جامعة بغداد
كلية التربية للعلوم الصرفة - ابن الهيثم
قسم الرياضيات

المرحلة الثانية

المعادلات التفاضلية الاعتيادية

Ordinary Differential Equations

Chapter 2

أساتذة المادة

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Examples:

1) Solve the O.D.E. $(y^2 + y)dx - (x^2 - x)dy = 0$

Solution:

$$\frac{dx}{x^2 - x} - \frac{dy}{y^2 + y} = 0$$

$$\underbrace{\int \frac{dx}{x^2 - x}}_{(1)} - \underbrace{\int \frac{dy}{y^2 + y}}_{(2)} = \int 0$$

Use the method of fragmentation (partial) of the fractions. نستخدم طريقة تجزئة الكسور

$$(1) \rightarrow \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

$$\rightarrow \frac{1}{x(x-1)} = \frac{Ax - A + Bx}{x(x-1)} = \frac{(A+B)x - A}{x(x-1)}$$

$$\rightarrow A + B = 0 \qquad \dots \quad (i)$$

$$-A = 1 \qquad \dots \quad (ii)$$

$From$: $(ii) \rightarrow A = -1$, substitute in (i)

$\rightarrow\ -1 + B = 0 \rightarrow B = 1$

$(2) \rightarrow \ \frac{1}{y(y+1)} \rightarrow A = 1$, and $B = \ -1$ (بنفس الأسلوب اعلاه)

$$\because \int \frac{1}{x(x-1)} dx - \int \frac{dy}{y(y+1)} = \int 0$$

$$\rightarrow \int \frac{-1}{x} dx + \int \frac{1}{x-1} - \int \frac{1}{y} dy + \int \frac{1}{y+1} dy = \int 0$$

$\rightarrow -\ln|x| + \ln|x-1| - \ln|y| + \ln|y+1| = c,$ (The general solution)

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2) Find the general solution of $2x^2y' - y(2x+y) = 0$

المعادلة متجانسة (اثبت ذلك؟) **(H.W.)**

ويمكن حل المعادلة المتجانسة بأسلوب اخر وذلك بتحويل المعادلة بصورة $f\left(\frac{y}{x}\right)$ حيث ان $v = \frac{y}{x}$

$$2x^2y' = 2xy + y^2 \rightarrow y' = \frac{2xy + y^2}{2x^2}$$

$$\rightarrow y' = \frac{y}{x} + \frac{1}{2}\left(\frac{y}{x}\right)^2$$

Let $v = \frac{y}{x} \rightarrow y = vx \rightarrow dy = vdx + xdv \rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\rightarrow v + x\frac{dv}{dx} = v + \frac{1}{2}v^2$$

$$\rightarrow\ x\frac{dv}{dx} = \frac{1}{2}v^2$$

$$\rightarrow\ \int \frac{dv}{\frac{1}{2}v^2} = \int \frac{dx}{x}$$

$$\rightarrow\ \frac{-2}{v} = \ln|x| + c$$

$$\rightarrow\ \frac{-2x}{y} = \ln|x| + c, \qquad \text{(The general solution)}$$

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(3) Solve the following diff. eq. $(\boldsymbol{x^2 + y^2})\boldsymbol{dx - 2xydy = 0}$

at $\boldsymbol{y(2) = 0}$.

<u>Solution:</u>

$\because M(x,y) =\ x^2 + y^2$

$\rightarrow\ M(tx,ty) =\ (tx)^2 + (ty)^2 = t^2(x^2 + y^2) = t^2M(x,y)$

$\because\ N(x,y) =\ -2x\,y$

$$\rightarrow N(tx, ty) = -2(tx)(ty) = -2t^2 x\, y = t^2 N(x, y)$$

(اذن المعادلة التفاضلية متجانسة)

$$[(x^2 + y^2)dx - 2xydy = 0] \div x^2$$

$$\left(1 + \left(\frac{y}{x}\right)^2\right)dx - \left(2\left(\frac{y}{x}\right)\right)dy = 0$$

