Lecture 9.

Simplify Trigonometric Expressions

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Simplify Trigonometric Expressions

It is important to remember the following laws;

$$\text{Tan}\,\theta = \frac{\sin\theta}{\cos\theta},\quad \text{Sec}\,\theta = \frac{1}{\cos\theta},\quad \text{Csc}\,\theta = \frac{1}{\sin\theta},\quad \text{C}ot\theta = \frac{\cos\theta}{\sin\theta}.$$

and

$$\text{Sin}^2\,\theta + \text{Cos}^2\,\theta = 1.$$

$$1 + \text{Tan}^2\,\theta = \text{Sec}^2\,\theta.$$

$$1 + Cot^2\theta = \text{Csc}^2\,\theta.$$

Examples: Simplify the following expressions;

1. $\text{Sec}\,\theta\,\text{Sin}\,\theta\,\text{Cos}\,\theta.$

Solution.

$$\text{Sec}\,\theta\,\text{Sin}\,\theta\,\text{Cos}\,\theta = \frac{1}{\text{Cos}\,\theta}\cdot\text{Sin}\,\theta\cdot\text{Cos}\,\theta$$
$$= \text{Sin}\,\theta.$$

2. $\text{Tan}^2\,\theta + \text{Sin}^2\,\theta + \text{Cos}^2\,\theta.$

Solution.

Based on $\text{Sin}^2\,\theta + \text{Cos}^2\,\theta = 1,$ so

$$\text{Tan}^2\,\theta + \text{Sin}^2\,\theta + \text{Cos}^2\,\theta = \text{Tan}^2\,\theta + 1$$
$$= \text{Sec}^2\,\theta.$$

3. $\text{Sin}^2\theta + \text{Csc}\,\theta + \text{Sec}\,\theta.$

Solution.

$$\text{Sin}^2\theta\,\text{Csc}\,\theta\,\text{Sec}\,\theta = \text{Sin}^2\theta \cdot \frac{1}{\text{Sin}\,\theta} \cdot \frac{1}{\text{Cos}\,\theta}$$

$$= \frac{\text{Sin}\,\theta}{\text{Cos}\,\theta}$$

$$= \text{Tan}\,\theta.$$

4. $Cot\theta[\text{Tan}\,\theta + Cot\theta].$

Solution.

$$Cot\theta[\text{Tan}\,\theta + Cot\theta] = Cot\theta\,\text{Tan}\,\theta + Cot^2\theta$$

$$= \frac{\cos\theta}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta} + Cot^2\theta$$

$$= 1 + Cot^2\theta$$

$$= \text{Csc}^2\theta.$$

5. $\text{Sec}^2\theta[1 - \text{Sin}^2\theta].$

Solution.

Based on $\text{Sin}^2\theta + \text{Cos}^2\theta = 1$, so

$$\begin{aligned}\text{Sec}^2\theta[1-\text{Sin}^2\theta] &= \text{Sec}^2\theta[\text{Sin}^2\theta+\text{Cos}^2\theta-\text{Sin}^2\theta]\\ &= \text{Sec}^2\theta\,\text{Cos}^2\theta\\ &= \frac{1}{\text{Cos}^2\theta}\text{Cos}^2\theta\\ &= 1.\end{aligned}$$

6. $\text{Cos}\theta+\text{Sin}\theta\,\text{Tan}\theta.$

Solution.

$$\begin{aligned}\text{Cos}\theta+\text{Sin}\theta\,\text{Tan}\theta &= \text{Cos}\theta+\text{Sin}\theta\frac{\text{Sin}\theta}{\text{Cos}\theta}\\ &= \frac{\text{Cos}^2\theta}{\text{Cos}\theta}+\frac{\text{Sin}^2\theta}{\text{Cos}\theta}\\ &= \frac{\text{Cos}^2\theta+\text{Sin}^2\theta}{\text{Cos}\theta}\\ &= \frac{1}{\text{Cos}\theta}\\ &= \text{Sec}\theta.\end{aligned}$$

7. $\dfrac{\text{Sec}\theta}{\text{Sin}\theta[Cot\theta+\text{Tan}\theta]}.$

Solution.

$$\frac{\text{Sec}\,\theta}{\text{Sin}\,\theta[Cot\theta+\text{Tan}\,\theta]}=\frac{\text{Sec}\,\theta}{\text{Sin}\,\theta\left(\frac{\text{Cos}\,\theta}{\text{Sin}\,\theta}+\frac{\text{Sin}\,\theta}{\text{Cos}\,\theta}\right)}$$

$$=\frac{\text{Sec}\,\theta}{\text{Sin}\,\theta\left(\frac{\text{Cos}\,\theta}{\text{Sin}\,\theta}+\frac{\text{Sin}\,\theta}{\text{Cos}\,\theta}\right)}$$

$$=\frac{\text{Sec}\,\theta}{\text{Cos}\,\theta+\frac{\text{Sin}^2\,\theta}{\text{Cos}\,\theta}}$$

$$=\frac{\text{Sec}\,\theta}{\frac{\text{Cos}^2\,\theta+\text{Sin}^2\,\theta}{\text{Cos}\,\theta}}$$

$$=\frac{\text{Sec}\,\theta}{\frac{1}{\text{Cos}\,\theta}}$$

$$=\frac{\text{Sec}\,\theta}{\text{Sec}\,\theta}=1.$$