

Operations Research I

First Semester

For the 3rd class students / Mathematics Department / College of Science for Women

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Preface

These lecture notes are for the course “Operations Research I” for the 3rd grade- first semester in Mathematics Department / College of Science for Women / University of Baghdad.

The author claims no originality. These lecture notes are collected from references listed in the “Bibliography”.

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ضمن فقرة :

Management Science : Operations Research and Management Decision

Ch.1: Introduction to Operations Research (OR)

1.1 Development of Operations Research (OR)

1.1.1 Before World War II

No science has ever been born on a specific day. Operations research is no exception. Its roots extend to even early 1800; it was in 1885 when F.W. Taylor (1856 - 1915) emphasized the application of scientific analysis to methods of production and management. He is considered as the father of “industrial engineering”. H.L. Gantt (1861 – 1919) has a great contribution in production and management and introduced the Gantt chart in 1910 to represent production schedules. In 1917, A. K. Erlang (1878-1929) published his work on the problem of congestion of telephone traffic which contains his formulae for call loss and waiting time. He is considered as the inventor of “queuing theory”.

1.1.2 World War II

The modern field of OR arose during the World War II. The military management in England called on a team of scientists to study the strategic and tactical problem of air and land defense. The objective was to determine the most effective utilization of limited military resources. The application included the effective use of newly invented radar, allocation of British Air Force planes to missions and the determination of best patterns for searching submarines. This group of scientists formed the first OR team.

The name **operations research** (or **operational research**) was because the team was searching out research on (military) operations. The encouraging results of those efforts led to the formation of more such teams in British armed services and the use of such scientific teams soon spread to the Western Allies: the US, Canada, and France.

1.1.3 After World War II

Immediately after the war, the success of military teams attracted the attention of industrial managers who were seeking solutions to their problems. As the industrial boom following the war was running its course, the problems caused by the increasing complexity and specialization in organizations was again coming to forefront. By 1950s, OR is used to a variety of organizations in business, industry, and government. The rapid spread of OR soon followed.

At least two other factors that played a key role in the rapid growth of OR during this period can be identified. One was the substantial progress that was

made early in improving the techniques to OR. Many of the standard tools of OR, such as linear programming, dynamic programming, queuing theory, and inventory theory were relatively well developed before the end of 1950s.

A second factor that gave a great impetus to the growth of the field was the onslaught of the computer revolution. A large amount of computations is usually required to deal most effectively with the complex problems typically considered by OR. Doing this by hand would often be out of the question. Therefore the development of electronic, digital computers, especially after the 1980s, with their ability to perform arithmetic calculations thousands or even millions of times faster than a human being can, was a tremendous boon to OR. In 1980s and afterward, several good software packages for doing OR was developed.

Today, OR is recognized worldwide as a modern decision-aiding science that has proved to be of great value to management, business, and industry.

1.2 Definition of Operation Research

Many definitions of OR have been suggested from time to time. We can define OR as follows:

Definition (1.1):

Operations research, in the most general sense, can be characterized as the application of scientific methods, techniques, and tools to problems involving the operations of systems so as to provide those in control of the operations with optimum solutions to the problem.

By a ***system***, we mean an organization of independent components that work together to accomplish the goal of the system. For example a car manufacturing company.

1.3 Prescriptive or Optimization Models

Most of the models in OR are ***prescriptive*** or ***optimization*** models. A ***prescriptive model*** “prescribes” behavior for an organization that will enable it to meet its goal(s). The components of a prescriptive model include:

- ***Objective function***: it is a function we wish to maximize or minimize.
- ***Decision variables***: the variables whose values are under our control and influence the performance of the system.
- ***Constraints***: restrictions on the values of decision variables.

In short, an ***optimization model*** seeks to find values of the decision variables that optimize (minimize or maximize) an objective function

among the set of all values for the decision variables that satisfy the given constraints.

1.4 Phases in Solving OR Problems

Any OR analyst has to follow certain sequential steps to solve the problems on hand. The steps are described below:

1.4.1 Formulation of the Problem

This step involves defining the scope of the problem under investigation. The OR team should identify three principal elements of the decision problem:

- 1- Description of the decision alternatives.
- 2- Determination of the objective of the study.
- 3- Specification of the limitations under which the modeled system operates.

1.4.2 Model Construction

This step is to translate the problem definition into mathematical relationships, i.e. to construct a model. In OR study, it is usually a **mathematical model**. A model helps to analyze a system, make the problem more meaningful, and clarifies important relationships among the variables. It also tells us to which of the variables are more important than the others. It must not be forgotten that a model is only an approximation of the reality (real situation). Hence it may not include all the variables.

1.4.3 Solve the Model

This step entails the use of well-defined optimization algorithm to find optimal, or best, solution. That is to find the values of decision variables.

Since a model is an approximation of the real system or problem, the optimum solution for the model does not guarantee an optimum solution for the real problem. However, if the model is well formulated and tested, solution from the model will provide a good approximation to the solution of the real problem.

1.4.4 Testing the Model and the Solution Derived from it

As already discussed, a model is never a perfect representation of reality. But, if properly formulated and correctly manipulated, it may be useful in predicting the effect of changes in control variables on the overall system effectiveness. The usefulness of a model is tested by determining how well it predicts the effect of these changes. Such an analysis is usually called **sensitivity analysis**. Sensitivity analysis is particularly needed when the parameters of the model cannot be estimated accurately. In these cases, it is important to study the

behavior of the optimum solution in the neighborhood of the estimated parameters.

1.4.5 Establishing Controls Over the Solution

Since life is not static, a solution which we felt was optimum today may not be so tomorrow, since the values of the variables (parameters) may change, new parameters may emerge and the structural relationship between the variables may undergo a change.

The solution derived from a model goes out of control if the values of one or more uncontrolled variables vary or relationship between variables undergoes a change. Therefore, controls must be established to indicate the limits within which the model and its solution can be considered as reliable. Also tools must be developed to indicate as to how and when the model or its solution will have to be modified to take the changes into account.

1.4.6 Implementation

The solution obtained above should be translated into operating procedures which can be easily understood and applied by those who control the operations. Necessary changes should be implemented by OR team.