جامعة بغداد
كلية التربية للعلوم الصرفة - ابن الهيثم
قسم الرياضيات

المرحلة الثانية

المعادلات التفاضلية الاعتيادية

Ordinary Differential Equations

Chapter 2

أساتذة المادة

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1. $(x^2+1-4xy-2y^2)dx-(2x^2+4xy-y^3+2)dy=0$

2. $ysec^2x\ dx+tan\ x\ dy=0$

3. $\cos y\ dx+(y^2-x\sin y)\ dy=0$

4. $e^x y^2 dx+2e^x y dy=0$

2.5: Integral factors:

If the equation ($M(x,y)dx+N(x,y)dy=0$) is not exact, then we multiply both sides by a factor $I(x,y)$ that turns it into an exact, and this factor is called the integral factor.

(اذا كانت المعادلة غير تامة، عندها نقوم بضرب طرفي المعادلة ب *I(x,y)* لتحويلها الى تامة)

ملاحظة: 1.يسمى *I(x,y)* عامل التكامل.

2. وظيفة عامل التكامل هي تحويل المعادلة غير التامة الى معادلة تامة

3. يختلف عامل التكامل من معادلة الى اخرى حسب شكل المعادلة كما مبين بالجدول الاتي:

N.	$M(x,y)dx+N(x,y)dy$	I(x,y) عامل التكامل	التعويض المناسب Z

1 i ii	اذا كان $\frac{M_y - N_x}{N} = f(x) \Rightarrow$ $\frac{M_y - N_x}{-M} = f(y) \Rightarrow$	$I(x) = e^{\int f(x)dx}$ $I(y) = e^{\int f(y)dy}$	تصبح المعادلة تامة وتحل حسب التامة
2	في حال ان $\frac{M_y - N_x}{N} \neq f(x)$ $\frac{M_y - N_x}{-M} \neq f(y)$	نضرب المعادلة ب $x^m y^n$ ونعوض في $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ لايجاد قيم لمعرفة عامل التكامل المطلوب	تصبح المعادلة تامة وتحل حسب التامة
3	$ydx - xdy$ Or: $xdy - ydx$	$(xy)^{-1}, x^{-2}, y^{-2}$ Or: $(ax^2 + bxy + cy^2)^{-1}$	$ln\frac{x}{y}$ or $\frac{x}{y}$ or $\frac{y}{x}$ or: $\int \left\{a + b\left(\frac{y}{x}\right) + c(\frac{y}{x})^2\right\} d(\frac{y}{x})$
4	$pydx + qxdy$	$x^{p-1}y^{q-1}$	$z = x^p y^q$
5	$ydx + xdy$	1	$z = xy$
6	$pxdx + qydy$	1	$z = \frac{1}{2}(px^2 + qy^2)$
7	$dy + P(x)ydx$ or : $dx + \propto (y)xdy$	$I = e^{\int p(x)dx}$	$z = y.e^{\int p(x)dx}$

	وتسمى المعادلة من هذا النوع **بالمعادلة الخطية**	$\boldsymbol{I = e^{\int \propto(y)dy}}$	$\boldsymbol{z = xe^{\int \propto(y)dy}}$

Now, we will explain each of these cases using examples

Case 1:

Multiplying the equation $(\boldsymbol{M(x,y)dx + N(x,y)dy = 0})$ by I(x,y) we get:

$(\boldsymbol{I\,.\,M(x,y)dx + I\,.\,N(x,y)dy = 0})$, where I, M, N are functions of x and y.

Deriving IM w.r.t. y and IN w.r.t. x

$$\rightarrow \frac{\partial(I\,M)}{\partial y} = I\,M_y + I_y M$$

$$\frac{\partial(I\,N)}{\partial x} = I\,N_x + I_x N$$

$$\rightarrow \frac{\partial(I\,M)}{\partial y} = \frac{\partial(I\,N)}{\partial x},$$ (بأعتبار ان I هو عامل التكامل)

then $I\,M_y + I_y M = I\,N_x + I_x N$

$$\rightarrow I[M_y - N_x] = I_x\, N - I_y M \quad \ldots\ldots\ldots\ldots\ldots \qquad (*)$$

Now we have two cases

1. ***If*** $\boldsymbol{I(x,y) = I(x)}$***, that is I is a function just for*** $\boldsymbol{(x)}$***.***

