"Functions"

الدوال

Definition: A function $f: X \to Y$ is a relationship between two sets (nonempty sets) such that associates each elements in X with exactly one element in Y.

i.e. $\forall x \in X, \exists y \in Y$ s.t. $f(x) = y$.

$$X \xrightarrow[\text{input (Domain)}]{} \boxed{f} \xrightarrow[\text{output (Range)}]{} Y$$

Definition (Domain): The set of all possible elements (input values) is called domain of f ($Dom(f)$ or D_f).

Definition (Range): The set of all corresponding elements in the second set (output values) is called range of f ($Rang(f)$ or R_f).

i.e. $f: X \to Y$ be a function, then:

The set X is called domain f ($Dom(f) = D_f$).
The set Y is called co-domain f ($Co\text{-}Dom(f)$),
$f(x)$ is called range f ($Rang(f) = R_f$).

"Some kinds of Functions"

بعض أنواع الدوال

1. Constant Function الدالة الثابتة

A function $f: X \to Y$ is called constant if $f(x) = c, \forall x \in X, c \in \mathbb{R}$ (set of real numbers).

for example: $f(x) = 1$, $f(x) = -1$, $f(x) = 10$, $f(x) = -100$, $f(x) = 0$ (Zero function).

$D_f = Dom(f) = \mathbb{R} = (-\infty, \infty)$
$R_f = Rang(f) = c$

2. Identity Function الدالة الذاتية

A function $f: X \to Y$ is called identity if $f(x) = x, \forall x \in X$.

$D_f = \mathbb{R} = (-\infty, \infty)$
$R_f = \mathbb{R} = (-\infty, \infty)$

3. Polynomial Function (متعددة الحدود) الدالة كثيرة الحدود

A function $f: X \to Y$ is called polynomial if
$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$
$a \neq a_n, a_{n-1}, \dots, a_1, a_0 \in \mathbb{R}$
$f(x)$ is polynomial of degree (n)

for example: $f(x) = 2x^3 + 5x^2 + 4x + \frac{1}{2}$ (Polynomial of degree 3)
$f(x) = x^7 + 5x^6 + 8x^4 + x + 12$ (Polynomial of degree 7).

$D_f = \mathbb{R}$ and $R_f = \mathbb{R}$ (for polynomial function).

4. Linear Function الدالة الخطية

A function $f: X \longrightarrow Y$ is called linear if
$f(x) = ax + b$ s.t. $a, b \in \mathbb{R}$ and $a \neq 0$.

for example: $f(x) = x+1$, $f(x) = 5x+3$, $f(x) = -2x-4$,
$f(x) = 3x$, $f(x) = -x+5$.

$D_f = \mathbb{R}$ and $R_f = \mathbb{R}$.

5. Quadratic Function الدالة التربيعية

A function $f: X \longrightarrow Y$ is called quadratic if
$f(x) = ax^2 + bx + c$, $a, b, c \in \mathbb{R}$ and $a \neq 0$.

for example: $f(x) = x^2$, $f(x) = 5x^2+1$, $f(x) = x^2-2x+5$,
$f(x) = -4x^2+2x-1$, $f(x) = x^2+3x$.

To find the domain and the range:
لايجاد مجال ومدى الدالة التربيعية

$Dom(f) = D_f = \mathbb{R}$ (مجال الدالة التربيعية هو مجموعة الاعداد الحقيقية $\mathbb{R}$)

لايجاد المدى توجد حالتان:

case 1:
If $a > 0$ (موجب)

$R_f \geqslant f\left(\frac{-b}{2a}\right)$

case 2: If $a < 0$ (سالب)

$R_f \leqslant f\left(\frac{-b}{2a}\right)$

Examples: Find the domain and the range for the following functions.

① $f(x) = x^2 + 2x + 1$

sol.

$D_f = \mathbb{R} = (-\infty, \infty)$

لايجاد المدى (Range)

$a = 1 > 0$ (موجب)

$f\left(\frac{-b}{2a}\right) = f\left(\frac{-2}{2}\right)$
$= f(-1)$
$= (-1)^2 + 2(-1) + 1$
$= 0$

$\Rightarrow R_f \geqslant 0$ $\left(R_f \geqslant f\left(\frac{-b}{2a}\right)\right)$

$\therefore R_f = [0, \infty) = \mathbb{R}^+ \cup \{0\}$.

② $f(x) = -x^2 + 2x + 1$

sol.