Let $v = \frac{y}{x} \rightarrow y = vx \rightarrow dy = v\,dx + x\,dv \rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy} \rightarrow \frac{dy}{dx} = \frac{x^2}{2xy} + \frac{y^2}{2xy}$$

$$\rightarrow \frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{y}\right) + \frac{1}{2}\left(\frac{y}{x}\right)$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1}{v}\right) + \frac{1}{2}v$$

$$x\frac{dv}{dx} = \frac{1}{2v} + \frac{1}{2}v - v$$

$$\rightarrow x\frac{dv}{dx} = \frac{1}{2v} - \frac{1}{2}v$$

$$\rightarrow x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1}{v} - v\right)$$

$$\to x\frac{dv}{dx}=\frac{1}{2}(\frac{1-v^2}{v})$$

$$\to \frac{dv}{\frac{1}{2}\frac{(1-v^2)}{v}}=\frac{dx}{x}$$

$$\to \int 2\frac{v}{1-v^2}dv=\frac{dx}{x}$$

$$\to -\ln|1-v^2|=\ln|x|+c$$

$$\to -\ln\left|1-\frac{y^2}{x^2}\right|=\ln|x|+c,$$ (The general solution)

(الحل الخاص: واجب؟)

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4) Solve: $(\boldsymbol{x^2-y^2})\boldsymbol{dy}-\boldsymbol{2xydx=0},\ \boldsymbol{when\ x=0, y=1}$

<u>Solution:</u> (The equation is homo. of degree 2) (واجب؟)

Let $y=vx\to dy=vdx+xdv$

$$(x^2-v^2x^2)(vdx+xdv)-2xvxdx=0$$

$$(x^2v-v^3x^2)dx+(x^3-v^2x^3)dv-2x^2vdx=0$$

$$(-v^3x^2-x^2v)dx+(x^3-v^2x^3)dv=0$$

$$\left(-x^2(v^3+v)\right)dx + x^3(1-v^2)dv = 0 \qquad * \frac{1}{(v^3+v)x^3}$$

$$\frac{-1}{x}dx + \frac{1-v^2}{v^3+v}dv = 0$$

$$-\ln|x| + \int \frac{1}{v^3+v}dv - \int \frac{v\,.\,v}{v(v^2+1)}dv = c$$

$$-\ln|x| + \int \frac{1}{v(v^2+1)}dv - \frac{1}{2}\ln|1+v^2| = c \qquad \ldots \quad (*)$$

$$\frac{1}{v(v^2+1)} = \frac{A}{v} + \frac{Bv+C}{v^2+1} = \frac{A(v^2+1)+Bv^2+cv}{v(v^2+1)}$$

$$= \frac{A(v^2+1)+Bv^2+Cv}{v(v^2+1)} = \frac{(A+B)v^2+cv+A}{v(v^2+1)}$$

$\rightarrow A + B = 0 \qquad \ldots (1)$

$C \quad = 0 \qquad \ldots (2)$

$A \quad = 1 \qquad \ldots (3)$

Sub. (3) in (1) $\rightarrow B = -1$

$$\rightarrow \int \frac{1}{v}dv + \int \frac{-v}{v^2+1}dv - \frac{1}{2}\ln|1+v^2| = \ln|x| + c$$

$$\ln|v| - \frac{1}{2}\ln|1+v^2| - \frac{1}{2}\ln|1+v^2| = \ln|x| + c$$

$$\ln|v| - \ln|1+v^2| = \ln|x| + c$$

$$\ln\frac{v}{v^2+1} = \ln x + c$$

$$\ln\frac{v}{1+v^2} - \ln x = c$$

$$\ln\frac{v}{x(1+v^2)} = c$$

$$\frac{v}{x(1+v^2)} = e^c$$

When $v = \frac{y}{x}$

$$\frac{\frac{y}{x}}{x\left(1+\left(\frac{y}{x}\right)^2\right.} = e^c \rightarrow \frac{y}{x}.\frac{x}{x^2+y^2} = e^c \rightarrow \frac{y}{x^2+y^2} = e^C(\,The\ gen.sol.)$$

When $x = 0\ and\ y = 1 \rightarrow 1 = e^c \rightarrow c = \ln 1 = 0$

$$\therefore \frac{y}{x^2+y^2} = 1, (since\ e^0 = 1) \quad (\,The\ particular\ sol.) \text{ (الحل الخاص)}$$

5) Find the general solution of O.D.E.