Then

$I_x = \dfrac{dI}{dx}$, $I_y = 0$ (عامل التكامل دالة ل x فقط)

Sub. in (*), we get

$$I[M_y - N_x] = N\ \frac{dI}{dx}$$

$$\rightarrow \frac{dI}{I} = \frac{M_y - N_x}{N}\ dx$$

Let $P(x) = \frac{M_y - N_x}{N}$ (function a lone for x)

وبتكامل الطرفين نحصل على:

$$\ln I = \int P(x)dx$$

$\rightarrow I(x) = e^{\int P(x)dx} \rightarrow I(x) = e^{\int (\frac{M_y - N_x}{N})dx}$ (عامل التكامل)

2. *If $I(x,y) = I(y)$, that is I is a function just for (y).Then*

$I_y = \dfrac{dI}{dy}$, $I_x = 0$ (عامل التكامل دالة ل y فقط)

Sub. in (*) we get:

$I[M_y - N_x] = -I_y M$

$\frac{I_y}{I} = \frac{M_y - N_x}{-M}$

$I(y) = e^{\int P(y)dy}$ where $P(y) = \frac{M_y - N_x}{-M}$ (function a lone for y)

$\rightarrow I(y) = e^{\int (\frac{M_y - N_x}{-M})dx}$ (عامل التكامل)

Ex1: Solve $(3x^3 + 2y)dx + \left(2x\ln 3x + \frac{3x}{y}\right)dy = 0$

Sol.: $M = 3x^3 + 2y$, $N = 2x\ln 3x + \frac{3x}{y}$

$M_y = 2$, $N_x = 2 + 2\ln 3x + \dfrac{3}{y}$

$M_y - N_x = -\left(2\ln 3x + \dfrac{3}{y}\right) \neq 0$

$\therefore$ *The equation is not exact.*

$$\frac{M_y - N_x}{N} = \frac{2\ln 3x + \frac{3}{y}}{x(2\ln 3x + \frac{3}{y})} = -\frac{1}{x} = P(x)$$

$\rightarrow I(x) = e^{\int -\frac{1}{x}dx} = e^{-\ln x} = e^{\ln\frac{1}{x}} = \frac{1}{x}$ (عامل التكامل)

(بضرب طرفي المعادلة في $\frac{1}{x}$ تصبح تامة)

$$\left(3x^2 + \frac{2y}{x}\right)dx + \left(2\ln 3x + \frac{3}{y}\right)dy = 0$$

$$\because \quad \frac{\partial M}{\partial y} = \frac{2}{x} = \frac{\partial N}{\partial x} \quad \rightarrow \quad The\ eq. is\ exact$$

الان نقوم بحل المعادلة التفاضلية التامة (H.W.)

Ex 2: Solve $\boldsymbol{y(2x+y)dx + (3x^2 + 4xy - y)dy = 0}$

Sol.: $M = y\,(2x+y), \quad N = 3x^2 + 4xy - y$

$M_y = 2x + 2y, \quad N_x = 6x + 4y$

$M_y - N_x = -4x - 2y = -2(2x+y) \neq 0$

The equation is not exact.

We must find the integration factor.

$$\frac{M_y - N_x}{-M} = \frac{-2(2x+y)}{-y(2x+y)} = \frac{2}{y}$$

$$\rightarrow I = e^{\int \frac{2}{y} dy} = e^{2\ln y} = e^{\ln y^2} = y^2$$ (عامل التكامل)

(نضرب طرفي المعادلة بعامل التكامل حتى تصبح تامة)

$$y^3(2x+y)dx + y^2(3x^2+4xy-y)dy = 0$$

$$M = 2xy^3 + y^4, \quad N = 3x^2y^2 + 4xy^3 - y^3$$

$$M_y = 6xy^2 + 4y^3, \quad N_x = 6xy^2 + 4y^3$$

Solution:

1. Integrating M(x,y) for x: نكامل بالنسبة ل x

$$M = \frac{\partial F}{\partial x} = 2xy^3 + y^4$$

$$F(x,y) = 2\frac{x^2}{2}y^3 + xy^4 + h(y) \qquad \dots \quad (*)$$

$$= x^2y^3 + xy^4 + h(y)$$

2. Deriving for y: نشتق بالنسبة ل y

$$\frac{\partial F}{\partial y} = 3x^2y^2 + 4xy^3 + h'(y)$$

3. Set $\frac{\partial F}{\partial y} = N$ نساوي. $\frac{\partial F}{\partial y}$ الى N

$$3x^2y^2 + 4xy^3 + h'(y) = 3x^2y^2 + 4xy^3 - y^3$$

$$\rightarrow h'(y) = -y^3$$

$$\rightarrow h(y) = \frac{-y^4}{4} = -\frac{1}{4}\,y^4$$

$$\rightarrow F = x^2y^3 + xy^4 - \frac{1}{4}\,y^4$$

The solution is : $x^2y^3 + xy^4 - \frac{1}{4}\,y^4 = c$

<u>Case (2):</u> If the above two integration factors state are not exist.

