Lecture 4. Types of Functions and Graphs of them

Prof. Dr. Ruma Kareem K. Ajeena

ا.د. رومى كريم خضر عجينة

Dr. Suhad Ahmed Ahmed

م.د. سهاد احمد احمد

College of Education for Pure Science - Ibn Al-Haitham

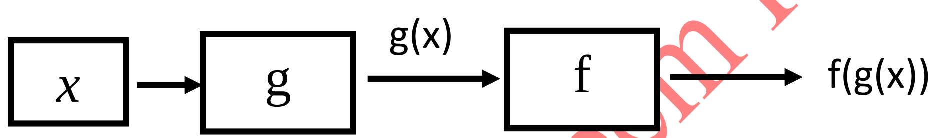
University of Baghdad

ruma.usm2015@gmail.com

Composite Function

Definition. If f(x) and g(x) are functions, the composite function $f \circ g(x)$ is defined by

$$f \circ g(x) = f(g(x)).$$



The domain of $f \circ g(x)$ consists of the numbers x in domain of g for which g(x) lies in the domain of f.

Example 1. Let f(x) = 3x-4 and g(x) = x^2-3. Compute $f \circ g(x)$ and $g \circ f(x)$.

Solution.

$$\begin{aligned} f \circ g(x) &= f(x^2 - 3) \\ &= 3(x^2 - 3) - 4 \\ &= 3x^2 - 9 - 4 \\ &= 3x^2 - 13. \end{aligned}$$

$$g \circ f(x) = g(f(x))$$
$$= g(3x-4)$$
$$= (3x-4)^2 - 3$$
$$= 9x^2 - 24x + 16 - 3$$
$$= 9x^2 - 24x + 13.$$

Example 2. If f(x) = √x and g(x) = x+1. Compute

1. $f \circ g(x)$. 2. $g \circ f(x)$. 3. $f \circ f(x)$. 4. $g \circ g(x)$.
   And determine the domain of them.

Solution.

1. $f \circ g(x) = f(g(x)) = f(x+1) = \sqrt{x+1}$. Its domain [-1,∞).
2. $g \circ f(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{x} + 1$. Its domain [0,∞).
3. $f \circ f(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = x^{1/4}$. Its domain [0,∞).
4. $g \circ g(x) = g(g(x)) = g(x+1) = x+1+1 = x+2$. Its domain (-∞,∞).

Inverse of Functions

Example 1. Let f(x) = 2x-7 find $f^{-1}(x)$.

Solution.

f(x) = 2x-7

y = 2x-7

x=2y-7

x+7 = 2y

$\frac{x+7}{2} = y \rightarrow f^{-1}(x) = \frac{x+7}{2}$.

Example 2. Let f(x) = x^3 +8 find $f^{-1}(x)$.

Solution.

f(x) = x^3 +8

y = x^3 +8

x = y^3 +8

x-8 = y^3

$\sqrt[3]{x-8} = y \rightarrow f^{-1}(x) = \sqrt[3]{x-8}$.

Example 3. Let f(x) = $\sqrt{x+2}-5$ find $f^{-1}(x)$.

Solution.

$f(x) = \sqrt{x+2}-5$

$y = \sqrt{x+2}-5$

$x = \sqrt{y+2}-5$

$x+5 = \sqrt{y+2}$

$(x+5)^2 = (\sqrt{y+2})^2$

$x^2+10x+25 = y+2$

$x^2+10x+25 - 2 = y$

$x^2+10x+23 = y \longrightarrow f^{-1}(x) = x^2+10x+23.$

Example 4. Let f(x) = $\sqrt[3]{x+4}-2$ find $f^{-1}(x)$.

Solution.

$f(x) = \sqrt[3]{x+4}-2$

$y = \sqrt[3]{x+4}-2$

$x = \sqrt[3]{y+4}-2$

$x+2 = \sqrt[3]{y+4}$

$(x+2)^3 = (\sqrt[3]{y+4})^3$

$(x+2)^3 = y+4$

$(x+2)^3 - 4 = y \rightarrow f^{-1}(x) = (x+2)^3 - 4.$

Example 5. Let $f(x) = \frac{3x-7}{4x+3}$, find $f^{-1}(x)$.

Solution.

$f(x) = \frac{3x-7}{4x+3}$

$y = \frac{3x-7}{4x+3}$

$x = \frac{3y-7}{4y+3}$

$x(4y+3) = 3y - 7$

$4xy + 3x = 3y - 7$

$3x + 7 = 3y - 4xy$

$3x + 7 = y\,(3 - 4x)$

$\frac{3x+7}{3-4x} = y \rightarrow f^{-1}(x) = \frac{3x+7}{3-4x}.$

Graphs of the Functions

Definition. Function graph includes three steps:

1. Make a table of pairs from the function.
2. Plot the corresponding points to determine the graph.
3. Complete the sketch by connecting the points.