$$(2x + y - 1)dx + (x + y - 2)dy = 0$$

Solution:

(نلاحظ انها معادلة تفاضلية ذات معاملات خطية)

$\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} \neq 0 \rightarrow$ (اذن المستقيمان متقاطعان)

الان يجب استخراج نقطة التقاطع:

$2x + y + 1 = 0$ ... (1)

$x + y - 2 = 0$ ... (2)

$Eq.(1) - Eq.(2) \rightarrow x + 1 = 0 \rightarrow x = -1$

$From: Eq.(2): -2 + y - 1 = 0 \rightarrow y = 3$

$\therefore (h, k) = (-1,3)$ [The intersection point]

Let $x = x_1 - 1 \rightarrow dx = dx_1$

Let $y = y_1 + 3 \rightarrow dy = dy_1$

Sub of O.D.E.$\rightarrow$

$$(2(x_1-1)+(y_1+3)-1)dx_1+\big((x_1-1)+y_1+3-2\big)dy_1=0$$

$$(2x_1+y_1)dx_1+(x_1+y_1)dy_1=0 \qquad \dots (*)$$

الان أصبحت المعادلة التفاضلية متجانسة

Let $v=\frac{y_1}{x_1}\rightarrow y_1=vx_1\rightarrow dy_1=vdx_1+x_1dv$

Sub. in (*) $\rightarrow$

$$(2x_1+vx_1)dx_1+(x_1+vx_1)(vdx_1+x_1dv)=0$$

$$2x_1dx_1+vx_1dx_1+x_1vdx_1+x_1^2dv+v^2x_1dx_1+vx_1^2dv=0$$

$$x_1(2+2v+v^2)dx_1+x_1^2(1+v)dv=0$$

$$\int\frac{1}{x_1}dx_1+\int\frac{2(1+v)}{2(2+2v+v^2)}dv=\int 0$$

$$\ln|x_1|+\frac{1}{2}\ln|2+2v+v^2|=c$$

$$\ln|x_1|+\frac{1}{2}\ln\left|2+2(\frac{y_1}{x_1})+\frac{y_1^2}{x_1^2}\right|=c$$

$$\ln|x+1|+\frac{1}{2}\ln\left|2+2(\frac{y-3}{x+1})+\frac{(y-3)^2}{(x+1)^2}\right|=c\,,(\mathit{The\ general\ sol.})$$

6) Find the general solution of

$$(4x + 2y + 3)dx + (6x + 3y - 2)dy = 0$$

(The Differential Eq. with linear coefficients)

Solution:

$4x + 2y + 3 = 0 \rightarrow 2(2x + y) + 3 = 0$

$6x + 3y - 2 = 0 \rightarrow 3(2x + y) - 2 = 0$

Let $z = 2x + y \rightarrow dz = 2dx + dy \rightarrow dy = dz - 2dx$

نقوم الان بالتعويض بالمعادلة التفاضلية

قبل ذلك سنقوم بالإثبات بأن المستقيمان متوازيان باستخدام الميل، حيث ان:

ميل المستقيم الاول : $m_1 = \frac{-\text{معامل}x}{\text{معامل}y}$ & ميل المستقيم الثاني: $m_2 = \frac{-\text{معامل}x}{\text{معامل}y}$

If $m_1 = m_2 \rightarrow$ المستقيمان متوازيان

If $m_1 \neq m_2 \rightarrow$ المستقيمان متقاطعان

$\because m_1 = \frac{-4}{2} = -2, m_2 = \frac{-6}{3} = -2$

$\therefore m_1 = m_2, so\ the\ two\ lines\ are\ Parllel$ (متوازيان)

الان نبدأ بالتعويض:

$$(2z+3)dx + (3z-2)(dz-2dx) = 0$$

$$2zdx + 3dx + 3zdz - 6zdx - 2dz + 4dx = 0$$

$$-4zdx + 7dx + 3zdz - 2dz = 0$$

$$(-4z+7)dx + (3z-2)dz = 0$$

$$dx + \frac{(3z-2)}{(-4z+7)}dz = 0$$

$$dx + \left(\frac{-3}{4} + \frac{4}{4}.\frac{\frac{13}{4}}{-4z+7}\right)dz = 0$$

$$x - \frac{3}{4}z - \frac{13}{16}\ln|-4z+7| = c$$

$$x - \frac{3}{4}(2x+y) - \frac{13}{16}\ln|-4(2x+y)+7| = c, \qquad (The\ gen.\ sol.).$$