بمعنى انه عامل التكامل (I) ليس دالة الى x ولا دالة الى y بشكل منفصل فعلينا هنا استخراج عامل التكامل.

Suppose that the integrating factor $\boldsymbol{I(x,y) = x^m\, y^n}$, and we find the value of m and that maxes the differential equation exact.

اذن الهدف هو الوصول الى قيمة m,n حتى نحصل على عامل التكامل.

وسوف نوضح ذلك بأستخدام المثال الاتي:

Ex3: solve the diff. eq.

$$\boldsymbol{(x^2 + xy^2)\frac{dy}{dx} - 3xy + 2y^3 = 0}$$

Sol: (يتم بالبداية ترتيب المعادلة).

$$\boldsymbol{(2y^3 - 3xy)dx + (x^2 + xy^2)dy = 0} \qquad \dots \quad (*)$$

$$\frac{\partial M}{\partial y} = 6y^2 - 3x \neq \frac{\partial N}{\partial x} = 2x + y^2 \quad \rightarrow The\ diff.\ eq.\ is\ not\ exact.$$

الان نبحث عن عامل التكامل

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{6y^2 - 3x - 2x - y^2}{-(2y^3 - 3xy)} \neq I(y)$$

And so

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{6y^2 - 3x - 2x - y^2}{x^2 + xy^2} \neq I(x)$$

Let $I(x,y) = x^m y^n\ is\ integrating\ factor$

$$(2x^m y^{n+3} - 3x^{m+1}y^{n+1})dx + (x^{m+2}\, y^n + x^{m+1}\, y^{n+2})dy = 0$$

$$\frac{\partial M}{\partial y} = 2(n+3)x^m y^{n+2} - 3(n+1)x^{m+1}y^n$$

$$= x^m y^n[(2n+6)y^2 - (3n+3)x]$$

$$\frac{\partial N}{\partial x} = (m+2)x^{m+1}y^n + (m+1)x^m y^{n+2}$$

$$= x^m y^n[(m+2)x + (m+1)y^2]$$

$$Since\ it\ is\ exact \rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\rightarrow\ x^m y^n[(2n+6)y^2 - (3n+3)x] = \ x^m y^n[(m+2)x + (m+1)y^2]$$

$$\rightarrow\ (2n+6)y^2 - (3n+3)x = \ (m+2)x + (m+1)y^2$$

(نقوم الان بتساوي المعاملات)

$$m+2 = \ -3n-3 \qquad \rightarrow m+1 = \ -3n-4 \qquad (1)$$

$$m+1 = \ 2n+6 \qquad \rightarrow m+1 = \ 2n+6 \qquad (2)$$

بمساواة المعادلتين (1) و(2) نحصل على:

$$\rightarrow\ 2n+6 = \ -3n-4$$

$\rightarrow\ 5n = -10$

$\rightarrow\ n = -2$

From (1) $\rightarrow m + 1 = -3(-2) - 4 \rightarrow m = 1$

$\therefore I(x, y) = xy^{-2}$

$(2xy - 3x^2y^{-1})dx + (x^3y^{-2} + x^2)dy = 0$

$$\frac{\partial M}{\partial y} = 2x + 3x^2y^{-2}$$

$$\frac{\partial N}{\partial x} = 2x + 3x^2y^{-2}$$

$\because\ \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow The\ eq.\ become\ \ exact.$ (حل المعادلة التامة يترك للطالب)

<u>Remark:</u> Some integral factors can be deduced for certain amounts of differential rule like cases 3-6 in the table.

