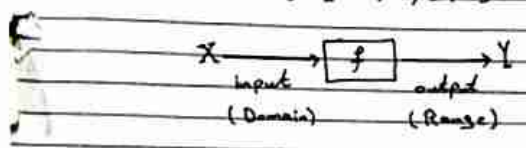


# "Functions"

تعريف

Definition: A function  $f: X \rightarrow Y$  is a relationship between two sets (non-empty sets) such that associates each elements in  $X$  with exactly one element in  $Y$ .

i.e.  $\forall x \in X, \exists y \in Y$  s.t.  $f(x) = y$ .



Definition (Domain): The set of all possible elements (input values) is called domain of  $f$  ( $\text{Dom}(f)$  or  $D_f$ ).

Definition (Range): The set of all corresponding elements in the second set (output values) is called range of  $f$  ( $\text{Rang}(f)$  or  $R_f$ ).

i.e.  $f: X \rightarrow Y$  be a function, then:

- The set  $X$  is called domain  $f$  ( $\text{Dom}(f) = D_f$ ).
- The set  $Y$  is called co-domain  $f$  ( $\text{co-Dom}(f)$ ).
- $f(x)$  is called range  $f$  ( $\text{Rang}(f) = R_f$ ).

□

# "Some kinds of Functions"

انواع مختلفة من الدوال

## 1. Constant Function دالة ثابتة

A function  $f: X \rightarrow Y$  is called constant if  $f(x) = c, \forall x \in X, c \in \mathbb{R}$  (set of real numbers).

For example:  $f(x) = 1, f(x) = -1, f(x) = 10, f(x) = -100, f(x) = 0$  (Zero function).

$D_f = \text{Dom}(f) = \mathbb{R} = (-\infty, \infty)$   
 $R_f = \text{Rang}(f) = \mathbb{C}$

## 2. Identity Function دالة هوية

A function  $f: X \rightarrow Y$  is called identity if  $f(x) = x, \forall x \in X$ .

$D_f = \mathbb{R} = (-\infty, \infty)$   
 $R_f = \mathbb{R} = (-\infty, \infty)$

## 3. Polynomial Function (مبتدأ) دالة كثيرة الحدود

A function  $f: X \rightarrow Y$  is called polynomial if  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$   
 $a_0, a_1, \dots, a_n \in \mathbb{R}$

$f(x)$  is polynomial of degree  $(n)$   
 for example,  $f(x) = 2x^3 + 5x^2 + 4x + \frac{1}{2}$  (Polynomial of degree 3)  
 $f(x) = x^7 + 5x^6 + 8x^4 + x + 11$  (Polynomial of degree 7).

$D_f = \mathbb{R}$  and  $R_f = \mathbb{R}$  (for polynomial function).

#### 4. Linear Function علاقة خطية

A function  $f: X \rightarrow Y$  is called linear if  
 $f(x) = ax + b$  s.t.  $a, b \in \mathbb{R}$  and  $a \neq 0$ .

for example:  $f(x) = x + 1$ ,  $f(x) = 5x + 3$ ,  $f(x) = 2x - 4$ ,  
 $f(x) = 3x$ ,  $f(x) = -x + 5$ .

$D_f = \mathbb{R}$  and  $R_f = \mathbb{R}$ .

#### 5. Quadratic Function علاقة تربيعية

A function  $f: X \rightarrow Y$  is called quadratic if  
 $f(x) = ax^2 + bx + c$ ,  $a, b, c \in \mathbb{R}$  and  $a \neq 0$ .

for example:  $f(x) = x^2$ ,  $f(x) = 5x^2 + 1$ ,  $f(x) = x^2 - 2x + 5$ ,  
 $f(x) = -4x^2 + 2x - 1$ ,  $f(x) = x^2 + 3x$ .

To find the domain and the range:

نجد المجال والمدى

$\text{Dom}(f) = D_f = \mathbb{R}$  (المجال التربيعية هو  $\mathbb{R}$ )

Case 1: إذا كان  $a > 0$  (مفتوح)

$$R_f \geq f\left(\frac{-b}{2a}\right)$$

Case 2: إذا كان  $a < 0$  (مغلق)

$$R_f \leq f\left(\frac{-b}{2a}\right)$$

Examples: Find the domain and the range  
 for the following functions.