2.4 Exact Differential Equation:

Theorem: If M and N have continuous partial derivative in a rectangular region R then the equation

$$M(x,y)\,dx + N(x,y)\,dy \ = 0$$

be an exact equation if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ (الشرط لكي تكون المعادلة التفاضلية تامة)

Ex 1: Determine whether the following equations are exact or not:

i. $\boldsymbol{(2x^2+5)dx + 3ydy = 0}$

ii. $\boldsymbol{x\cos y\,dx + y\cos x\,dy = 0}$

iii. $\boldsymbol{\cos y\,dx + (y^2 - x\sin y)\,dy = 0}$

Sol. (i): $\because \frac{\partial M}{\partial y} = 0 = \frac{\partial N}{\partial x} \rightarrow it\ is\ exact$

Sol. (ii): $\because \frac{\partial M}{\partial y} = -x\sin y, and\ \frac{\partial N}{\partial x} = \ -y\sin x$

$$\rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \rightarrow it\ is\ not\ exact$$

Sol. (iii): $\because \frac{\partial M}{\partial y} = -\sin y, and\ \frac{\partial N}{\partial x} = -\sin y \rightarrow\ it\ is\ exact$

Remark 1: An equation in the form

$$M(x)dx + N(y)dy = 0$$

is an exact equation.

For example: $x^2 dx + \sin y\, dy\ = 0$ is exact as follows:

$\because \frac{\partial M}{\partial y} = 0 = \frac{\partial N}{\partial x} \rightarrow it\ is\ exact$

Remark 2: Every separable eq.is an exact equation after separating its variables (كل معادلة قابلة للفصل هي معادلة تامة بعد فصل متغيراتها)

Remark 3: The exact diff. eq. for the function $f(x, y)$ is

$$d\ f(x, y) = \frac{df}{dx}dx + \frac{df}{dy}dy$$

if we make it equal zero, we get the exact diff. eq.

$$\frac{df}{dx}dx + \frac{df}{dy}dy = 0$$

The solution of this diff. eq. is $f(x, y) = c,$ (ثابت اختياري : c)

Solution of the Exact Diff. Eq.

Through some examples, we will show here two methods to find the general solution of the exact equation.

من خلال الامثلة سوف نوضح طريقتين لحل المعادلة التفاضلية التامة:

■ **Method 1**: By using Remark 3

Ex 2 : Solve $(\boldsymbol{x}+\mathbf{2}\boldsymbol{y})\boldsymbol{dx}+(\mathbf{2}\boldsymbol{x}+\boldsymbol{y})\boldsymbol{dy}=\mathbf{0}$

Solution: $\because \frac{\partial M}{\partial y}=2=\frac{\partial N}{\partial x} \rightarrow the\ diff.\ eq.\ is\ exact,$

and hence its solution is $f(x,y)=C$

$$\frac{\partial F}{\partial x}=x+2y \qquad \ldots \quad (1) \qquad (M)$$

$$\frac{\partial F}{\partial y}=2x+y \qquad \ldots \quad (2) \qquad (N)$$

We can use any of the above equations. to find F.

From (1): $\frac{\partial F}{\partial x}=x+2y \rightarrow F=\frac{1}{2}x^2+2xy+h(y) \quad \ldots \quad (3)$ (تكامل جزئي)

And to find $h(y)$, we use the fact that eq.(3) satisfied eq.(2), then:

$$2x + h'(y) = 2x + y \rightarrow h'(y) = y \rightarrow h(y) = \frac{1}{2}y^2$$

Substituting in eq.(3), we get

$$F(x, y) = \frac{1}{2}x^2 + 2xy + \frac{1}{2}y^2$$

Then the solution is: $x^2 + 4xy + y^2 = c$

Ex3 : Solve $\boldsymbol{(2xy - 3x^2)dx + (x^2 + 2y)dy = 0}$

<u>Solution:</u> $\because \frac{\partial M}{\partial y} = 2x = \frac{\partial N}{\partial x} \rightarrow the\ diff.\ eq.\ is\ exact,$

