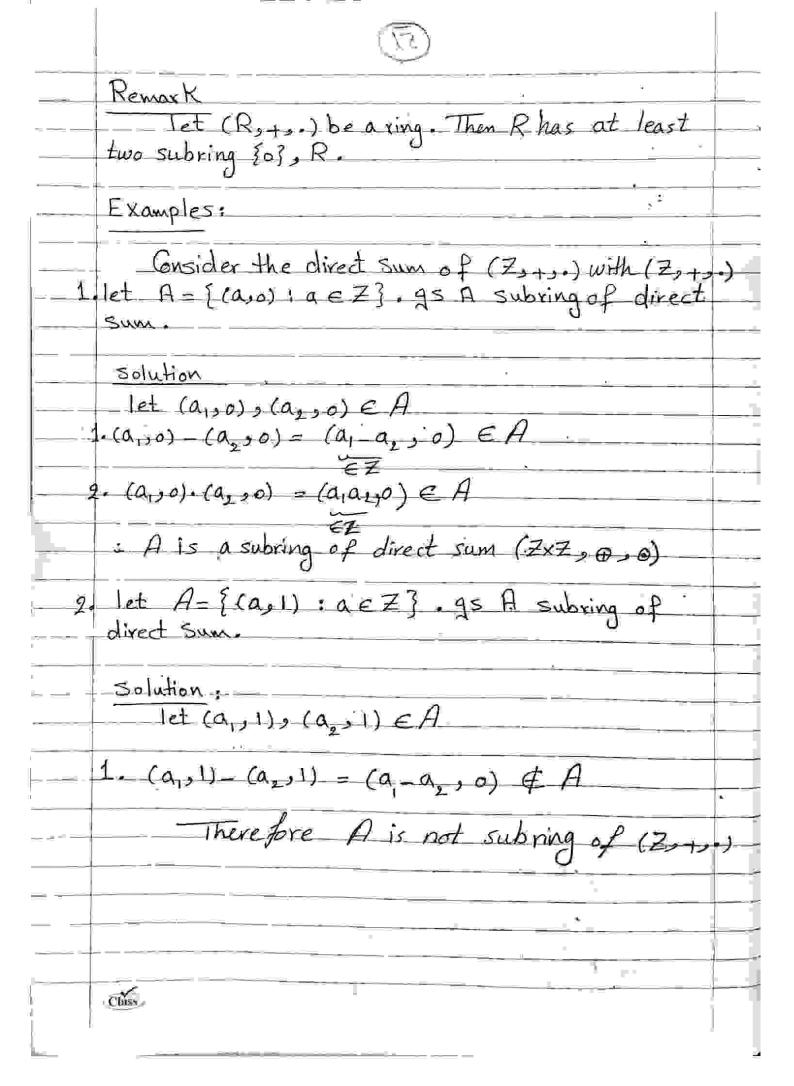
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الفصل الثاني

