

$$C = \begin{bmatrix} -1 \times 1 + 2 \times 0 + 0 \times 7 & -1 \times 3 + 2 \times -1 + 0 \times 5 \\ 4 \times 1 + 1 \times 0 + -2 \times 7 & 4 \times 3 + 1 \times -1 + -2 \times 5 \end{bmatrix} = \begin{bmatrix} -1 & -5 \\ -10 & 1 \end{bmatrix}$$

### General Properties for Matrix Multiplication

1. Matrix multiplication is associative, that is

$$\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$$

2. Matrix multiplication is distributive

$$(\mathbf{A} + \mathbf{B})\mathbf{C} = \mathbf{AC} + \mathbf{BC}$$

3. Matrix multiplication is not commutative in general

$$\mathbf{AB} \neq \mathbf{BA}$$

in fact, unless the two matrices are square, reversing the order in the product will cause the matrices to be *nonconformal*. The order of the terms in the product is therefore very important. In the product  $\mathbf{C} = \mathbf{AB}$ ,  $\mathbf{A}$  is said to *pre-multiply*  $\mathbf{B}$ , while  $\mathbf{B}$  is said to **post-multiply**  $\mathbf{A}$ .

4. If  $\mathbf{A}^2 = \mathbf{I}$ , then this does not imply that  $\mathbf{A} = +\mathbf{I}$  or  $\mathbf{A} = -\mathbf{I}$ .
5.  $\mathbf{AB} = \mathbf{0}$  does not imply that either  $\mathbf{A} = \mathbf{0}$  or that  $\mathbf{B} = \mathbf{0}$ .
6. Even if  $\mathbf{A} \neq \mathbf{0}$  and  $(\mathbf{AB} = \mathbf{AC})$  it does not imply that  $\mathbf{B} = \mathbf{C}$ .
7.  $\mathbf{AI} = \mathbf{IA}$ .
8.  $\mathbf{A}^0 = \mathbf{I}$ .
9.  $\mathbf{0A} = \mathbf{0}$ .

### 3/ **Determinants**

#### **The Determinant of a Matrix**

Every *square* matrix can be associated with a real number called its *determinant*.

##### **Definition of the Determinant of a $2 \times 2$ Matrix**

The **determinant** of the matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

is given by

$$\det(A) = |A| = a_{11}a_{22} - a_{21}a_{12}.$$

##### **EXAMPLE 1**

##### **The Determinant of a Matrix of Order 2**

Find the determinant of each matrix.

$$(a) A = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} \quad (b) B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \quad (c) C = \begin{bmatrix} 0 & 3 \\ 2 & 4 \end{bmatrix}$$

**SOLUTION** (a)  $|A| = \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = 2(2) - 1(-3) = 4 + 3 = 7$

(b)  $|B| = \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} = 2(2) - 4(1) = 4 - 4 = 0$

(c)  $|C| = \begin{vmatrix} 0 & 3 \\ 2 & 4 \end{vmatrix} = 0(4) - 2(3) = 0 - 6 = -6$

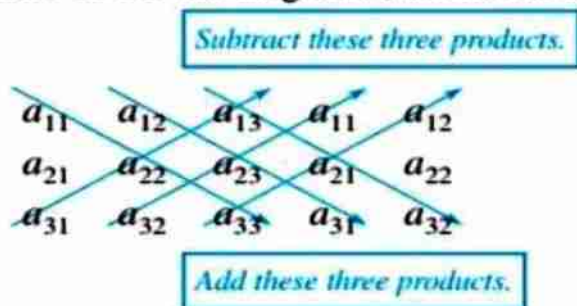
##### **REMARK:**

The determinant of a matrix of order 1 is defined simply as the entry of the matrix. For instance, if  $A = [-2]$ , then

$$\det(A) = -2.$$

## The Determinant of a Matrix of Order 3

There is an alternative method commonly used for evaluating the determinant of a  $3 \times 3$  matrix  $A$ . To apply this method, copy the first and second columns of  $A$  to form fourth and fifth columns. The determinant of  $A$  is then obtained by adding (or subtracting) the products of the six diagonals, as shown in the following diagram.



Try confirming that the determinant of  $A$  is

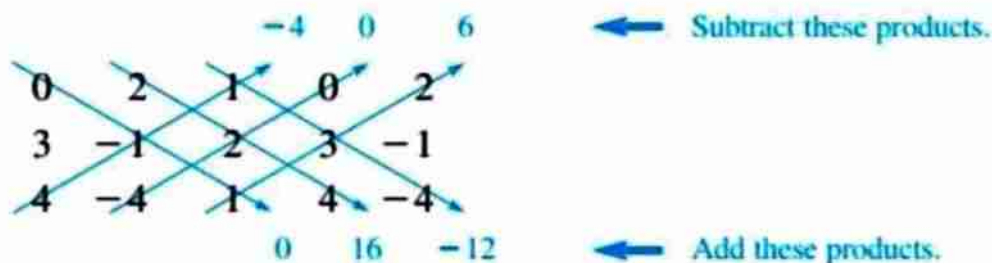
$$|A| = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}.$$

### EXAMPLE 5 The Determinant of a Matrix of Order 3

Find the determinant of

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & -4 & 1 \end{bmatrix}.$$

**SOLUTION** Begin by recopying the first two columns and then computing the six diagonal products as follows.



Now, by adding the lower three products and subtracting the upper three products, you can find the determinant of  $A$  to be  $|A| = 0 + 16 + (-12) - (-4) - 0 - 6 = 2$ .