and hence its solution is $f(x, y) = c$

$$\frac{\partial F}{\partial x} = 2xy - 3x^2 \qquad \dots \ (1) \qquad (M)$$

$$\frac{\partial F}{\partial y} = x^2 + 2y \qquad \dots \ (2) \qquad (N)$$

From (1): $F(x, y) = x^2y - x^3 + h(y)$ (نكامل بالنسبة x)

$$N = \frac{\partial F}{\partial y} = x^2 + h'(y) = x^2 + 2y$$

$$\rightarrow h'(y) = 2y \rightarrow h(y) = y^2$$

Then $F(x, y) = x^2y - x^3 + y^2$,

and the general solution is: $x^2y - x^3 + y^2 = c$

Ex 4: Solve $\boldsymbol{(1 + xy^2)dx + (x^2y + y)dy = 0}$

<u>Solution:</u> $\because \frac{\partial M}{\partial y} = 2xy = \frac{\partial N}{\partial x} \rightarrow the\ diff.eq.is\ exact,$

$\frac{\partial F}{\partial x} = 1 + xy^2$ ... (1) (M)

$\frac{\partial F}{\partial y} = x^2y + y$ ... (2) (N)

From (1): $F(x, y) = x + \frac{1}{2}x^2y^2 + h(y)$ (نكامل بالنسبة x)

$x^2y + h'(y) = x^2y + y \rightarrow h'(y) = y \rightarrow h(y) = \frac{1}{2}y^2$

And the general solution is: $2x + x^2y^2 + y^2 = c$

Ex 5: Solve the initial-value problem

$$\boldsymbol{\frac{3x^2y+1}{y}dx - \frac{x}{y^2}dy = 0,\ y(2) = 1}$$

Solution: we can rewrite the equation as following

$$(3x^2 + y^{-1})dx - xy^{-2}dy = 0$$

$$\frac{\partial M}{\partial y} = -y^{-2}$$

$$\frac{\partial N}{\partial x} = -y^{-2}$$

$$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow the\ diff.eq.is\ exact,$$

$$\frac{\partial F}{\partial x} = 3x^2 + y^{-1} \qquad \dots\ (1) \qquad (M)$$

$$\frac{\partial F}{\partial y} = -xy^{-2} \qquad \dots\ (2) \qquad (N)$$

From (2): $F(x,y) = xy^{-1} + g(x)$ (نكامل بالنسبة y)

Then: $\frac{\partial F}{\partial x} = y^{-1} + g'(x) = 3x^2 + y^{-1}$

$$\rightarrow g(x) = x^3, then\ F(x,y) = xy^{-1} + x^3$$

And the general solution is: $xy^{-1} + x^3 = c.$

When y(2)=1, we get :

$$2(1)^{-1} + 2^3 = c \Rightarrow c = 10$$

Sub. in the general solution above, we get:

$$xy^{-1} + x^3 = 10$$

And this is the particular solution

ملخص الطريقة :

1.نضع $M = \frac{\partial F}{\partial x}$ ونكاملها جزئيا ل x فنحصل على F(x,y) حيث تتضمن دالة اختيارية ل y ولتكن h(y)

2.نشتق F(x,y) بالنسبة ل y ونساويها ب N للحصول على قيمة الدالة h(y)

3.نعوض h(y) في معادلة F(x,y) عن فنحصل على الحل العام

ملاحظة:يمكن البدء مع $N = \frac{\partial F}{\partial y}$ ونحصل على النتيجة نفسها

METHOD 2: We choose a point (a,b) that it is within the domain of the function and satisfies M(x,y) and N(x,y) and we substitute in the following:

$$F(x,y) = \int_a^x M(t,y)dt + \int_b^y N(a,t)dt = c$$

We find this integral to obtain the general solution of the diff. eq.