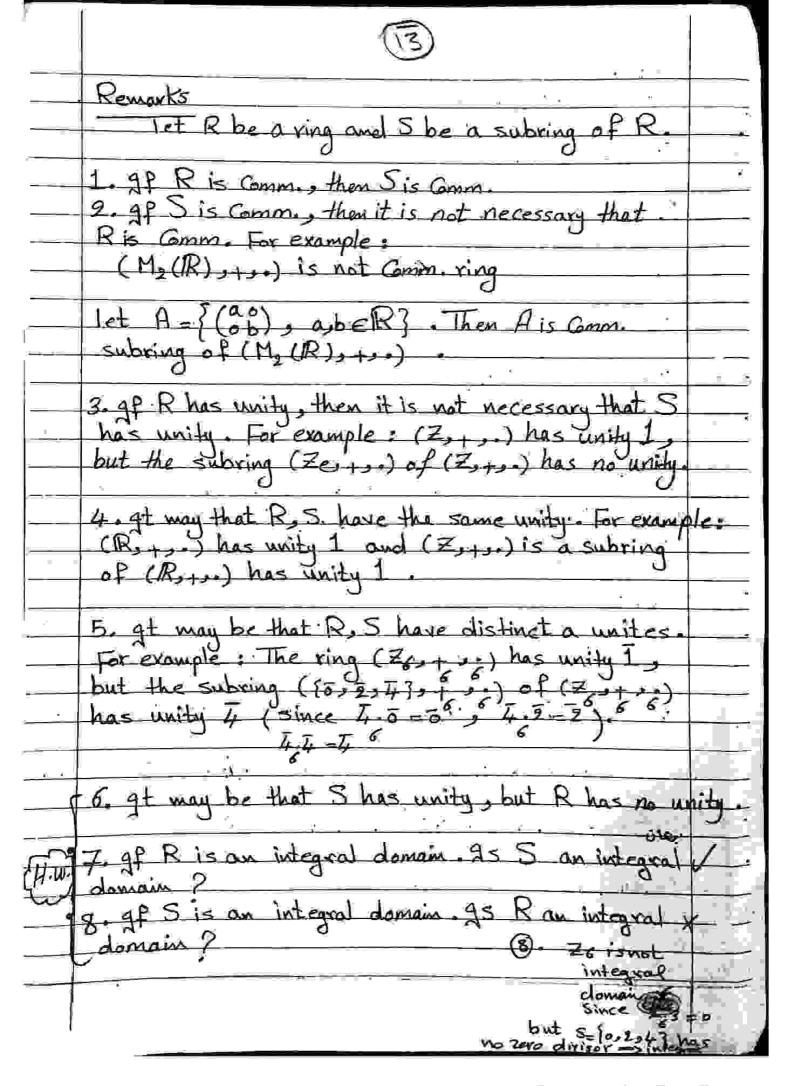
Chapter Two

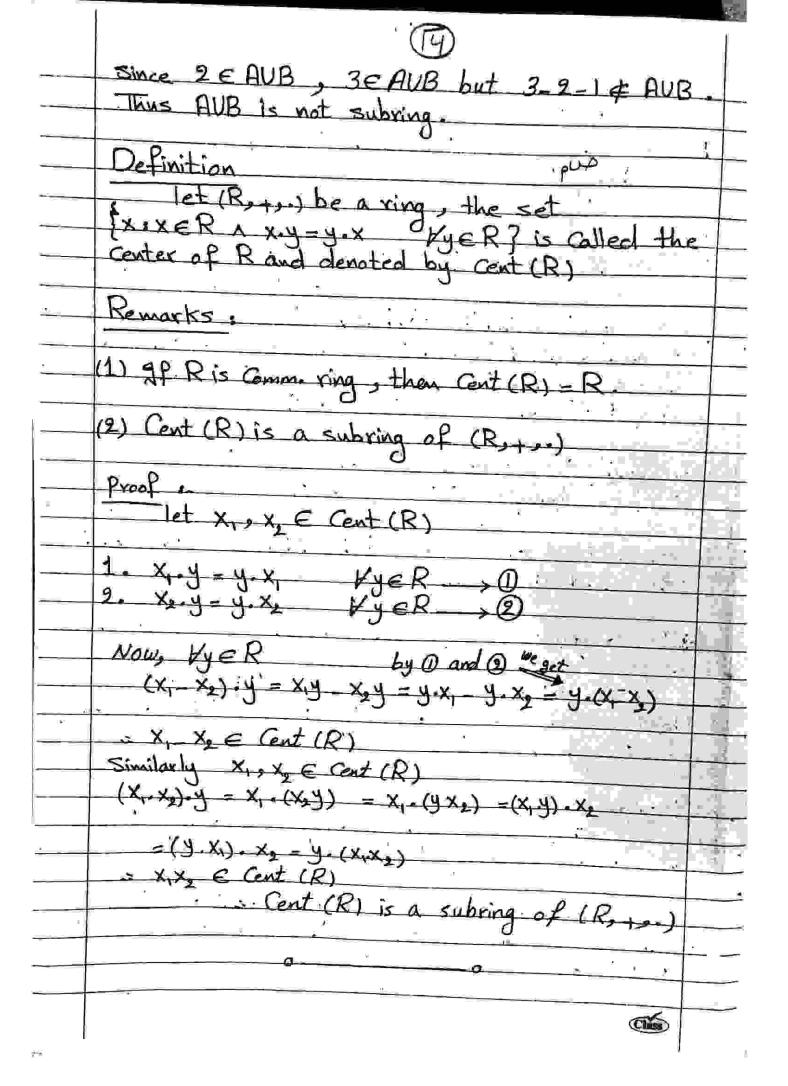
	Since S + Ø , then Ix & S
	$0 = X - X \in S \longrightarrow 0 \in S$ by Gnolition ①
	Vaes, o-aes by Condition @
	\Rightarrow -a \in S
	Now, HasheS, a+b=a-(-b) eS
	+ on S is Comm. (since, 5 c R and + is) Comm. on R
	+ on S is associative (since SCR and + asso.)
	· is closed on 5 (is given andition 2)
	e is asso. on S (SCR and asso. on R)
-	· distr. over + on S (S \in R, dist. over +)
	Thus (5,1,1) is a ring.
E	Example Consider (Z6-66) ring.
	Then all subrings of (Z6++++) are:
1	· (76, 16) is a subring of (76, 16)
	Class