$D_f = \mathbb{R} = (-\infty, \infty)$

$a = -1 < 0$ (سالب)

$f\left(\frac{-b}{2a}\right) = f\left(\frac{-2}{-2}\right)$
$= f(1)$
$= -(1)^2 + 2 + 1 = 2$

$\Rightarrow R_f \leqslant 2$ $\left(R_f \leqslant f\left(\frac{-b}{2a}\right)\right)$

$\therefore R_f = (-\infty, 2]$.

③ $f(x) = x^2$

Sol:

$D_f = \mathbb{R} = (-\infty, \infty)$

$R_f = [0, \infty) = \mathbb{R}^+ \cup \{0\}$.

④ $f(x) = 5x^2 + 1$

$D_f = \mathbb{R} = (-\infty, \infty)$

$R_f = [1, \infty)$.

6. Square Root Function   دالة الجذر التربيعي

A function $f : X \to Y$ is called square root if $f(x) = \sqrt{x}$, $x \in X$.

for examples: $f(x) = \sqrt{2x}$, $f(x) = \sqrt{x+9}$,
$f(x) = \sqrt{16 - x^2}$, $f(x) = \sqrt{x^2 - 3x}$.

لايجاد المجال (Domain) و المدى (Range) لدالة الجذر التربيعي :

Examples: Find the domain and the range for the following functions.

① $f(x) = \sqrt{x}$

Sol:

$x \geq 0 \Rightarrow D_f = [0, \infty) = \mathbb{R}^+ \cup \{0\}$

$R_f = [0, \infty) = \mathbb{R}^+ \cup \{0\}$.

② $f(x) = \sqrt{x+9}$

Sol:

$x + 9 \geq 0 \Rightarrow x \geq -9 \Rightarrow D_f = [-9, \infty)$.

$R_f = [0, \infty)$.

③ $f(x) = \sqrt{x-3}$

Sol:

$x - 3 \geq 0 \Rightarrow x \geq 3 \Rightarrow D_f = [3, \infty)$

$R_f = [0, \infty)$

④ $f(x) = \sqrt{16 - x^2}$

Sol:

$16 - x^2 \geq 0$

$x^2 \leq 16$

$|x| \leq 4$

$-4 \leq x \leq 4$

$\therefore D_f = [-4, 4]$

$R_f = [0, 4]$.

7. Rational Function الدالة الكسرية

A function $f: X \longrightarrow Y$ is called rational if

$$f(x) = \frac{g(x)}{h(x)} \quad s.t. \quad h(x) \neq 0, \ \forall x \in X.$$

for example: $f(x) = \frac{1}{x}$, $f(x) = \frac{7}{x-4}$,

$$f(x) = \frac{1}{\sqrt{x}}, \quad f(x) = \frac{x}{\sqrt{x+1}}.$$

لايجاد مجال الدالة الكسرية:

الحالة الأولى: اذا كان البسط عددا ثابت فان مجال الدالة الكسرية هو جميع الأعداد الحقيقية ماعدا الأعداد التي تجعل المقام يساوي صفر.

الحالة الثانية: اذا كان البسط دالة غير ثابتة (أي نوع من الدوال السابقة)

مجال الدالة الكسرية = مجال البسط $\cap$ مجال المقام ماعدا الأعداد التي تجعل المقام يساوي صفر

Examples: Find the domain and the range for the following functions.

① $f(x) = \frac{1}{x}$

sol.

$x = 0 \Rightarrow D_f = \mathbb{R} - \{0\}$

$R_f = \mathbb{R} - \{0\}.$

② $f(x) = \frac{7}{x+4}$

sol.

$x + 4 = 0 \Rightarrow x = -4 \Rightarrow D_f = \mathbb{R} - \{-4\}$

$R_f = \mathbb{R} - \{0\}.$

③ $f(x) = \frac{1}{\sqrt{x}}$

sol. $x > 0 \Rightarrow D_f = (0, \infty) = \mathbb{R}^+$

$R_f = \mathbb{R} - \{0\}.$

④ $f(x) = \frac{x^2 - 4}{x - 2}$

sol.

$D_f = \mathbb{R} \cap \mathbb{R} - \{2\} = \mathbb{R} - \{2\}$

لايجاد المدى (Range):

$$f(x) = \frac{x^2-4}{x-2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = x + 2$$

$f(2) = 2 + 2 = 4$

$\therefore R_f = \mathbb{R} - \{4\}$

8. Absolute Value Function دالة القيمة المطلقة

A function $f: X \to Y$ is called absolute value if $f(x) = |x|, \ \forall x \in X.$