Lecture 3. Functions

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Functions

Definition. A function from a set T to a set S is a rule that assigns a unique element f(*x*) ∈ S for each element *x* ∈ T.

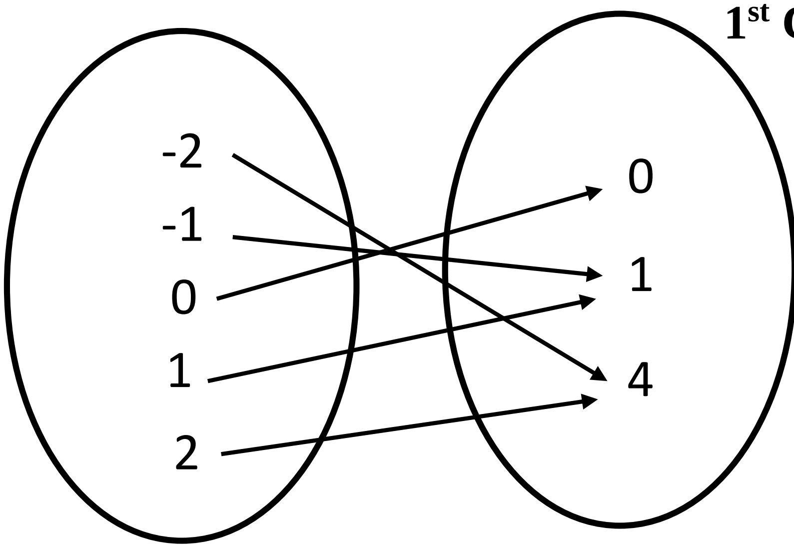
- The set T of all possible values is called **domain** of the function.
- The set of all values of f(x) as x varies in T is called the **range** of the function.

Some Examples of Functions:

Example 1. $f(x) = x^2$ is a function.

If the domain f is [-2,2] then

f(-2) = 4, f(-1) = 1, f(0) =0, f(1) = 1 and f(2) = 4. So the range is {0,1,4}.



Example 2 . Find the domain and range for the following functions.

1. f(x) = x^2. 2. f(x) = 1/x. 3. f(x) = $\sqrt{x}$. 4. f(x) = $\sqrt{4-x}$.
2. f(x) = $\sqrt{1-x^2}$.

Solution:

1. The domain is (-∞, ∞), since for any real number x gives the value f(x) = y = x^2. The range of y = x^2 of any real number x is nonnegative and zero, so y ≥ 0. Thus, the range is [0, ∞).
2. The domain is (-∞, 0) ∪ (0, ∞). The range (-∞, 0) ∪ (0, ∞).

3. The domain is [0, ∞). The range is [0, ∞).
4. In f(x) = $\sqrt{4-x}$, the quantity 4-x ≥ 0, so x ≤ 4. The domain is (-∞, 4]. The range of $\sqrt{4-x}$ is the set of all nonnegative number, so it is [0, ∞).
5. The domain is [-1,1]. The range is [0,1].

Even and Odd Functions

Definition. (Even and Odd Functions). A function y = f(x) is an **even function of x** if f(-x) = f(x), and its **odd function of x** if f(-x) = -f(x), ∀ x in domain.

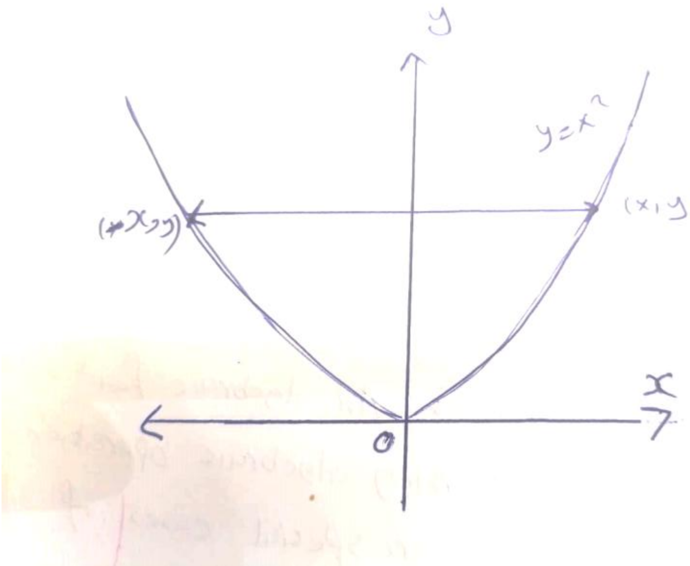
Example. Recognizing Even and Odd Functions:

f(x) = x^2, f(x) = x^2 +1, f(x) = x^3, f(x) = x,

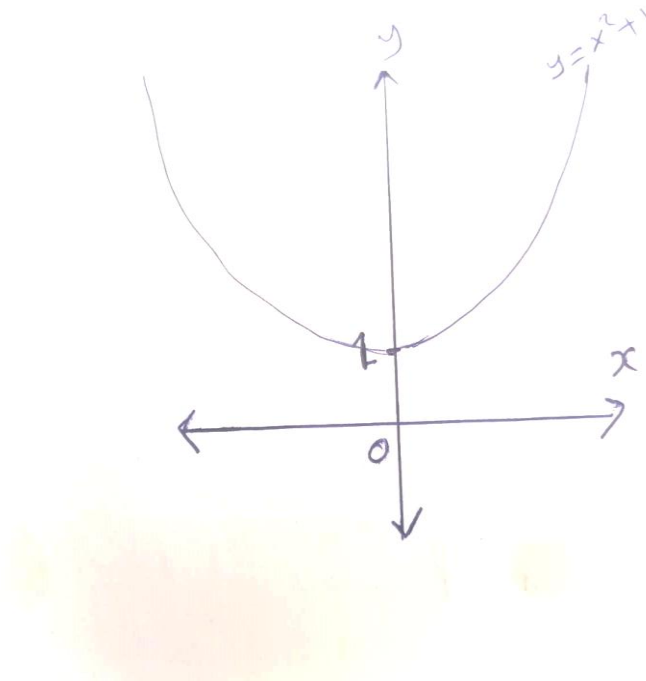
f(x) = x+1.