(2)($\{\bar{0}\}, \frac{1}{6}, \frac{1}{6}$ (3) $A = \{\bar{0}, \bar{2}, \frac{1}{6}, 1$					
+6 0 <u>2</u> - 0 <u>0 <u>0</u> - 0 <u>0</u> - </u>	Carried Control		6 0 0 0 2 0 4 0	2. 4 5 5 7 2 7 4	
+ closed	on A		- cl	osed on	A
-2 = 4 = 2 = 	×			s. over	+ on 1
@ B={0,3	3) . To pro) ()
(A) B= {\vec{0}{0}} = \vec{3}{0}			· () 0		95)
+ ₆	3 · Το pro		close	d on B	96)
-16 -	5 3 5 3	ve (B) to	close	d on B	



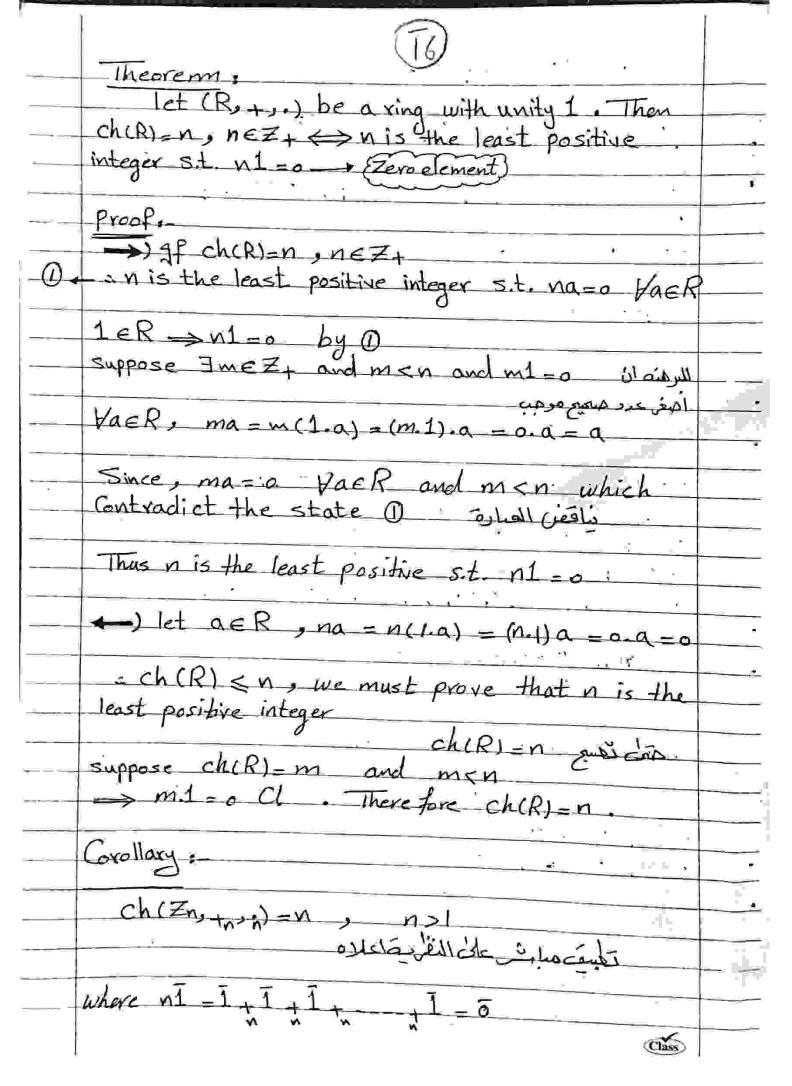
Class





Class

(E) · [
na + ma = (a + a + + a) + (a + a + + a)	
n-times m-times	-
= a+ a+ a++a	
(n+m)-times	
= (n+m)a	
$\frac{\text{Caselii}): n \in \mathbb{Z}, m = 0}{n}$	-
Gase(iii): NEZ-, mEZ- H.W.	
· · · · · · · · · · · · · · · · · · ·	*
Case(IV): NEZ, MEZ-H.W.	
Definition : your	
let (R, +, 0) be a ring, if there exists apositive	
n s.t. na=0 YaER, then the least positive	
integer with this property is Called characteristic	
of R (simply ch(R)).	
gf no such positive integer exists, implies chiR)=	o
Examples:	
1. Consider (Zoto)	
- I - I - I - I - I - I - I - I - I - I	
Haez, only na = o , then n=0	
$= \sum_{i=1}^{n} (h(Z) = 0)$	
2. Each ring (Q, +, 0) , (R, +, 0) , (Ze, +, 0)	····
has chara. Zero	بيس
3. Consider (M2(R),+,0)	
Class	

