# Lecture 10. Rules of Differentiation

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## Rules of Differentiations:

There are several rules of differentiations that are given as follows.

1. If $f(x)=c,$ where $c$ is a constant then $f'(x)=0.$

   For example. 1) $f(x) = 3,\ f'(x)=0.$

   2) $y = f(x) = -5,\ y'=0.$

2. If $f(x)=cx,$ then $f'(x)=c.$

   For example. 1) $f(x) = 2x$ then $f'(x)=2.$

   2) $f(x) = -3x$ then $f'(x)=-3.$

3. If $f(x)=cx^n,$ then $f'(x)=cnx^{n-1}.$

   For example. 1) $f(x) = 2x^3$ then $f'(x)=2\cdot 3x^2=6x^2.$

   2) $f(x) = -5x^4$ then $f'(x)=-20x^3.$

4. If $f(x)$ is a polynomial given by $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + ax + a_0$

   Then $f'(x)=a_nnx^{n-1}+a_{n-1}(n-1)x^{n-2}+\ldots+a.$

   For example. 1) $f(x) = 3x^3 - 2x + 3$ then $f'(x)=9x^2-2.$

   2) $y = x^3 + x^2 - x + 2$ then $y'=3x^2+2x-1.$

   3) $f(x) = x^{-2} + x^{-5} + x^2 + 2x - 1$ then

   $f'(x)=-2x^{-3}-5x^{-6}+2x+2.$

   4) $f(x) = x^2 + x^{-3} + x^{1/2} + x^{3/2} + 2$ then

   $$f'(x)=2x-3x^{-4}+\frac{1}{2}x^{-\frac{1}{2}}+\frac{3}{2}x^{\frac{1}{2}}.$$

5. $f(x) = h(x)\,g(x)$ then $f'(x) = h(x)g'(x) + g(x)h'(x)$.

For example. If $f(x) = (2x + 5)(x^2 + 2)$ then

$$\begin{aligned} f'(x) &= (2x+5)(2x) + (x^2+2)2 \\ &= 4x^2 + 10x + 2x^2 + 4 \\ &= 6x^2 + 10x + 4. \end{aligned}$$

6. If $f(x) = (x^2 + 2)^2$ then $f'(x) = 2(x^2+2)2x = 4x^3 + 8x$.

Another example, If $y = (x^3 - 2x)^3$ then

$$y' = 3(x^3 - 2x)(3x^2 - 2) = (9x^2 - 6)(x^3 - 2x).$$

7. If $f(x) = h(x)/g(x)$, where $g(x) \neq 0$ then

$$f'(x) = \frac{g(x)h'(x) - h(x)g'(x)}{[g(x)]^2}.$$

For example. 1) Let $f(x) = \dfrac{x^2+2}{x-5}$, find $f'(x)$.

Solution.

$$\begin{aligned} f'(x) &= \frac{(x-5)(2x) - (x^2+2)(1)}{(x-5)^2} \\ &= \frac{2x^2 - 10x - x^2 - 2}{x^2 - 10x + 25} \\ &= \frac{x^2 - 10x - 2}{x^2 - 10x + 25}. \end{aligned}$$

2) Let $f(x) = \dfrac{3x^3 - 2x}{x^2 + 3}$, find $f'(x)$.

$$f'(x)=\frac{(x^2+3)(9x^2-2)-(3x^3-2x)(2x)}{(x^2+3)^2}$$
$$=\frac{9x^4-2x^2+27x^2-6-6x^3+4x}{x^4+6x^2+9}$$
$$=\frac{9x^4-6x^3+25x^2+4x-6}{x^4+6x^2+9}.$$

8. If $f(x)=\sqrt{x}=x^{\frac{1}{2}}$ then $f'(x)=\frac{1}{2\sqrt{x}}$.

If $f(x)=\sqrt[3]{x}=x^{\frac{1}{3}}$ then $f'(x)=\frac{1}{3\sqrt{x^2}}$.

If $f(x)=\sqrt[4]{x}=x^{\frac{1}{4}}$ then $f'(x)=\frac{1}{4\sqrt{x^3}}$.