①  $f(x) = x^2 + 2x + 1$

sol:

$$D_f = \mathbb{R} = (-\infty, \infty)$$

(Range) نجد المدى

$$a = 1 > 0 \text{ (مفتوح)}$$

$$\begin{aligned} f\left(\frac{-b}{2a}\right) &= f\left(\frac{-2}{2}\right) \\ &= f(-1) \\ &= (-1)^2 + 2(-1) + 1 \\ &= 0 \end{aligned}$$

$$\Rightarrow R_f \geq 0 \quad (R_f \geq f\left(\frac{-b}{2a}\right))$$

$$\therefore R_f = [0, \infty) = \mathbb{R}^+ \cup \{0\}$$

②  $f(x) = -x^2 + 2x + 1$

sol:

$$D_f = \mathbb{R} = (-\infty, \infty)$$

$$a = -1 < 0 \text{ (مغلق)}$$

$$\begin{aligned} f\left(\frac{-b}{2a}\right) &= f\left(\frac{-2}{-2}\right) \\ &= f(1) \\ &= -(1)^2 + 2 + 1 = 2 \end{aligned}$$

$$\Rightarrow R_f \leq 2 \quad (R_f \leq f\left(\frac{-b}{2a}\right))$$

$$\therefore R_f = (-\infty, 2]$$

$$③ f(x) = x^2$$

sol:

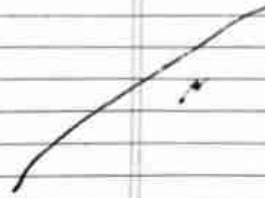
$$Df = \mathbb{R} = (-\infty, \infty)$$

$$Rf = [0, \infty) = \mathbb{R}^+ \cup \{0\}$$

$$④ f(x) = 5x^2 + 1$$

$$Df = \mathbb{R} = (-\infty, \infty)$$

$$Rf = [1, \infty)$$



### 6. Square Root Function دالة الجذر التربيعي

A function  $f: X \rightarrow Y$  is called square root if  
 $f(x) = \sqrt{x}$ ,  $x \in X$ .

for examples:  $f(x) = \sqrt{x}$ ,  $f(x) = \sqrt{x+9}$ ,  
 $f(x) = \sqrt{16-x^2}$ ,  $f(x) = \sqrt{x^2-3x}$ .

المجال (Range) والمجال (Domain) للجدول  
التربيعي:

Examples: Find the domain and the range for  
the following functions:

$$① f(x) = \sqrt{x}$$

sol:

$$x \geq 0 \Rightarrow Df = [0, \infty) = \mathbb{R}^+ \cup \{0\}$$

$$Rf = [0, \infty) = \mathbb{R}^+ \cup \{0\}$$

$$② f(x) = \sqrt{x+9}$$

sol:

$$x+9 \geq 0 \Rightarrow x \geq -9 \Rightarrow Df = [-9, \infty)$$

$$Rf = [0, \infty)$$

$$③ f(x) = \sqrt{x-3}$$

sol:

$$x-3 \geq 0 \Rightarrow x \geq 3 \Rightarrow Df = [3, \infty)$$

$$Rf = [0, \infty)$$

$$④ f(x) = \sqrt{16-x^2}$$

sol:

$$16-x^2 \geq 0$$

$$x^2 \leq 16$$

$$|x| \leq 4$$

$$-4 \leq x \leq 4$$

$$\therefore Df = [-4, 4]$$

$$Rf = [0, 4]$$

7. Rational Function الدالة الكسرية

A function  $f: X \rightarrow Y$  is called rational if

$$f(x) = \frac{g(x)}{h(x)} \text{ s.t. } h(x) \neq 0, \forall x \in X.$$

for example,  $f(x) = \frac{1}{x}, f(x) = \frac{7}{x-4},$

$$f(x) = \frac{1}{\sqrt{x}}, f(x) = \frac{x}{\sqrt{x+1}}.$$

لديهم بال دالة الكسرية :

المثال الأول : إذا كان البسط عددياً فإن دالة الدالة الكسرية هو جميع الأعداد الحقيقية ما عدا الأعداد التي تجعل المقام يساوي صفر.

المثال الثاني : إذا كان البسط دالة غير ثابتة (أي  $x^2$ ) فهو من الدوال المثلثية.

بال دالة الكسرية = حال البسط  $A$  بال المقام  $B$  ما عدا الأعداد التي تجعل المقام يساوي صفر

Examples: Find the domain and the range for the following functions:

①  $f(x) = \frac{1}{x}$

sol.

$$x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$$

$$R_f = \mathbb{R} - \{0\}.$$

②  $f(x) = \frac{7}{x+4}$

sol.

$$x+4 \neq 0 \Rightarrow x \neq -4 \Rightarrow D_f = \mathbb{R} - \{-4\}$$

$$R_f = \mathbb{R} - \{0\}.$$

③  $f(x) = \frac{1}{\sqrt{x}}$

sol.

$$x > 0 \Rightarrow D_f = (0, \infty) = \mathbb{R}^+$$

$$R_f = \mathbb{R} - \{0\}.$$

④  $f(x) = \frac{x^2-4}{x-2}$

sol.

$$D_f = \mathbb{R} \cap \mathbb{R} - \{2\} = \mathbb{R} - \{2\}$$

لديهم الدالة (Range)

$$f(x) = \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} = x+2$$

$$f(2) = 2+2 = 4$$

$$\therefore R_f = \mathbb{R} - \{4\}$$

8. Absolute value Function الدالة القيمة المطلقة

A function  $f: X \rightarrow Y$  is called absolute value if  $f(x) = |x|, \forall x \in X.$