$$(g \circ f)(x) = g(f(x))$$
$$= g(x^2+2)$$
$$= \sqrt{x^2+2-4}$$
$$= \sqrt{x^2-2}$$

$(f \circ g)(-2) = -2-2 = -4$.
$(g \circ f)(2) = \sqrt{(2)^2-2} = \sqrt{2}$.

H.w.

① Let $f(x) = |x|$, $g(x) = 3x-5$. Find $(f \circ g)(x)$, $(f \circ g)(\frac{1}{3})$, $(g \circ f)(x)$, $(g \circ f)(\frac{-1}{3})$.

② Let $f(x) = 2x+1$, $g(x) = 2x^2+3x$. Find $(f \circ g)(x)$, $(f \circ g)(-1)$, $(g \circ f)(x)$, $(g \circ f)(1)$.

"Graphs of Functions"

رسم الدوال

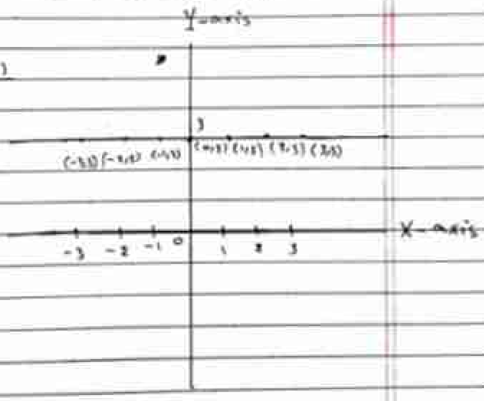
If f is a function with domain (D_f), the graph of f consists of all ordered pairs (x, y) where $y = f(x)$ in the cartesian plan (X-axis and Y-axis).

i.e. $f: X \to Y$, with Domain $= D_f$, the graph of f is $\{(x,y) : x \in X, y = f(x) \in Y\}$.

Examples: Draw (Graph) the following functions.

① $f(x) = 3$

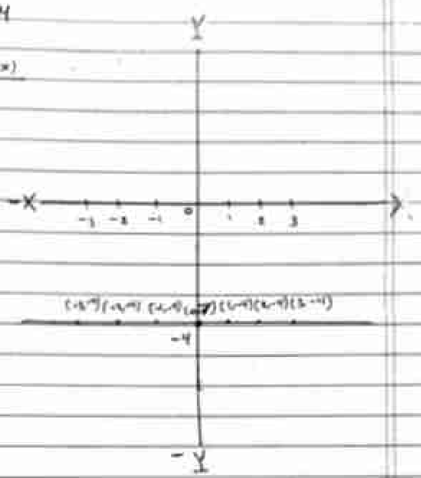
x	$y = f(x)$
⋮	⋮
-3	3
-2	3
-1	3
0	3
1	3
2	3
3	3
⋮	⋮



$Dom(f) = \mathbb{R}$
$Rang(f) = 3$

② $f(x) = -4$

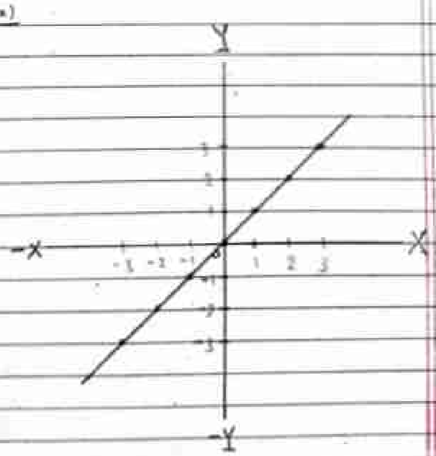
x	$y = f(x)$
⋮	⋮
-3	-4
-2	-4
-1	-4
0	-4
1	-4
2	-4
3	-4
⋮	⋮



