One Dimension Motion

- **Outline**

- **1-1 Position, Distance, and Displacement**
- **1-2 Average Speed and Velocity**
- **1-3 Instantaneous Velocity**
- **1-4 Acceleration**
- **1-5 Motion with Constant Acceleration**
- **1-6 Applications of the Equations of Motion**
- **1-7 Freely Falling Objects**

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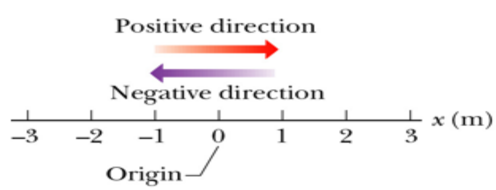
Classification of Physics Quantities

Vector : quantity with both magnitude (size) and direction
Scalar : quantity with magnitude only

Motion in One Dimensional (Position x)

- **Motion: A change in position, over time, relative to a reference point.**

- **For motion along a straight line, the direction is represented simply by + and – signs.**

Direction: **positive (i.e ; + sign): Right or Up.** **negative (i.e ; - sign): Left or Down.**	

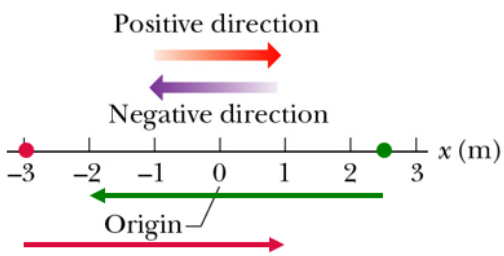
- **Any motion involves four concepts**

- **Displacement**
- **Velocity**
- **Time**
- **Acceleration**

- **These concepts can be used to study objects in motion.**

Displacement

- ❑ **Displacement : is a change of position in time which has both magnitude and direction (i.e : vector quantity, + or – sign)**
  - **Displacement:** $\Delta x = x_f(t_f) - x_i(t_i)$
    - **f stands for final and i stands for initial.**



$x_1(t_1) = +2.5$ m
$x_2(t_2) = -2.0$ m
$\Delta x = -2.0 \text{ m} - 2.5 \text{ m} = -4.5$ m

$x_1(t_1) = -3.0$ m
$x_2(t_2) = +1.0$ m
$\Delta x = +1.0 \text{ m} + 3.0 \text{ m} = +4.0$ m

- ❑ **Distance (*d*) :how far you have traveled, regardless of direction (length of the path traveled).**
- ❑ **Displacement (*d*) : how far you have traveled, includes direction (length and direction from start to finish).**
- ❑ **Units**

- **Displacement & Distance . . .   meter (m)**
- **Time . . .   second (s)**
- **Velocity & Speed . . .   (m/s)**

Example (1) : **A car (X) moves with ($-25\ m/s$) and another car (Y) moves with ($25\ m/s$) , explain whether both cars have the same velocity or not .**

Ans **:Both cars have different velocities , despite of the magnitude of their average velocities they have different directions .**

Furthermore **we can say that they have the same speed which has the same magnitude.**

Velocity

- ❑ **Velocity: is the rate of change of position which has both magnitude and direction (i.e: vector quantity).**

- **Average velocity** $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$ — displacement
- **Average speed** $s_{avg} = \frac{\text{total distance}}{\Delta t}$ — displacement
- **Instantaneous velocity** $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ — displacement

Acceleration

- **Acceleration :** is the rate at which velocity changes over time. Acceleration has both magnitude and direction.
- **Acceleration has a dimension of length/time^2: [m/s^2].**

- **An object speeds up (accelerating) if the velocity and acceleration point in the same direction (either positive or negative);**
- **An object slows down (decelerating) if the velocity and acceleration point in opposite directions**

- **Instantaneous Acceleration: The acceleration of an object at any point in time.**
  - **Definition:**
    - **Average acceleration** $a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$
    - **Instantaneous acceleration** $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2 v}{dt^2}$

Special Case: Motion with Constant Acceleration

$$v = v_0 + at$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Ex (2): A car accelerates from a velocity of 16 ms^{-1} to a velocity of 40 ms^{-1} in a distance of 500 m. Calculate the acceleration of the car.

$$v^2 = v_0^2 + 2a(x - x_0)$$

$40^2 = 16^2 + 2 \times a \times 500$

$$a = \frac{1600 - 256}{1000} = 1.344 \text{ ms}^{-2}$$

Ex (3): A car decelerates from a velocity of 36 ms^{-1}. The magnitude of the deceleration in 3 ms^{-2}. Calculate the time required to cover a distance of 162 m.

