Computer Science Department
1st Class: Mathematics

Lecture 5. Limits of Function Values

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Definition. Let f(x) be a function defined on an open interval about x_0 except possibly at x_0 itself. If f(x) close to L, for all x close to x_0, we say f(x) approaches to the limit L as x approaches to x_0 and we write

$$\lim_{x \to x_0} f(x) = L.$$

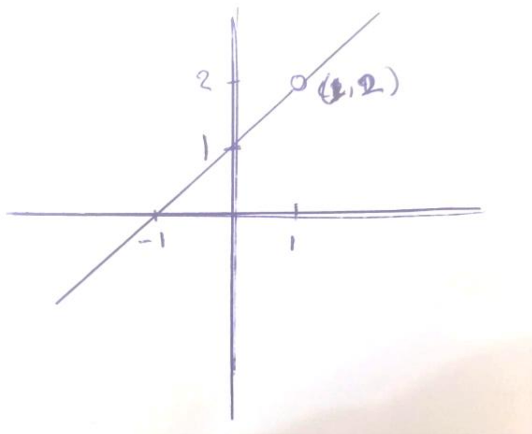
Example 1. Let f(x) = $\frac{x^2-1}{x-1}$. Compute $\lim_{x \to 1} f(x)$.

Solution.

f(x) = $\frac{x^2-1}{x-1} = \frac{(x-1)(x+1)}{x-1} = x+1$, for $x \neq 1$.

$\lim_{x \to 1} f(x) = 1+1 = 2.$

The graph of f = y = x+1 with the point (1,2) take a hole point.



Example 2. Find the limits by calculating f(x_0).

1. $\lim_{x\to 2} 4.$
2. $\lim_{x\to -13} 4.$
3. $\lim_{x\to 3} x.$
4. $\lim_{x\to 2} (5x-3).$
5. $\lim_{x\to -2} \frac{3x+4}{x+5}.$

Solution:

1. $\lim_{x\to 2} 4 = 4.$
2. $\lim_{x\to -13} 4 = 4.$
3. $\lim_{x\to 3} x = 3.$
4. $\lim_{x\to 2} (5x-3) = 5(2)-3 = 7.$
5. $\lim_{x\to -2} \frac{3x+4}{x+5} = \frac{-6+4}{-2+5} = \frac{-2}{3}.$

Calculating Limits using the Limits Laws

Theorem 1. (Limits Laws):

If L, M and k ∈ R and $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$ then

1. Sum rule.

$$\lim_{x\to c}(f(x)+g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = L + M.$$

2. Difference rule.

$$\lim_{x\to c}(f(x)-g(x))=\lim_{x\to c} f(x)-\lim_{x\to c} g(x)=L-M.$$

3. Product rule.

$$\lim_{x\to c}(f(x)\cdot g(x))=\lim_{x\to c} f(x)\cdot\lim_{x\to c} g(x)=L\cdot M.$$

4. Constant multiple rule.

$$\lim_{x\to c}(k\cdot f(x))=k\cdot\lim_{x\to c} f(x)=k\cdot L.$$

5. Quotient rule.

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}=\frac{L}{M}, M\neq 0.$$

6. Power rule.

$$\lim_{x\to c}(f(x))^{\frac{r}{s}}=L^{\frac{r}{s}},\ L^{\frac{r}{s}}\in R. \text{ And if } s\in Z_e \text{ then L} > 0.$$

Example.

1. $\lim_{x\to c}(x^3+4x^2-3)=\lim_{x\to c} x^3+\lim_{x\to c} 4x^2-\lim_{x\to c} 3$

$$=c^3+4c^2-3.$$

2. $$\lim_{x\to c}\frac{x^4+x^2-1}{x^2+5}=\frac{\lim_{x\to c}(x^4+x^2-1)}{\lim_{x\to c}(x^2+5)}$$

$$=\frac{\lim_{x\to c} x^4+\lim_{x\to c} x^2-\lim_{x\to c} 1}{\lim_{x\to c} x^2+\lim_{x\to c} 5}$$

$$=\frac{c^4+c^2-1}{c^2+5}.$$

3. $\lim_{x\to -2}\sqrt{4x^2-3} = \sqrt{\lim_{x\to -2}(4x^2-3)}$

$$= \sqrt{\lim_{x\to -2}4x^2 - \lim_{x\to -2}3}$$

$$= \sqrt{4(-2)^2-3}$$

$$= \sqrt{13}.$$

Theorem 2. (Limits of Polynomials):

If $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + ax + a_0$ then

$$\lim_{x\to c} P(x) = P(c) = a_n c^n + a_{n-1}c^{n-1} + \ldots + ac + a_0 .$$

Theorem 3. (Limits of Rational Functions):

If P(X) and Q(x) are polynomials and Q(c) $\neq$ 0 then

$$\lim_{x\to c}\frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}.$$

Example 1. Compute $\lim_{x\to -1}\dfrac{x^3+4x^2-3}{x^2+5}$.

Solution.

$$\lim_{x\to -1} \frac{x^3+4x^2-3}{x^2+5} = \frac{(-1)^3+4(-1)^2-3}{(-1)^2+5} = \frac{0}{6} = 0.$$

Example 2. Compute $\lim_{x\to 1} \dfrac{x^2+x-2}{x^2-x}$.

Solution.

$$\lim_{x\to 1} \frac{x^2+x-2}{x^2-x} = \lim_{x\to 1} \frac{(x+2)(x-1)}{x(x-1)} = \lim_{x\to 1} \frac{(x+2)}{x} = 1+2 = 3.$$

Example 3. Compute $\lim_{x\to 0} \dfrac{\sqrt{x^2+100}-10}{x^2}$.

Solution.

$$\lim_{x\to 0} \frac{\sqrt{x^2+100}-10}{x^2} = \lim_{x\to 0} \frac{\sqrt{x^2+100}-10}{x^2} \cdot \frac{\sqrt{x^2+100}+10}{\sqrt{x^2+100}+10}$$

$$= \lim_{x\to 0} \frac{x^2+100-100}{x^2(\sqrt{x^2+100}+10)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x^2+100}+10}$$

$$= \frac{1}{\sqrt{0^2 + 100} + 10}$$

$$= \frac{1}{10 + 10} = \frac{1}{20} = 0.05.$$