$D_f = \mathbb{R}$
$R_f = -4$

③ $f(x) = x$

x	$y = f(x)$
⋮	⋮
-3	-3
-2	-2
-1	-1
0	0
1	1
2	2
3	3
⋮	⋮

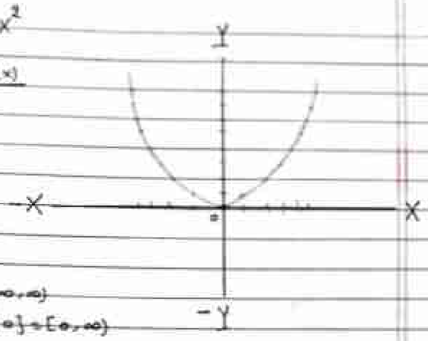


0·/10

$D_f = \mathbb{R}$
$R_f = \mathbb{R}$

④ $f(x) = x^2$

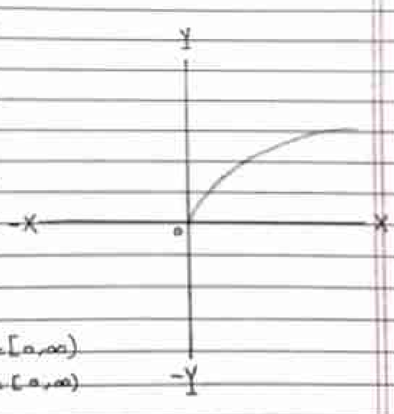
x	$y = f(x)$
⋮	⋮
-2	4
-1	1
0	0
1	1
2	4
⋮	⋮



$D_f = \mathbb{R} = (-\infty, \infty)$
$R = \mathbb{R}^+ \cup \{0\} = [0, \infty)$

⑤ $f(x) = \sqrt{x}$

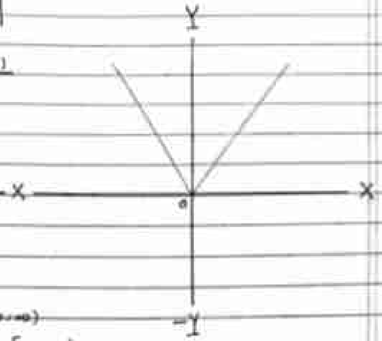
x	$y = f(x)$
0	$\sqrt{0} = 0$
1	$\sqrt{1} = 1$
2	$\sqrt{2} = 1.4$
3	$\sqrt{3} = 1.7$
4	$\sqrt{4} = 2$
⋮	⋮



$D_f = \mathbb{R}^+ \cup \{0\} = [0, \infty)$
$R_f = \mathbb{R}^+ \cup \{0\} = [0, \infty)$

⑥ $f(x) = |x|$

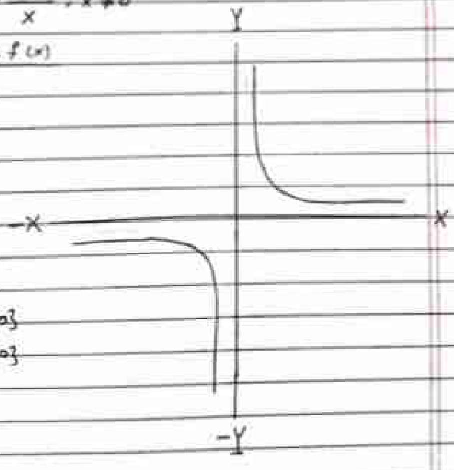
x	$y = f(x)$
⋮	⋮
-2	2
-1	1
0	0
1	1
2	2
⋮	⋮



$D_f = \mathbb{R} = (-\infty, \infty)$
$R_f = \mathbb{R}^+ \cup \{0\} = [0, \infty)$

⑦ $f(x) = \frac{1}{x}$, $x \neq 0$

x	$y = f(x)$
⋮	⋮
-2	$\frac{-1}{2}$
-1	-1
1	1
2	$\frac{1}{2}$
⋮	⋮



$D_f = \mathbb{R} - \{0\}$
$R_f = \mathbb{R} - \{0\}$

"Shifting (Moving) Graph of Functions"
تزحيف رسم الدالة

Let $f: \mathbb{R} \to \mathbb{R}$ be a function, $c > 0$, then:

1. $y = f(x) + c$, the graph shift (up) c units
2. $y = f(x) - c$, the graph shift (down) c units
3. $y = f(x + c)$, the graph shift to the (left) c units
4. $y = f(x - c)$, the graph shift to the (right) c units.