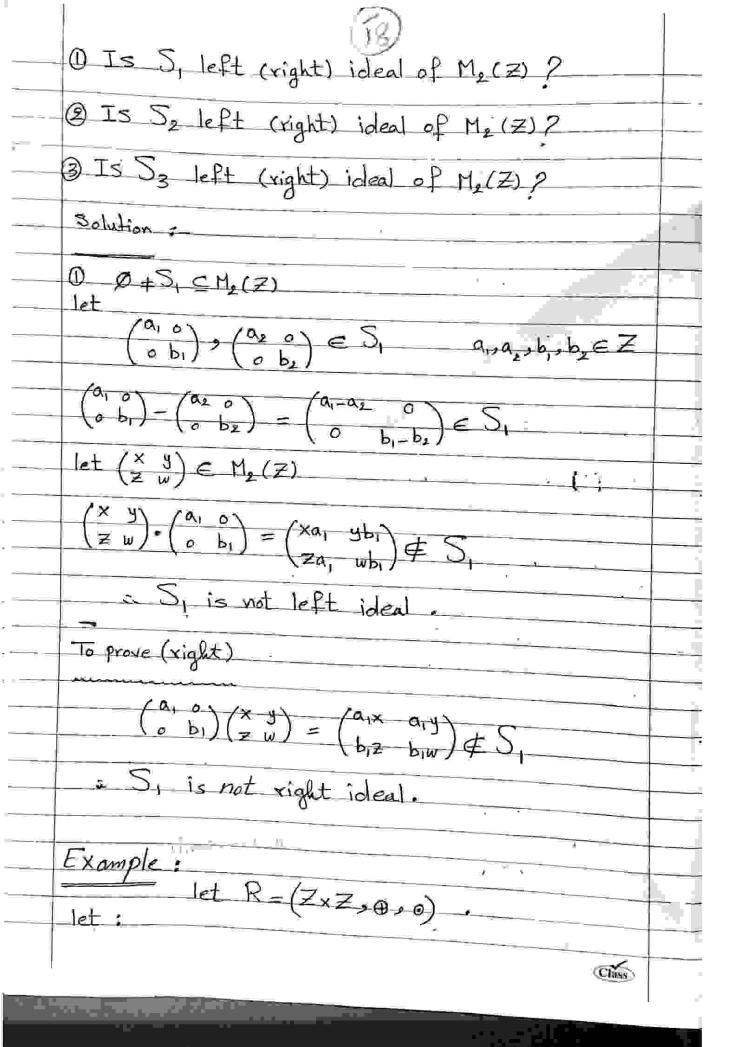


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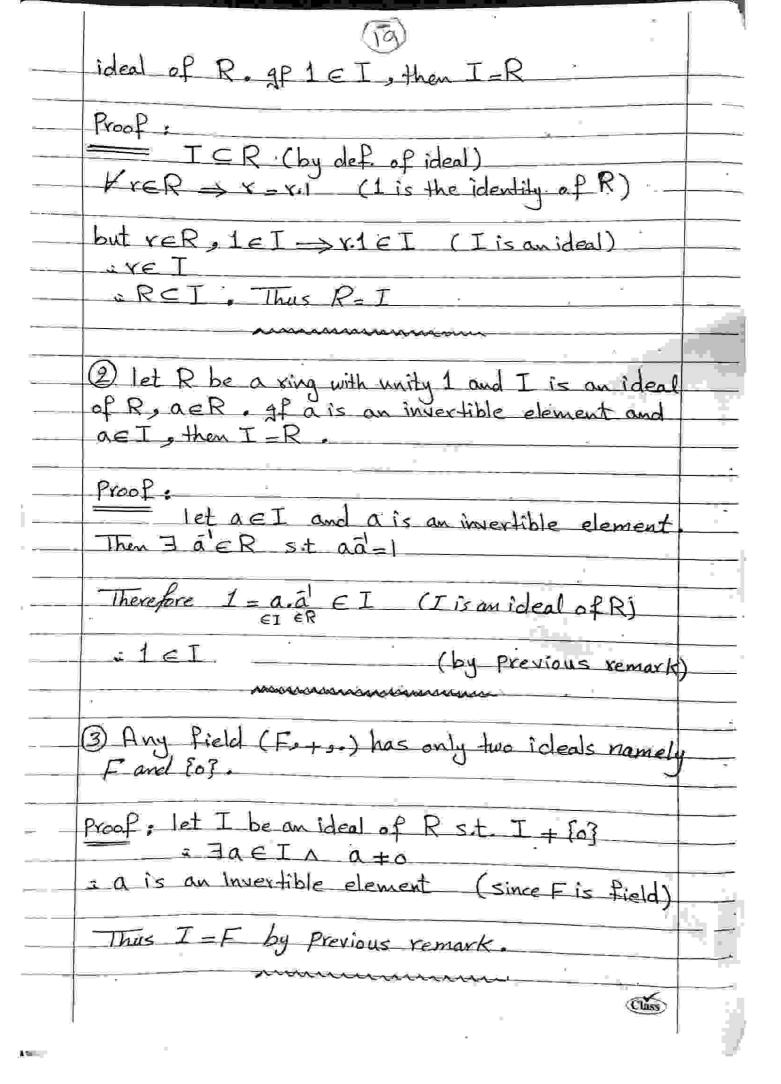
الفصل الثالث

