix}$$

Find **A - B**

$$A-B=\begin{bmatrix}3-(-3) & 2-5\\ 4-8 & -9-(-1)\\ (-2)-2 & -3-8\end{bmatrix}=\begin{bmatrix}6 & -3\\ -4 & -8\\ -4 & -11\end{bmatrix}$$

- **Multiplication of a Matrix by a Scalar Quantity**

If **A** is a matrix and *k* is a scalar quantity, the product

B = *k***A**

Where **B** is defined to be the matrix of the same dimensions as **A** whose elements are simply all scaled by the constant *k*,

$$b_{ij} = ka_{ij}$$

Example:

$$if\ A=\begin{bmatrix}-3 & 5\\ 8 & -1\\ 2 & 8\end{bmatrix} and\ k=3, then\ kA=\begin{bmatrix}-9 & 15\\ 24 & -3\\ 6 & 24\end{bmatrix}$$

- **Multiplication of two Matrices**

  To multiply two matrices their dimensions must meet the condition that allow them to conform, that is the number of columns of the first matrix must be the same as the number of rows as the second one.
  Let **A** be a matrix with dimensions $m \times n$, and **B** is another matrix with dimensions $n \times p$, that is the number of columns in **A** is the same as the number of rows in **B**, then the product

  $$\mathbf{C} = \mathbf{AB}$$

  Where **C** is a matrix with dimensions of $m \ x \ p$.

$$A = \begin{bmatrix} ? & ? \\ ? & ? \\ ? & ? \end{bmatrix}, B = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \end{bmatrix}, C = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

$m \ x \ n$ $\qquad$ $n \ x \ p$ $\qquad$ $m \ x \ p$

must match $\qquad$ *columns in result*

rows in result

Each element in the resulting matrix **C** will be calculated as

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

The element in position *ij* is the sum of the products of elements in the i^{th} row of the first matrix (**A**) and the corresponding elements in the j^{th} column of the second matrix (**B**).
For example if

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}, \ B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} \text{ then } C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$c_{21} = a_{21} \times b_{11} + a_{22} \times b_{21} + a_{23} \times b_{31}$$

Example:
If A and B are matrices given by

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 4 & 1 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 0 & -1 \\ 7 & 5 \end{bmatrix}$$

Then find **C=AB**

$$C = \begin{bmatrix} -1\times 1+2\times 0+0\times 7 & -1\times 3+2\times -1+0\times 5 \\ 4\times 1+1\times 0+-2\times 7 & 4\times 3+1\times -1+-2\times 5 \end{bmatrix} = \begin{bmatrix} -1 & -5 \\ -10 & 1 \end{bmatrix}$$

General Properties for Matrix Multiplication

1. Matrix multiplication is associative, that is

   $$\mathbf{A\,(BC)=(AB)\,C}$$

2. Matrix multiplication is distributive

   $$(\boldsymbol{A}+\boldsymbol{B})\boldsymbol{C} = \boldsymbol{AC}+\boldsymbol{BC}$$

3. Matrix multiplication is not commutative in general

   $$\mathbf{AB} \neq \mathbf{BA}$$

   in fact, unless the two matrices are square, reversing the order in the product will cause the matrices to be *nonconformal*. The order of the terms in the product is therefore very important. In the product **C = AB, A** is said to *pre-multiply* **B**, while **B** is said to **post-multiply A**.
4. If $\mathbf{A}^2 = \mathbf{I}$, then this does not imply that **A = +I or A = -I.**
5. **AB = 0** does not imply that either **A = 0** or that **B = 0.**
6. Even if $\mathbf{A} \neq \mathbf{0}$ and (**AB = AC**) it does not imply that **B =C.**
7. **AI = IA.**
8. $\mathbf{A}^0 = \mathbf{I}$.
9. **0A= 0.**