(يمكن استنتاج بعض عوامل التكامل لبعض المقادير من قواعد التفاضل مثل مشتقة حاصل قسمة دالتين ومشتقة حاصل ضرب دالتين وغيرهما والتي ستظهر في الحالات من 3-6 في الجدول

For example:

$\boldsymbol{d\left(\frac{y}{x}\right) = \frac{xdy-ydx}{x^2}}$, this means $(\frac{1}{x^2})$ is an integral factor of $xdy - ydx$.

Case 3: When the left side of the equation is in the form $\boldsymbol{xdy - ydx}$ or $\boldsymbol{ydx - xdy}$:

Ex 4: Prove that the integral factor of $\boldsymbol{xdy - ydx + f(x)dx = 0}$ is $\boldsymbol{x^{-2}}$

Proof:

$$\therefore [x\,dy - y\,dx + f(x)dx = 0] * \frac{1}{x^2}$$

$$\frac{xdy - ydx}{x^2} + \frac{f(x)}{x^2}\,dx = 0$$

$$\frac{1}{x}\,dy - \frac{y}{x^2}\,dx + \frac{1}{x^2}\,f(x)dx = 0$$

$$(\frac{1}{x^2}\,f(x) - \frac{y}{x^2})\,dx + \frac{1}{x}\,dy = 0$$

$$\because \frac{\partial M}{\partial y} = -\frac{1}{x^2} = \frac{\partial N}{\partial x} \rightarrow Exact$$

Then x^{-2} is an integral factor of the equation above

And the general solution is:

$$\int d(\frac{y}{x}) + \int \frac{1}{x^2} f(x)\ dx = 0$$

$$\to \frac{y}{x} + \int \frac{1}{x^2} f(x)\ dx = c$$

And by the same way one can prove that $(\frac{1}{y^2})$ is an integral factor of

$xdy - ydx + f(y)dy = 0.$

In general, we can convert $xdy - ydx$ or $ydx - xdy$ to exact diff. eq. by dividing on one of the following:

x^2 , y^2 , xy , $x^2 + y^2$, $x^2 - y^2$, ...

And the general solution of this amounts is $ax^2 + bxy + cy^2$

<u>Ex 5:</u> Prove that $\frac{1}{ax^2+bxy+cy^2}$ is an integral factor of $(xdy - ydx)$, set $a \neq 0, b \neq 0, c \neq 0$ at the same time.

$$xdy - y\,dx \,.\frac{1}{ax^2 + bxy + cy^2} = \frac{xdy - y\,dx}{ax^2 + bxy + cy^2}$$

$$= \frac{\frac{xdy - y\,dx}{x^2}}{a + b\frac{y}{x} + c\left(\frac{y}{x}\right)^2} = \frac{d(\frac{y}{x})}{a + b\frac{y}{x} + c\left(\frac{y}{x}\right)^2}$$

Let $\frac{y}{x} = z$

$\frac{dz}{a+bz+cz^2} = d\,f(z)$ (تفاضل تام)

Ex 6: Find the general solution of

$$\boldsymbol{xdy - ydx = x^4y^2dx}$$

Sol: $\left(\underbrace{-y - x^4y^2}_{M}\right)dx + \underbrace{x}_{N}\,dy = 0$

$$\frac{\partial M}{\partial y} = -1 - 2x^4y, \quad \frac{\partial N}{\partial x} = 1$$

$$\because \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \quad \rightarrow \quad The\ Diff.\ Eq.\ is\ not\ exact.$$

لذلك نحتاج الى عامل التكامل: لإيجاد الحل العام نضرب الطرفين ب $(\frac{1}{x^2})$

$$\frac{xdy - ydx}{x^2} = x^4\frac{y^2}{x^2}dx$$

$$d(\frac{y}{x}) = x^4\left(\frac{y}{x}\right)^2 dx$$

$Let\ z = \frac{y}{x} \rightarrow\ dz = x^4 z^2 dx$

$$\rightarrow \int z^{-2}\, dz = \int x^4\, dx$$

$$\rightarrow \frac{-1}{z} = \frac{x^5}{5} + c$$

$$\rightarrow \frac{-x}{y} = \frac{x^5}{5} + c \qquad \text{(الحل العام)}$$

طريقة حل أخرى: يمكن أيضا" ان نقسم على y^2

$$\frac{xdy - ydx}{y^2} = x^4 dx$$

$$\rightarrow \int d(\frac{-x}{y}) = \int x^4 dx$$

$$\rightarrow \frac{-x}{y} = \frac{x^5}{5} + c \qquad \text{(الحل العام)}$$

Ex 7: Find the general solution of

$$\boldsymbol{xdy - ydx = y^3(x^2 + y^2)dy}$$

Sol:

$$\underbrace{-y}_{M}\,dx + \left[\underbrace{x - y^3(x^2+y^2)}_{N}\right]dy = 0$$

$$\frac{\partial M}{\partial y} = -1, \quad \frac{\partial N}{\partial x} = 1 - 2y^3x$$

$$\because \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \quad \rightarrow \quad The\ diff.\ eq.\ is\ not\ exact.$$