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

162 = 36t + ½ x (-3)x t^2

1.5 t^2 – 36 t + 162 = 0

t^2 - 24 t + 108 = 0

(t-6) (t-18) = 0

t = 6 or 18

Free Fall Acceleration

Freefall acceleration: Earth gravity is the only force acting on an object (no air resistance).

- **Free-fall acceleration is independent of mass.**
- **Magnitude: $|a| = g = 9.8$ m/s^2**
- **Downward Direction a = 9.8 m/s^2**
- **Up Direction $a = -g = -9.8$ m/s^2**
- **v = 0 m/s at the highest point**

$$v = v_0 + at$$

$$y - y_0 = \frac{1}{2}(v_0 + v)t$$

$$y - y_0 = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a(y - y_0)$$

Ex (4): A ball is thrown vertically upwards with an initial speed 30 ms^{-1} . Calculate the height reached.

a = -g = - 10 ms^{-2}

$$v^2 = v_0^2 + 2a(y - y_0)$$

0 = 30^2 -2 x 10 x h

$$h = \frac{900}{20} \; 45 \, m$$

Ex (5): A ball is thrown vertically upwards with a speed of 40 ms^{-1}. Calculate the time interval between the instants that the ball is 20 m above the point of release.

a = - g = - 10 ms^{-2}

$$y - y_0 = v_0 t + \frac{1}{2} a t^2$$

20 = 40 t – 0.5 x 10 x t^2

5 t^2 - 40 t +20 = 0

t^2 - 8 t +4 = 0

using the quadratic formula

$$t = \frac{8 \mp \sqrt{48}}{2} = 7.62 \; or \; 0.54$$

the ball is 20 m above the point of release twice , at t = 0.54 s (on way up) and t = 7.62 s (on way down).

The required time interval is

7.62 - 0.54 = 7.08 s

Motion in two dimensions

- **Motions in each dimension are independent components**
- **Constant acceleration equations**

$$\vec{v} = \vec{v}_0 + \vec{a}t \qquad \vec{r} - \vec{r} = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

- **Constant acceleration equations hold in each dimension**

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

$$x - x_0 = v_{0x} t + \tfrac{1}{2} a_x t^2 \qquad y - y_0 = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$v_x{}^2 = v_{0x}{}^2 + 2a_x(x - x_0) \qquad v_y{}^2 = v_{0y}{}^2 + 2a_y(y - y_0)$$

  - $t = 0$ **beginning of the process;**
  - $\vec{a} = a_x\hat{i} + a_y\hat{j}$ **where a_x and a_y are constant;**
  - **Initial velocity** $\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$ **initial displacement** $\vec{r}_0 = x_0\hat{i} + y_0\hat{j}$

Projectile Motion

projectile : is a particle moving near the Earth's surface under the influence of its weight only (directed downward).

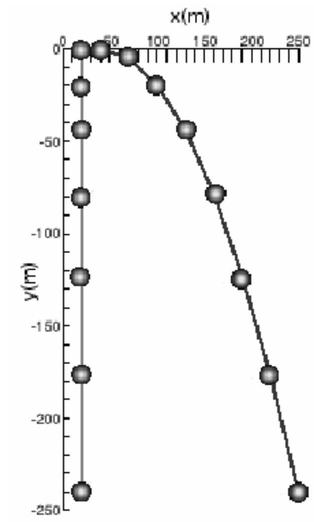
- X and Y motions happen independently, so we can treat them separately

$$v_x = v_{0x} \qquad v_y = v_{0y} - gt$$

$$x = x_0 + v_{0x}t \qquad y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

Horizontal **Vertical**

- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$
- Horizontal motion + Vertical motion
- Horizontal: $a_x = 0$, constant velocity motion
- Vertical: $a_y = -g = -9.8$ m/s^2
- x and y are connected by time t
- y(x) is a parabola