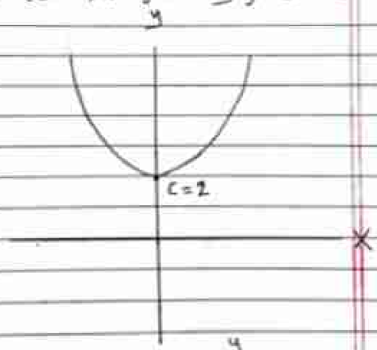
Examples: Graph (Draw) the following functions:

① $y = x^2 + 2$

sol.

$y = x^2 + 2$

i.e. $y = f(x) + c$

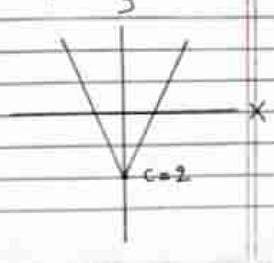


② $y = |x| - 2$

sol.

$y = |x| - 2$

i.e. $y = f(x) - c$

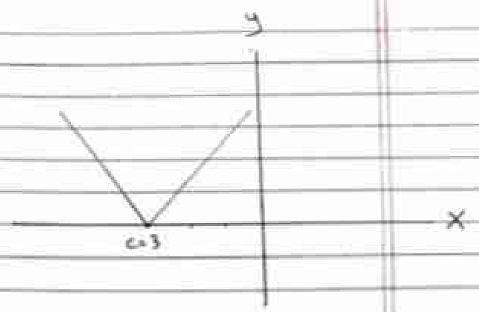


③ $y = |x+3|$

sol.

$y = |x+3|$

i.e. $y = f(x+c)$

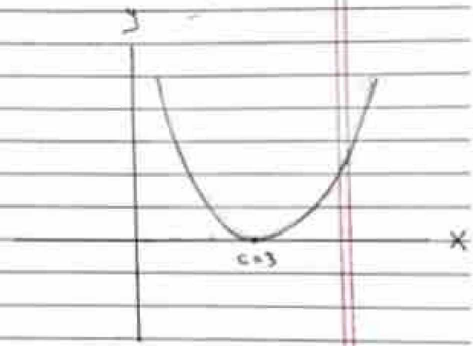


④ $y = (x-3)^2$

sol.

$y = (x-3)^2$

i.e. $y = f(x-c)$



Hw. Graph the following functions.

1. $f(x) = \frac{-1}{x}$
2. $y = \sqrt{x} + 2$
3. $y = |x| - 1$
4. $y = |x-2|$
5. $y = (x+2)^2$.

"Limits of Functions"

غايات الدوال

Definition: Let $f: X \to Y$ be a function, then

$$\lim_{x \to a} f(x) = L$$

is read (the limit of $f(x)$ as x approaches to a).
It means $f(x)$ gets closer and closer to L as x gets closer and closer to a.

Properties of Limits: خواص الغايات

1. $\lim_{x \to a} c = c$
2. $\lim_{x \to a} x = a$
3. $\lim_{x \to a} (c\, f(x)) = c \lim_{x \to a} f(x)$, $c \in \mathbb{R}$
4. $\lim_{x \to a} (f(x) \mp g(x)) = \lim_{x \to a} f(x) \mp \lim_{x \to a} g(x)$
5. $\lim_{x \to a} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, $\lim_{x \to a} g(x) \neq 0$
6. $\lim_{x \to a} (f(x) \cdot g(x)) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
7. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$, n is any positive integer number and $\lim_{x \to a} f(x) \geq 0$.

Remarks:

1. $\lim f(x) = \frac{0}{[illegible]}$ impossible (undefined)