لذلك نحتاج الى عامل التكامل: لإيجاد الحل العام نضرب الطرفين ب $(\frac{1}{x^2+y^2})$

$$\frac{xdy - ydx}{x^2+y^2} = y^3dy$$

$$\frac{\frac{xdy - ydx}{x^2}}{1+\frac{y^2}{x^2}} = y^3dy$$

$$\frac{d(\frac{y}{x})}{1+\left(\frac{y}{x}\right)^2} = y^3dy$$

$$Let\ z = \frac{y}{x} \quad \rightarrow \quad \int \frac{dz}{1+z^2} = \int y^3dy$$

$$\rightarrow \quad \tan^{-1} z = \frac{1}{4}y^4 + c$$

$$\rightarrow \quad \tan^{-1}(\frac{y}{x}) = \frac{1}{4}y^4 + c$$ (الحل العام)

Case 4: When the equation contains the term($\boldsymbol{pydx + qxdy}$)

first we know that

$$d(x^p y^q) = qx^p y^{q-1} dy + p\, y^q x^{p-1} dx \qquad \text{(حاصل ضرب دالتين)}$$

$$= x^{p-1} y^{q-1}(qxdy + pydx)$$

$$= x^{p-1} y^{q-1}(pydx + qxdy)$$

لذلك يكون عامل تكامل المقدار $(pydx + qxdy)$ هو $(x^{p-1}y^{q-1})$

والتعويض المناسب هو $(z = x^p y^q)$

Ex8: Find the general solution of

$$\boldsymbol{xdy - 3ydx = x^4 y^{-1} dx}$$

Sol:

$$\left.\begin{array}{c} \boldsymbol{xdy - 3ydx = x^4 y^{-1} dx} \\ pydx + qxdy \end{array}\right] \quad \text{نقارن}$$

$$\therefore \quad p = -3, \quad q = 1$$

$$\therefore \quad Integrate\ factor\ is \quad x^{-3-1} y^{1-1} = x^{-4}$$

$$z = x^{-3}y \;\rightarrow\; z = \frac{y}{x^3} \rightarrow \; y = z\,.\,x^3$$

$$\rightarrow \quad dz = x^{-3}dy - 3yx^{-4}dx$$

By multiplying the end of equation by $\frac{\mathbf{1}}{\boldsymbol{x^4}}$

(نضرب طرفي المعادلة بعامل التكامل)

$$\underbrace{x^{-3}dy - 3x^{-4}ydx}_{dz} = y^{-1}dx$$

$$\rightarrow \quad dz = \frac{dx}{y} \quad \rightarrow \quad dz = \frac{dx}{z.x^3}$$

$$zdz = x^{-3}dx \quad \rightarrow \quad \frac{z^2}{2} = \frac{x^{-2}}{-2} + c$$

$$\rightarrow \quad \frac{(x^{-3}y)^2}{2} = \frac{x^{-2}}{-2} + c, \quad (The\ \ general\ \ solution)$$

Ex9: Find the general solution of

$$\boldsymbol{xdy - ydx = x^2y^3dx}$$

Sol:

$$\left.\begin{matrix} xdy - ydx = x^2y^3dx \\ pydx + qxdy \end{matrix}\right] \quad \text{نقارن}$$

$$\therefore \quad p = -1, \quad q = 1$$

$$\therefore \quad Integrate\ factor\ is \quad x^{-1-1}y^{1-1} = \frac{1}{x^2}$$

$$z = x^{-1}y \ \rightarrow \ z = \frac{y}{x}$$

By multiplying the end of equation by $\frac{\mathbf{1}}{\boldsymbol{x^2}}$

$$\frac{xdy - ydx}{x^2} = y^3 dx$$

$$d\left(\frac{y}{x}\right) = \frac{x^3}{x^3} y^3 dx$$ (نضرب ونقسم على x^3)