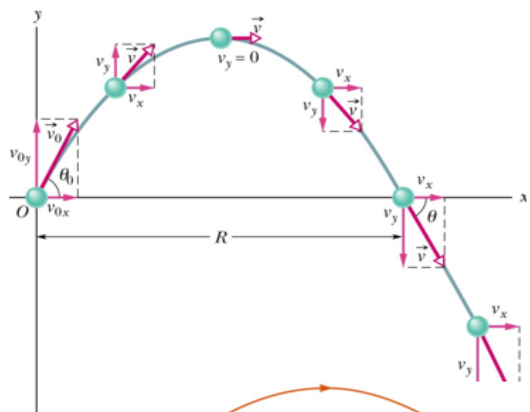


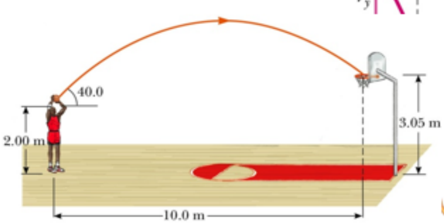
- 2-D problem and define a coordinate system.
- Horizontal: $a_x = 0$ and vertical: $a_y = -g$.
- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$.
- Velocity initial conditions:
  - v_0 can have x, y components.
  - v_{0x} is constant usually. $v_{0x} = v_0 \cos\theta_0$
  - v_{0y} changes continuously. $v_{0x} = v_0 \sin\theta_0$
- Equations:

Horizontal **Vertical**

$$v_x = v_{0x} \qquad v_y = v_{0y} - gt$$

$$x = x_0 + v_{0x}t \qquad y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$





Trajectory of Projectile Motion

- **Initial conditions ($t = 0$): $x_0 = 0, y_0 = 0$**
  $v_{0x} = v_0 \cos\theta_0$ and $v_{0y} = v_0 \sin\theta_0$
- **Horizontal motion:**

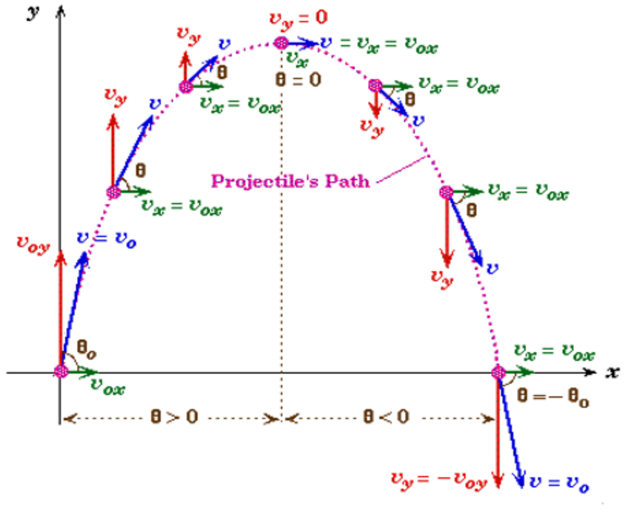
$$x = 0 + v_{0x}t \quad \Rightarrow \quad t = \frac{x}{v_{0x}}$$

- **Vertical motion:**

$$y = 0 + v_{0y}t - \tfrac{1}{2}gt^2$$

$$y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \frac{g}{2}\left(\frac{x}{v_{0x}}\right)^2$$

$$y = x\tan\theta_0 - \frac{g}{2v_0^2\cos^2\theta_0}x^2$$



- **Parabola;**
  - ***$\theta_0 = 0$ and $\theta_0 = 90$?***

What is *R* and *h* ?

- **Initial conditions ($t = 0$): $x_0 = 0, y_0 = 0$ $v_{0x} = v_0 \cos\theta_0$ and $v_{0x} = v_0 \sin\theta_0$, then**

$$x = 0 + v_{0x}t \qquad 0 = 0 + v_{0y}t - \tfrac{1}{2}gt^2$$

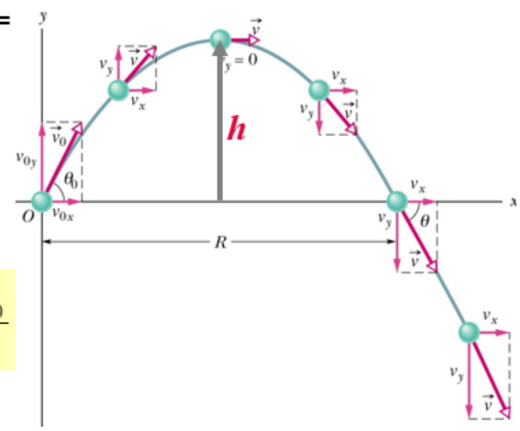
$$t = \frac{2v_{0y}}{g} = \frac{2v_0\sin\theta_0}{g}$$