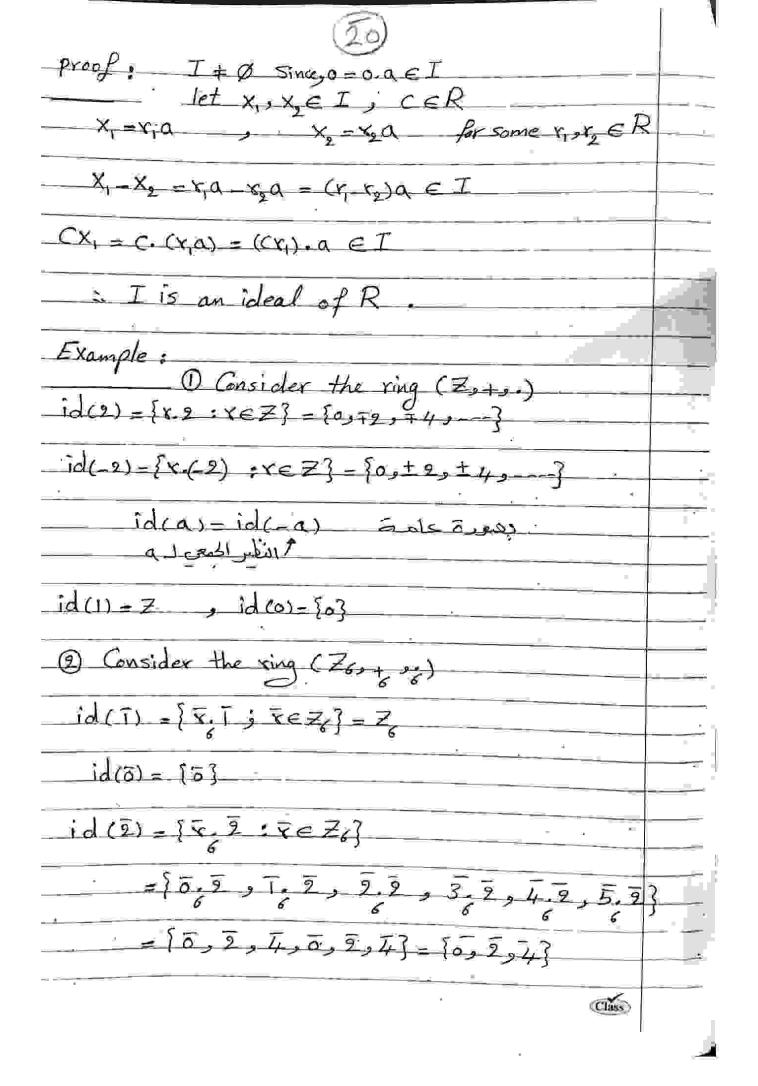
chapter three

	Chapter Three * (IDEAL) * (Sulfall)	
	Definition:	-
	let (Roto) be a ring and 5 a subring of R.	
	1. S is called a right ideal of R ⇒ a.r ∈ S. 2. S is called a left ideal of R ⇒ ra. ∈ S.	
	Vaes, YreR	
	3. S is Called left and right ideal of R (or two sided ideal) = are S and rae S	
	two sided ideal) (are S and ra E S	
	Yaes, YER.	
	Remark:	
	gn comme ving every left ideal is right	
	ideal and Conversely.	
	Example:	
	The subring (Ze+++) is an ideal of	
_	(7)	
	Since $g p a = 2n \in \mathbb{Z}e$ and $x \in \mathbb{Z}$, then $xa = a.x = (2n).x = 2(n.x) \in \mathbb{Z}e$	
	Y.a = a.r. = (211)	
	Theorem:	
	Theorem: let (Ro+so) be a ring, and let \$\psi + SCR. Then S is an ideal of R \iff	
	S is an ideal of K	
	1. a-bes Vabes	
	2. ares and raes reR	
	'L'	
	Class	
- 1		



