



Lecture 6. Limits of Function at Infinity

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Limits of Function at Infinity

Example 1. Compute $\lim_{x \rightarrow \infty} \frac{2x^4 - x^2 + 8}{-5x^4 + 7}$.

Solution.

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{2x^4 - x^2 + 8}{-5x^4 + 7} &= \lim_{x \rightarrow \infty} \frac{2\frac{x^4}{x^4} - \frac{x^2}{x^4} + \frac{8}{x^4}}{-5\frac{x^4}{x^4} + \frac{7}{x^4}} \\ &= \lim_{x \rightarrow \infty} \frac{2 - \frac{1}{x^2} + \frac{8}{x^4}}{-5 + \frac{7}{x^4}}\end{aligned}$$

$$\begin{aligned} &= \frac{2 - \frac{1}{\infty^2} + \frac{8}{\infty^4}}{-5 + \frac{7}{\infty^4}} \\ &= \frac{2 - 0 + 0}{-5 + 0} = -\frac{2}{5}. \end{aligned}$$

Example 2. Compute $\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 + 6}}{5 - 2x}$.

Solution:

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 + 6}}{5 - 2x} &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{3x^2}{x^2} + \frac{6}{x^2}}}{\frac{5}{x} - \frac{2x}{x}} \\&= \lim_{x \rightarrow \infty} \frac{\sqrt{3 + \frac{6}{x^2}}}{\frac{5}{x} - 2} \\&= \frac{\sqrt{3 + \frac{6}{\infty^2}}}{\frac{5}{\infty} - 2} = \frac{\sqrt{3 + 0}}{0 - 2} = -\frac{\sqrt{3}}{2}.\end{aligned}$$

Example 3. Compute $\lim_{x \rightarrow \infty} \frac{4x^2 + x^6}{1 - 5x^3}$.

Solution:

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{4x^2 + x^6}{1 - 5x^3} &= \lim_{x \rightarrow \infty} \frac{\frac{4x^2}{x^6} + \frac{x^6}{x^6}}{\frac{1}{x^6} - \frac{5x^3}{x^6}} \\&= \lim_{x \rightarrow \infty} \frac{\frac{4}{x^4} + 1}{\frac{1}{x^6} - \frac{5}{x^3}} \\&= \frac{\frac{4}{\infty^4} + 1}{\frac{1}{\infty^6} - \frac{5}{\infty^3}} = \frac{0 + 1}{0 - 0} = \frac{1}{0} = \infty.\end{aligned}$$

Example 4. Compute $\lim_{x \rightarrow -\infty} \frac{x^2 - 5x - 9}{2x^4 + 3x^3}$.

Solution:

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 5x - 9}{2x^4 + 3x^3} = \lim_{x \rightarrow -\infty} \frac{\frac{x^2}{x^4} - \frac{5x}{x^4} - \frac{9}{x^4}}{\frac{2x^4}{x^4} + \frac{3x^3}{x^4}}$$

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$$= \lim_{x \rightarrow -\infty} \frac{\frac{1}{x^2} - \frac{5}{x^3} - \frac{9}{x^4}}{2 + \frac{3}{x}}$$

$$= \frac{\frac{1}{-\infty^2} - \frac{5}{-\infty^3} - \frac{9}{-\infty^4}}{2 + \frac{3}{-\infty}} = \frac{0 - 0 - 0}{2 + 0} = \frac{0}{2} = 0.$$