Example 1. Sketch the graph of the following function

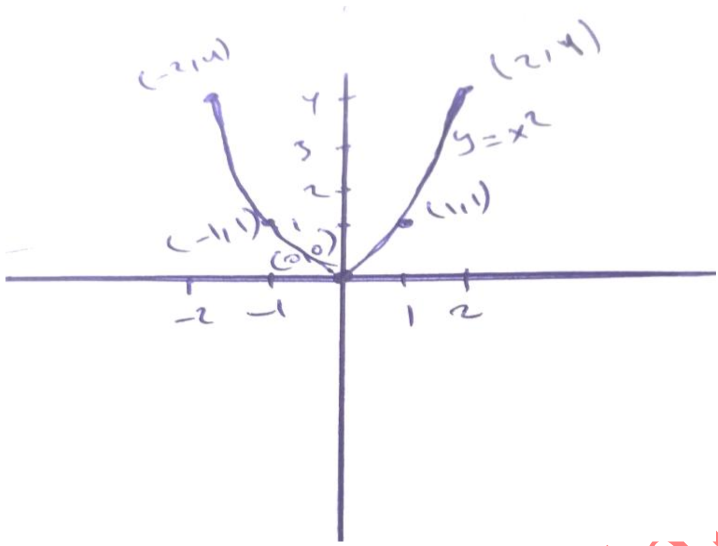
$y = x^2$.

Solution.

1. Make a table of pairs from the function.

x	$y = x^2$	(x,y)
-2	4	(-2,4)
-1	1	(-1,1)
0	0	(0,0)
1	1	(1,1)
2	4	(2,4)

2. Plot the corresponding points to determine the graph and complete the sketch by connecting the points.

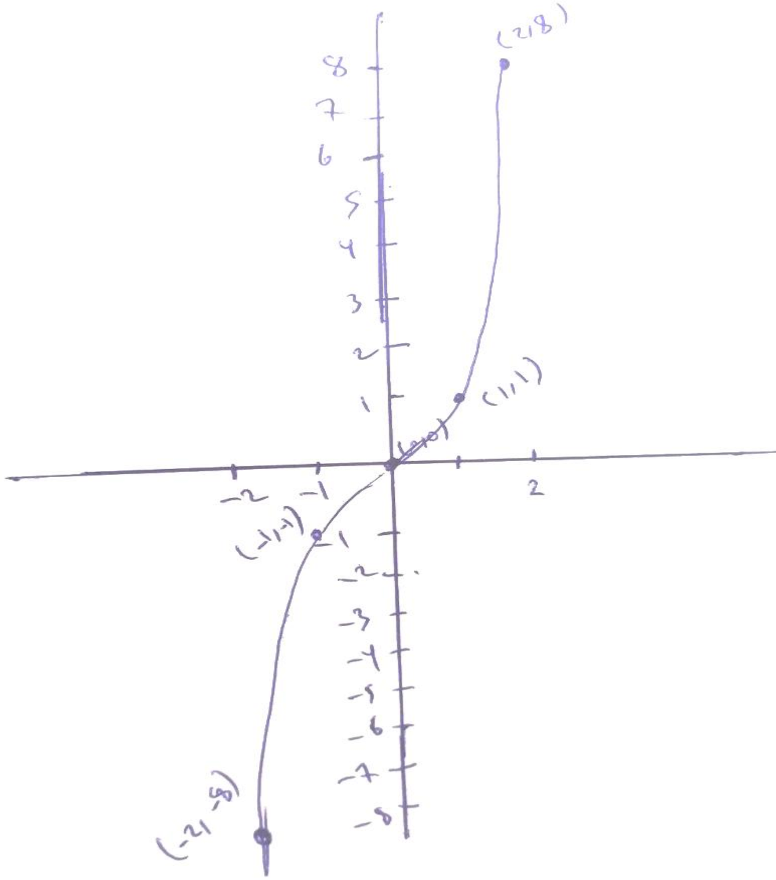


Example 2. Sketch the graph of the following $y = x^3$.

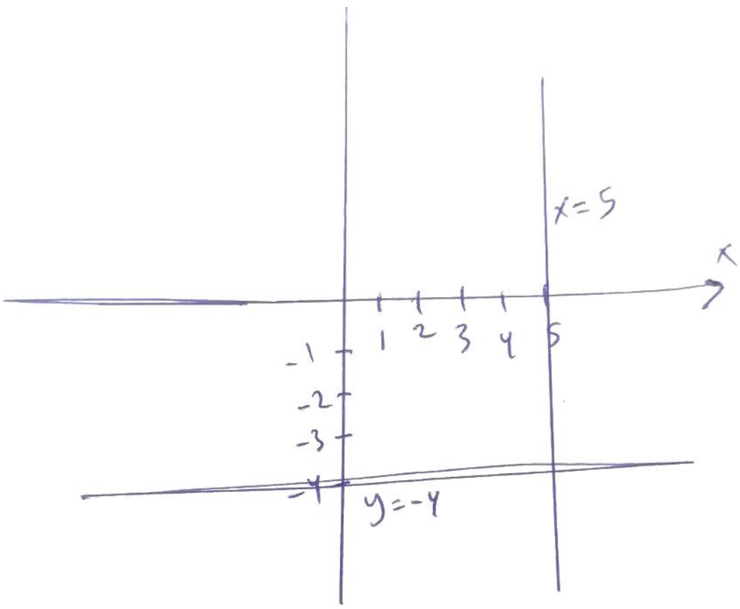
1. Make a table of pairs from the function.

x	$y = x^3$	(x,y)
-2	-8	(-2,-8)
-1	-1	(-1,-1)
0	0	(0,0)
1	1	(1,1)
2	8	(2,8)

2. Plot the corresponding points to determine the graph and complete the sketch by connecting the points.



Example 3. Sketch the graph of the following functions
x =5 and y = - 4.



H.W.

Sketch the graph of the following functions:

1. y = x.
2. y = |x|.
3. y = $\sqrt{x}$.