Lecture 7. Continuity

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Continuity

Definition. A function f is continuous function at x = a if

1. f(a) is defined.
2. $\lim_{x\to a} f(x)$ exists. i.e $\lim_{x\to +a} f(x) = \lim_{x\to -a} f(x)$.
3. $\lim_{x\to a} f(x) = f(a)$.

Example 1. Let $f(x) = \begin{cases} \sqrt{x+2}, & x < 2 \\ x^2 - 2, & 2 \le x < 3 \\ 2x+5, & x \ge 3. \end{cases}$

Show that f(x) is continuous function at x=2,3 or not.

Solution.

If x = 2 then

1. $f(2) = (2)^2 - 2 = 2$.
2. $\lim_{x\to 2^+} f(x) = 2^2 - 2 = 2$.

   And $\lim_{x\to 2^-} f(x) = \sqrt{2+2} = \sqrt{4} = 2$.

   So, $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$. Therefore, $\lim_{x\to 2} f(x)$ exists.
3. $\lim_{x\to 2} f(x) = f(2) = 2$.

Thus, f(x) is continuous at x =2.

If x = 3 then

1. f(3) =2(3)+5 = 11.
2. $\lim_{x\to 3^+} f(x) = 2(3)+5 = 11$

   and $\lim_{x\to 3^-} f(x) = 3^2 - 2 = 9 - 2 = 7.$

   So, $\lim_{x\to 3^+} f(x) \neq \lim_{x\to 3^-} f(x)$. Therefore, $\lim_{x\to 3} f(x)$ is not existed. Thus, f(x) is discontinuous at x = 3.

Example 2. Let $f(x) = \begin{cases} 2x+5, & x < -1 \\ x^2+2, & x > -1 \\ 5, & x = -1. \end{cases}$

Show that f(x) is continuous function at x = -1 or not.

Solution:

1. f(-1) = 5.
2. $\lim_{x\to -1^+} f(x) = (-1)^2 + 2 = 3$ and $\lim_{x\to -1^-} f(x) = 2(-1)+5 = 3.$

   So, $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x)$. Therefore, $\lim_{x\to -1} f(x)$ is existed.
3. $\lim_{x\to -1} f(x) \neq f(-1).$

Thus, f(x) is discontinuous at x = -1.

Example 3. Let f(x) = 1/x. Show that f(x) is continuous function at x = 0 or not.

Solution:

1. f(0) = 1/0 = ∞, so f(0) is not defined. Thus, f(x) is dis continuous at x = 0.