$$R = x - x_0 = v_{0x}t = \frac{2v_0\cos\theta_0 v_0\sin\theta_0}{g} = \frac{v_0^2\sin 2\theta_0}{g}$$

$$h = y - y_0 = v_{0y}t_h - \tfrac{1}{2}gt_h^2 = v_{0y}\frac{t}{2} - \frac{g}{2}\left(\frac{t}{2}\right)^2$$

$$h = \frac{v_0^2\sin^2\theta_0}{2g}$$

$$v_y = v_{0y} - gt = v_{0y} - g\frac{2v_{0y}}{g} = -v_{0y}$$

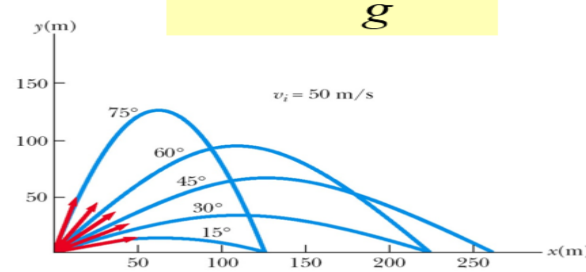


Horizontal	**Vertical**
$v_x = v_{0x}$	$v_y = v_{0y} - gt$
$x = x_0 + v_{0x}t$	$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

Projectile Motion
at Various Initial Angles

- Complementary values of the initial angle result in the same range
  - The heights will be different
- The maximum range occurs at a projection angle of 45°

$$R = \frac{v_0^{\ 2} \sin 2\phi}{g}$$



Ex (6): A ball is thrown vertically upwards with an initial speed of 30 ms ^{1}. Calculate the height reached.

$$v^2 = v_0^2 + 2a\,(y - y_0)$$

$0 = 30^2 - 2 \times 10 \times h$

where h is the maximum height reached

$$h = \frac{900}{20} = 45\,m$$

Ex (6): A ball is thrown vertically upwards with a speed of 40 ms ^{1} . Calculate the time interval between the instants that the ball is 20 m above the point of release.

$a = g = 10$ ms $^{-2}$

$$y - y_0 = v_0 t + \frac{1}{2} a t^2$$

$20 – 0 = 40\,t – 0.5 \times 10 \times t^2$

$5\,t^2 - 40\,t + 20 = 0$

$t^2 - 8\,t + 4 = 0$

$$t = \frac{8 \pm \sqrt{48}}{2} = 7.62 \ \text{ or } \ 0.54$$

The ball is 20 m above the point of release twice, at t = 0.54 s (on way up)

and *t =1.62* s (on way down).

The required time interval is 7.62 0.54 = 7.08 seconds.

Ex (7): A car accelerates from ($10\ m/s$) to ($20\ m/s$) through ($2\ sec$) , calculate its acceleration.

$$v_f = v_0 + a\,t$$

20 = 10 + a x 2

a = 5 ms^{-2}

Ex (8): A plane lands on a road with a velocity ($250\ km/h$) , having an acceleration of a magnitude ($4.8\ m/s2$) , calculate :

I. The distance traveled on the road till the plane stopped in meters .

II. The time needed till it stops in seconds .

1.

$$v_f^2 = v_i^2 + 2\,a\,\Delta X$$

$$0 = [\frac{250\ x\ 10^3}{60\ x\ 60}]^2 + 2\,(-4.8)\ x\ \Delta\, x$$

$$\Delta\, x = 502.3\ m$$

2.

$$v_f = v_0 + a\,t$$

$$0 = [\frac{250\ x\ 10^3}{60\ x\ 60}] + (-4.8)\ x\ \Delta\, t$$

$$\Delta\, t = 14.47\ s$$

Ex (9): A baseball is hit with a horizontal speed of 25 m/s.