Ex 6: Solve $(\boldsymbol{x}+\mathbf{2}\boldsymbol{y})\boldsymbol{dx}+(\mathbf{2}\boldsymbol{x}+\boldsymbol{y})\boldsymbol{dy}=\mathbf{0}$

Solution: $M(x,y)=x+2y \Rightarrow \frac{\partial M}{\partial y}=2$

$$N(x,y)=2x+y \Rightarrow \frac{\partial N}{\partial x}=2$$

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x} \Rightarrow \textit{the equation is exact}$$

Let (a,b)=(0,0),substituting in :

$$F(x,y)=\int_a^x M(t,y)dt+\int_b^y N(a,t)dt=c$$

$$=\int_0^x M(t,y)dt+\int_0^y N(0,t)dt=c$$

$$=\int_0^x (t+2y)dt+\int_0^y tdt=c$$

$$=\left(\frac{t^2}{2}+2yt\right)\Big|_0^x+\left(\frac{t^2}{2}\right)\Big|_0^y=c$$

Ex 7: Solve the following diff. eq.

$$(\boldsymbol{y}^2+\boldsymbol{x}\boldsymbol{y}^2+\mathbf{1})\boldsymbol{dx}+(\boldsymbol{x}^2\boldsymbol{y}+\mathbf{2}\boldsymbol{x}\boldsymbol{y}+\boldsymbol{y})\boldsymbol{dy}=\mathbf{0}$$

Solution:

$$\frac{\partial M}{\partial y} = 2y + 2xy$$

$$\frac{\partial N}{\partial x} = 2xy + 2y$$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow the\ diff.\ eq.\ is\ exact$, let (a,b)=(0,0) ,we get:

$$F(x,y) = \int_0^x (y^2 + ty + 1)\, dt + \int_0^y t\, dt = c$$

$$= xy^2 + \frac{1}{2}x^2y^2 + x + \frac{1}{2}y^2 = c$$

the solution is: $2xy^2 + x^2y^2 + 2x + y^2 = c.$

Ex 8: Solve $\boldsymbol{\frac{3x^2y+1}{y}dx - \frac{x}{y^2}dy = 0 \quad , y(2) = 1}$

Solution:

$$M(x,y) = \frac{3x^2y+1}{y} \Rightarrow \frac{\partial M}{\partial y} = -y^{-2}$$

$$N(x,y) = \frac{-x}{y^2} \Rightarrow \frac{\partial N}{\partial x} = -y^{-2}$$

Then the equation is exact.

Let (a,b)=(0,1) ,sub. in

$$F(x,y) = \int_a^x M(t,y)dt + \int_b^y N(a,t)dt = c$$

$$= \int_0^x M(t,y)dt + \int_1^y N(0,t)dt = c$$

$$= \int_0^x \frac{3t^2y+1}{y} dt + \int_1^y 0.\, dt = c$$

$$= \int_0^x (3t^2 + \frac{1}{y})dt = c$$

$$= \left(t^3 + \frac{t}{y}\right) \Big|_0^x = c$$

$= x^3 + \frac{x}{y} = c$ and this is the general solution

When y(2)=1, we get the following

$$2^3 + \frac{2}{1} = c \Rightarrow c = 10$$

Then the Particular solution is $x^3 + \frac{x}{y} = 10$

<u>H.W.:</u> Solve the following equations by using two methods for the exact Differential Equation:

1. $(x^2 + 1 - 4xy - 2y^2)dx - (2x^2 + 4xy - y^3 + 2)dy = 0$

2. $y sec^2 x\, dx + tan\, x\, dy = 0$

3. $\cos y\, dx + (y^2 - x \sin y)\, dy = 0$

4. $e^x y^2 dx + 2e^x y dy = 0$

2.5: Integral factors:

If the equation ($M(x,y)dx + N(x,y)dy = 0$) is not exact, then we multiply both sides by a factor $I(x,y)$ that turns it into an exact, and this factor is called the integral factor.

(اذا كانت المعادلة غير تامة، عندها نقوم بضرب طرفي المعادلة ب *I(x,y)* لتحويلها الى تامة)

ملاحظة: 1.يسمى *I(x,y)* عامل التكامل.

2. وظيفة عامل التكامل هي تحويل المعادلة غير التامة الى معادلة تامة

3. يختلف عامل التكامل من معادلة الى اخرى حسب شكل المعادلة كما مبين بالجدول الاتي:

N.	$M(x,y)dx + N(x,y)dy$	I(x,y) عامل التكامل	التعويض المناسب Z