Let $z = \frac{y}{x} \quad \rightarrow \quad dz = x^3 z^3 dx$

$$\rightarrow \quad \int z^{-3}\, dz = \int x^3 dx$$

$$\rightarrow \quad \frac{z^{-2}}{-2} = \frac{x^4}{4} + c$$

$$\rightarrow \quad \frac{-1}{2z^2} = \frac{x^4}{4} + c$$

$$\rightarrow \quad \frac{-1}{2(\frac{y}{x})^2} = \frac{x^4}{4} + c, \quad (The\ general\ solution)$$

Ex 10: Find the general solution of

$$xdy + 2ydx = e^x dx$$

Sol: p=2 , q=1

I=$x^{p-1}y^{q-1}$ حسب الحالة 4 من الجدول

$= x^{2-1}y^{1-1} = x$

$z = x^p y^q = x^2 y$ التعويض المناسب هو

Multiplying both sides by I=x ,we get:

$x^2 dy + 2xydx = xe^x dx$

$d(x^2 y) = xe^x dx \Rightarrow dz = xe^x dx$

Integrating both sides, we get:

$z = \int xe^x dx + c$

$= xe^x - \int e^x dx + c$

$= xe^x - e^x + c$

Replacing z to get the following general solution:

$x^2 y = xe^x - e^x + c$

Case 5: When the equation contains the term $(xdy + ydx)$

The integral factor in this case equal 1 this means that we just need some math operations here.

Ex11: Solve $\quad \boldsymbol{xdy + ydx = (5x - 2x^2y)dx}$

Sol: Let $z = xy \quad \rightarrow \quad dz = xdy + ydx$

$$\rightarrow \quad y = \frac{z}{x}$$

substituting in the original equation بالتعويض بالمعادلة التفاضلية:

$$\rightarrow \qquad dz = \left(5x - 2x^2\left(\frac{z}{x}\right)\right)dx$$

$$\rightarrow \qquad dz = (5 - 2z)xdx$$

$$\rightarrow \qquad \int \frac{dz}{(5-2z)} = \int xdx$$

$$\rightarrow \qquad -\frac{1}{2}\ln|5 - 2z| = \frac{1}{2}x^2 + c$$

$$\rightarrow \qquad -\frac{1}{2}\ln|5 - 2xy| = \frac{1}{2}x^2 + c$$, (The general solution)

Ex12: Solve $\quad \boldsymbol{x^2\frac{dy}{dx} + xy + \sqrt{1 - x^2y^2} = 0}$

Sol: $x^2dy + xydx + \sqrt{1-x^2y^2}dx = 0$

$$x(xdy + ydx) + \sqrt{1-x^2y^2}dx = 0$$

$$\frac{(xdy+ydx)}{\sqrt{1-x^2y^2}} + \frac{dx}{x} = 0$$

$\sin^{-1}(xy) + \ln|x| = c,$ (The general solution)

Case 6: When the equation contains the term ***(xdx+ydy)***

The integral factor in this case equal 1 this means that we just need some math operations here.

Ex13: **Solve** $\boldsymbol{xdx + ydy = 3\sqrt{x^2+y^2}\, y^2dy}$

Sol: $\frac{xdx+ydy}{\sqrt{x^2+y^2}} = 3y^2dy$

$$\frac{1}{2}\ \frac{2xdx+2ydy}{\sqrt{x^2+y^2}} = 3y^2dy$$

$$\frac{1}{2}\int (x^2+y^2)^{-1/2}(2xdx + 2ydy) = \int 3y^2dy$$

$\sqrt{x^2+y^2} = y^3 + c$ and this is the general solution

Homework: Solve the following diff. equations

14) Solve $\boldsymbol{xdx + ydy = y^2(x^2 + y^2)dy}$

15) $\boldsymbol{xdx + ydy = (x^2 + y^2)^3(xdy - ydx)}$

Case 7: Linear Differential Equation:

We have defined the general form of linear differential equation of order n to be:

$$\boldsymbol{a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)}$$

We remind you, that linearity means that all coefficients are functions of x only, and that y and its derivatives are raised to the first power.

Def: (Linear Differential Equation of First Order and First Degree)

A differential eq. of the form

$$a_1(x)\frac{dy}{dx} + a_0(x)y = g(x), \qquad a_1(x) \neq 0 \qquad .... \quad (1)$$

is said to be a linear first order differential equation.