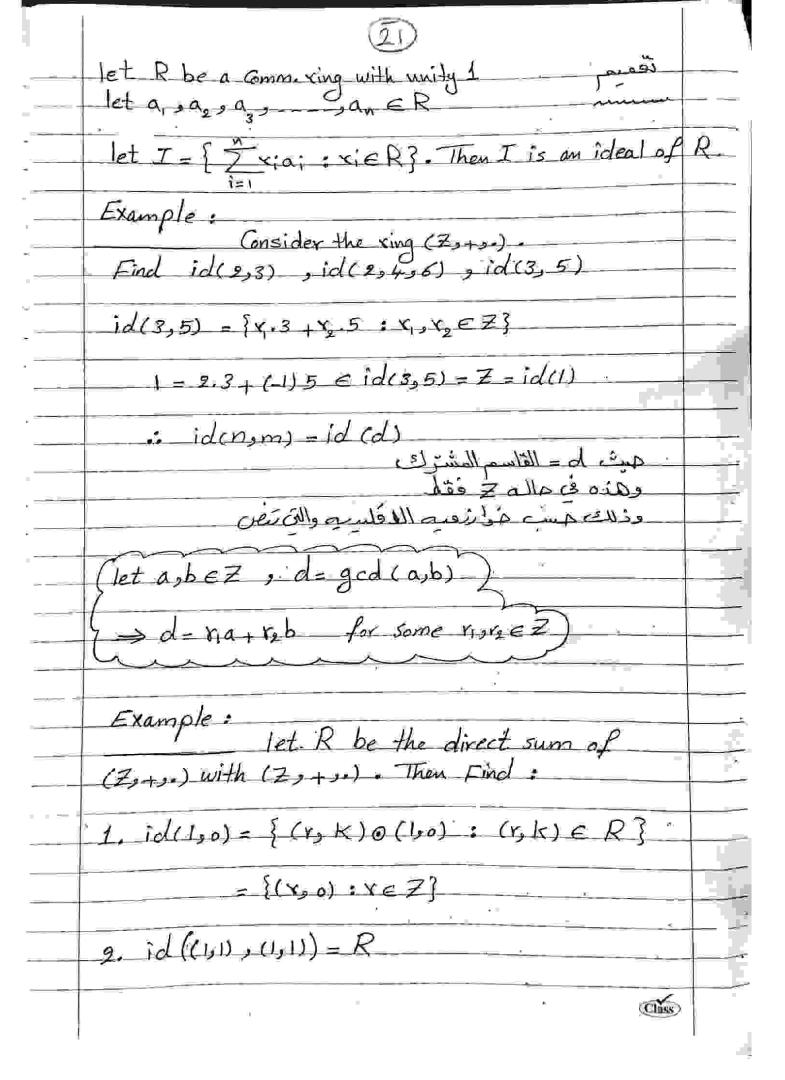
	-19-
	$S_1 = \{(a, a) : a \in \mathbb{Z}\}$
H 1	S ₂ ={(a,b); a∈Z,b∈Z}
-	Si= [(a,b); a EZ, b EZe]
	Sy= { (a,b); a \in Ze A b \in Ze}
	IS S, , S, , S, S, on ideal of R?
	Solution: 3 $\emptyset + S_2 \subseteq \mathbb{R}$
	let (a,,b,), (a2,b2) ∈ S2 st. a, a, ∈ Z b,,b2 ∈ Ze
	$(a_1,b_1)-(a_2,b_2)=(a_1-a_2,b_1-b_2)\in S_2$
	Since $a_1, a_2 \in \mathbb{Z} \longrightarrow a_1 - a_2 \in \mathbb{Z}$ $b_1, b_2 \in \mathbb{Z} \longrightarrow b_1 - b_2 \in \mathbb{Z}$
	let (x,y) ER = ZxZ
	$(x,y) \otimes (a_1 \Rightarrow b_1) \Rightarrow (xa_1 \Rightarrow yb_1) \in S_2$
	Since $x \in Z$, $a_1 \in Z \Longrightarrow xa_1 \in Z$
	yez, bieze ybieze and o is Gomm.
	we have So is an ideal of R=ZxZ
	Remarks:
	1. let (R,+,) be a ring with unity 1, I be an