1. What is its position and velocity after 2 s?
2. What are the velocity components after 2 s?

The horizontal displacement

$$x = x_0 + v_{0x}t$$

$x_0 = 0,\quad y_0 = 0$ at $t = 0$

x = 0 + (25 x 2) = 25 m/s

the vertical displacement

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

$y = -\ 0.5\ x\ 9.8\ x\ (2)^2$

$y = -\ 19.6\ m$

$$v_x = v_{0x} = 25\ m/s$$

$$v_y = v_{0y} - gt$$

y = 0 – (9.8 x 2) = - 19.6 m/s

Ex (10): A ball has an initial velocity of 160 ft/s at an angle of 30º with horizontal.

1. **Find its position and velocity after 2 s and after 4 s.**
2. **the vertical components of position after 2 s and after 4 s**
3. **the horizontal and vertical components of velocity after 2 and 4 s.**
4. **The displacement R_2, θ is found from the x_2 and y_2 component displacements**

- $v_{0x} = v_0 \cos\theta_0$ **and** $v_{0y} = v_0 \sin\theta_0$

$$v_{0x} = v_0 \cos \theta_0$$
$$v_{0x} = 160 \cos 30^0 = 139\ ft/s$$

$$v_{0y} = v_0 \sin \theta_0$$
$$v_{0y} = 160 \sin 30^0 = 80\ ft/s$$

- **Since v_x constant, the horizontal displacements after 2 and 4 seconds are:**

$$x = v_{0x}\, t$$
x = 139 x 2 = 277 ft

x = 139 x 4 = 544 ft

$$y = v_o t + \frac{1}{2}\ g t^2$$
$$y = 80\, t - 0.5\, x\, 32\, t^2$$
$$y = 80\, t - 16t^2$$
at 2 sec
$$y = 80\, x\, 2 - 16\,(2)^2 = 96\, ft$$

at 4 sec

$$y = 80\, x4 - 16\,(4)^2 = 16\, ft$$

- **Since v_x constant,**

$v_{0x} = 160 \cos 30^0 = 139\ ft/s$ *at all times*

$$v_y = v_{0y} - gt$$
g = - 32 ft/s²
$$v_y = v_{0y} - gt$$

$$v_y = 80 - 32\,t$$

$$v_{y2} = 80 - 32\,x2 = 16\,ft/s$$
$$v_{y4} = 80 - 32\,x4 = -48\,ft/s$$
$$R = \sqrt{x^2 + y^2}$$
$$R_2 = \sqrt{(277)^2 + (96)^2} = 293\,ft$$
$$tan\,\theta = \frac{y}{x} = \frac{96}{277} = 19.1^0$$

Ex(11): **A car travels with a velocity of 20m/s, the driver increased the velocity until it reaches 100km/h in 3s. Then the driver decided to stop, the car stopped after 4s. Find the average acceleration in both cases.**

$$v_f = \frac{100\,x\,1000}{60\,x\,60} = 27.8\,m/s$$

$$a_{avr,1} = \frac{\Delta v}{\Delta t} = \frac{27.8 - 20}{3} = 2.6\,m/s$$

$$a_{avr,2} = \frac{\Delta v}{\Delta t} = \frac{0 - 27.8}{4} = 6.95\,m/s$$

Ex(12): ***1.What are maximum height and range of a projectile if v_o = 28 m/s at 30^0?***
2. What is maximum height of the projectile 3. the range of the projectile

$$v_{0x} = v_0 \cos\theta_0$$
$$v_{0x} = 28\cos 30^0 = 24.2\,m/s$$
$$v_{0y} = v_0 \sin\theta_0$$
$$v_{0y} = 28\sin 30^0 = 14\,m/s$$
$$v_y = 0$$
$$v_y = v_{0y} - gt$$
$$0 = 14 - 9.8\,t$$
t = 1.43 s
$$y = v_o t + \frac{1}{2}\,g\,t^2$$
$$y = 14\,x\,1.43 - 0.5\,x\,9.8\,(1.43)^2$$
$$y = 20 - 10 = 10\,m$$
$$y = v_o t + \frac{1}{2}\,g\,t^2 \quad then \quad y = 0$$
$$0 = 14t - \frac{1}{2}\,x\,9.8\,x\,t^2 \quad then \quad t = 2.86\,s$$
$$X_r = v_{0x}\,t = 24.2\,x\,2.86 = 69.2\,m$$