Solution.

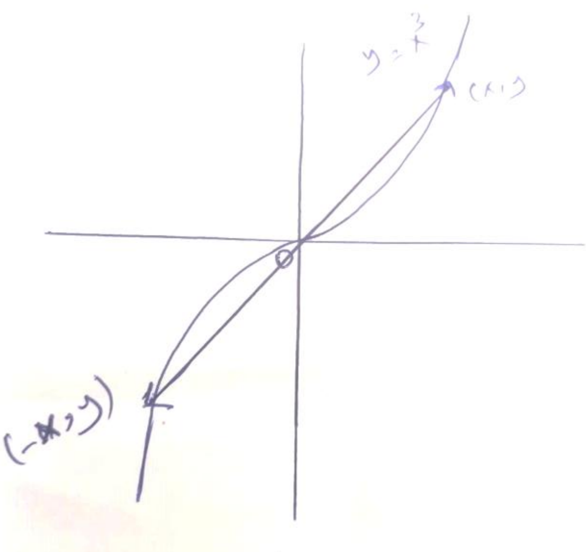
1. f(x) = x^2 is an even fun., since $(-x)^2 = x^2$, ∀ x in D. And it is symmetry about y-axis.



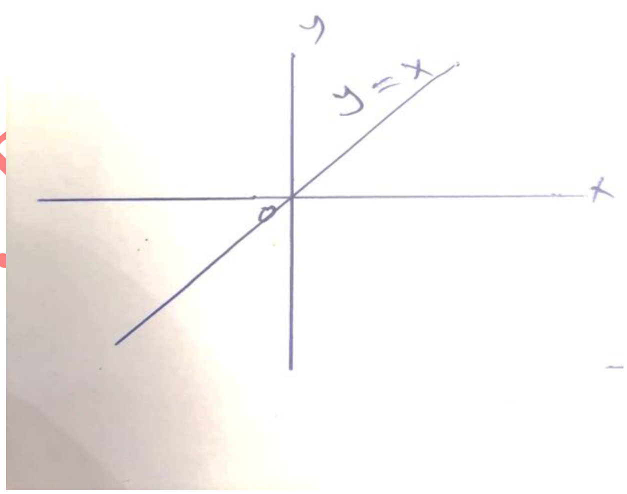
2. $f(x) = x^2 + 1$ is an even fun., since $(-x)^2 + 1 = x^2 + 1$, ∀ x in D. And it is symmetry about y-axis.



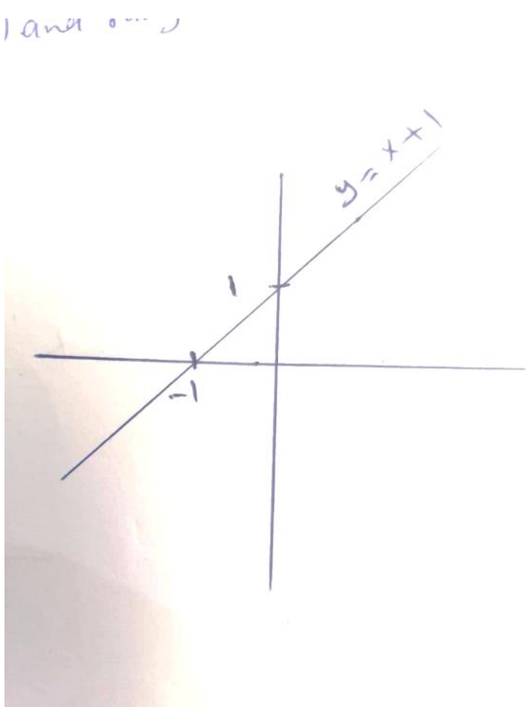
3. $f(x) = x^3$ is an odd fun., since $(-x)^3 = -x^3$, ∀ x in D. And it is symmetry about origin.



4. $f(x) = x$ is an odd fun., since $(-x) = -x$, ∀ x in D. And it is symmetry about origin.



5. $f(-x) = -x+1$ but $-f(x) = -x-1$. It is not odd.
   $f(-x) = -x+1 \neq x+1, \forall x \neq 0$. It is not even.



Types of Functions

1. Polynomials. A function P is a polynomial if

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

Where n is a nonnegative integer and $a_0, a_1, \ldots, a_n$ are real constants which are called the coefficients of P(x).
Note that:

a. All polynomials have domain $(-\infty,\infty)$.
b. If the leading coefficients $a_n \neq 0$ and $n > 0$ then n is called degree of P(x).
c. Linear functions are polynomials
d. Polynomials with degree 2 are written by

$$f(x) = ax^2 + bx + c$$

which are called quadratic functions.

2. Rational Functions. A rational function is a quotient of two polynomials:

$$f(x) = \frac{p(x)}{q(x)}$$

where p and q are polynomials. The domain of a rational function is the set of all real x for which $q(x) \neq 0$. For example, the function

$$f(x) = \frac{2x^2-3}{7x+4}$$

is a rational function with domain $\{x: x \neq -4/7\}$.