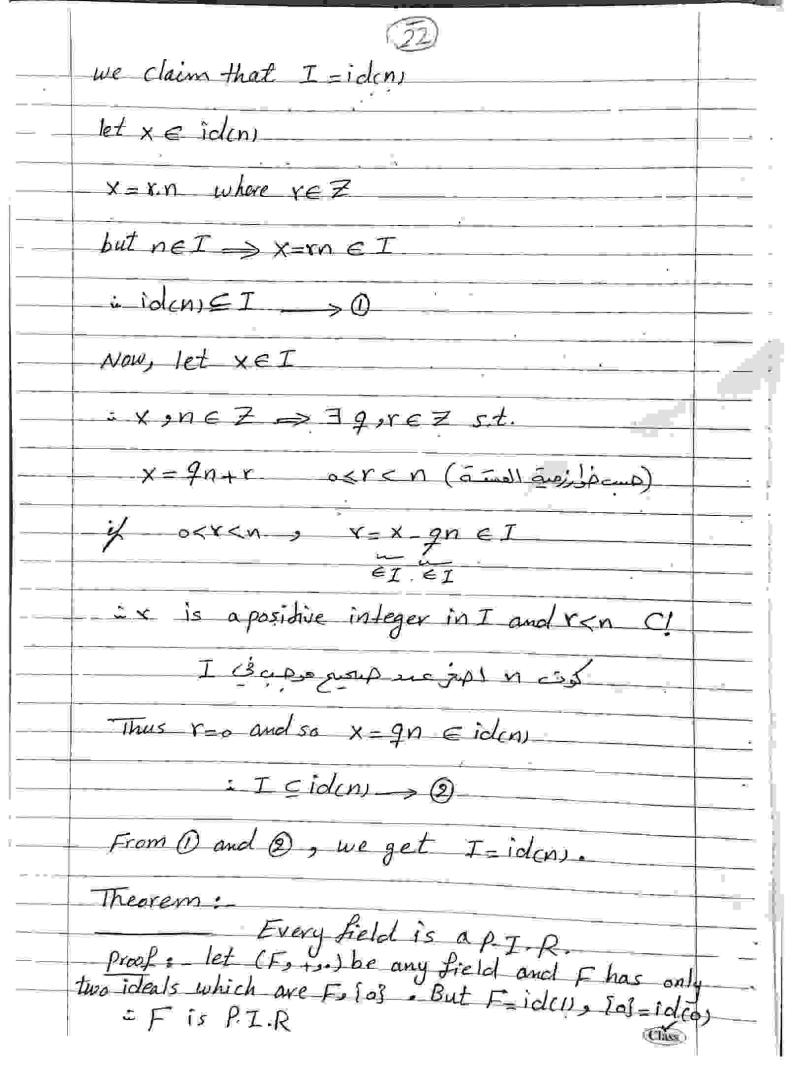
I = [Yia + Y2b : Y1, Y2 ER]. Then I is an idea

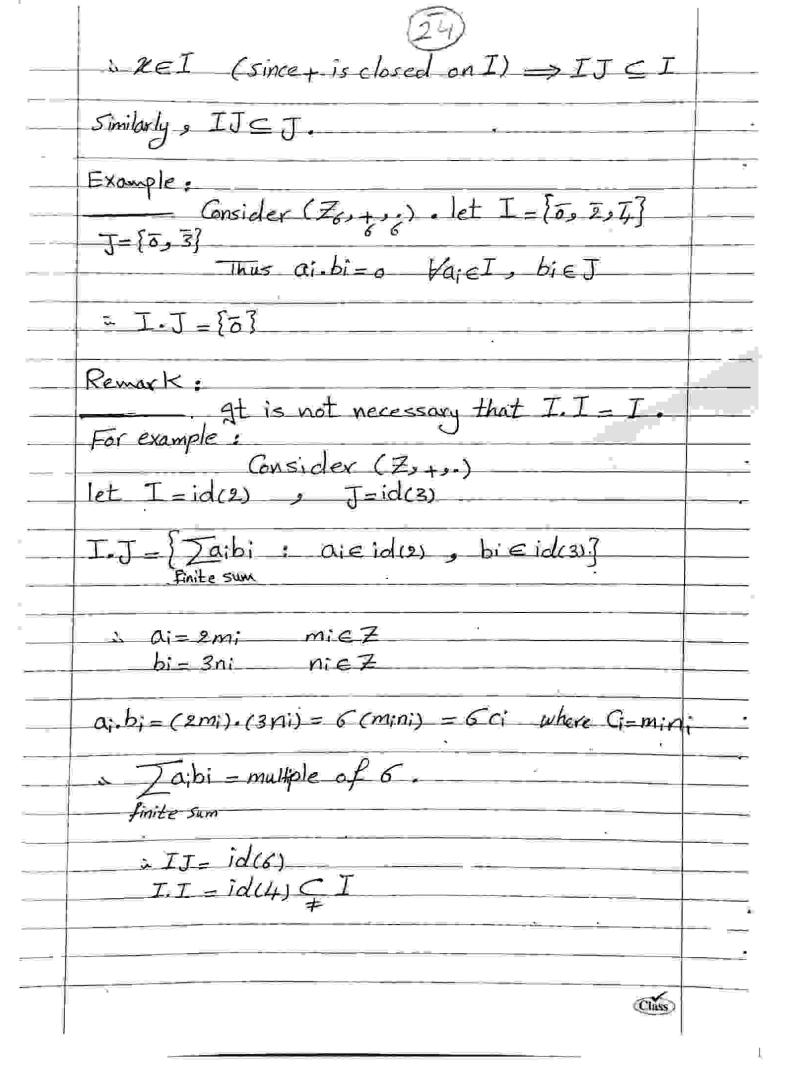
of R and this ideal is called the ideal generated by a and b, denoted by idea, b), <a,b>,



i (x, k,), (x, k,) \in R \} = \{ (x, \in 0), (0, k, 2) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	
Definition: let R be a Comm. ring with unity 1. R is alled a principle ideal ring (PI.R.) iff every ideal of R is a PI. Example: (Z ₈ , + ₈ , * ₈) is a P.I.R. Theorem: (Z ₇ + ₈ .) is a P.I.R. Proof: let I be any ideal of Z gf I = {0}, then I = id(0) is a P.I. gf I + {0}, then I me I a m + 0 Since me I me I me I so, I Contians positive integers I is a me o. S I of integers I is a me o. S I plus a measure	3. id ((100), (01)) = {((1,0 K1), (12, K2))@ ((100), (01)
Definition: let R be a Comm. ring with unity 1. R is Called a principle ideal ring (P.I.R.) if every ideal of R is a P.I. Example: (Z ₈ ,+ ₈ ,* ₈) is a P.I. R. Theorem: (Z ₊ +,.) is a P.I. R. Proof: let I be any ideal of Z gP I = {0}, then I = id(0) is a P.I. Af I + {0}, then I me I a m + 0 Since me I => me I so, I Contians positive integers I is a supplied to be a principal of the proof. S = Z ₊ . But Z ₊ is a supplied to principal supplied to the principal supplied	j (x, , k,) ∈ R }
Definition: let R be a Comm. ring with unity 1. R is Called a principle ideal ring (P.I.R.) if every ideal of R is a P.I. Example: (Zg,+g,*g) is a P.I.R. Theorem: (Z,+,*) is a P.I.R. Proof: let I be any ideal of Z gP I = {0}, then I = id(0) is a P.I. Af I + {0}, then I me I a m + 0 Since me I - me I so, I Contians positive integers I is a supposed. I is a supposed. S = Z+ . But Z+ is a supposed. I plus a supposed. I has a least element say n.	zx(v) U(c)xZ.
Example: = (Z ₈ , + ₈ , * ₈) is a P. I. R. Theorem: (Z ₃ + ₈ , * ₈) is a P. I. R. Proof: let I be any ideal of Z gP I = {0}, then I = id(0) is a P. I. gP I + {0}, then I me I a m + 0 Since me I = me I So, I Contians positive integers I is a some D substitute of the part of	Definition: let R be a Comm rive with unite 1.
Example: (Z ₈ , + ₈ , * ₈) is a P. I. R. Theorem: (Z ₈ , + ₈ , * ₈) is a P. I. R. Proof: let I be any ideal of Z gP I={0}, then I = id(0) is a P. I. Af I + {0}, then I me I a m + 0 Since me I = me I So, I Contians positive integers I is a some of Subility of Sasan	R is Called a principle ideal ring (P.I.R.) if every ideal of R is a P.I.
proof: let I be any ideal of Z gf I={0}, then I=id(0) is a f. I gf I + {0}, then I = id(0) is a f. I Af I + {0}, then I = id(0) is a f. I Since me I => -m e I so, I Contians positive integers I is a begin applied assumed six of the continuous of the continuo	
gf I={0}, then I=id(0) is a fI gf I + {0}, then I me I a m + 0 Since me I => -m e I so, I Contians positive integers I is a before some Substitution of the service of	
Af I + {o}, o then I we I n m + o Since me I -> -m e I So, I Contians positive integers I is a wo o s i S = Z + . But Z + is a wo o s i plus a supers i S has a least element say n.	
Since me I -> -m e I So, I Contians positive integers I is a mos SCZ+ But Z is a mos in plus saises is 5 has a least element say n.	gp I={o}, then I = id(o) is a PI
الله ک می معمری تے کل الاعداد العلمی قالوجودة فی آ کے S = Z+ . But Z+ is a m. o.s الله ک می معمری کی آن می ک می کان کی کان	AP I + {o}, then ∃m∈ I 1 m+0
I ch a specie is a mos single plant of sign o	Since me I ->-me I
ت S ⊆ Z+ . But Z+ is a w. o. s انه می کانسی کا	
	∴ S ⊆ Z+ . But Z+ is a w. o. s
	5 has a least element say no
w VI IS the least positive integer of	in n is the least positive integer of I.

 ${\bf a}_{i}$





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الفصل الرابع والخامس
Chapter Four And
Five