University of Baghda
College of Engineering
Civil Engineering Department

Title: Lecture 5: Secondary Consolidation Settlement & Time Rate of Consolidation

Subject: Soil Mechanics

Year: Third

Semester: 2

Speaker: Prof. Dr. Nesreen Kurdy Al-Obaidy

References:

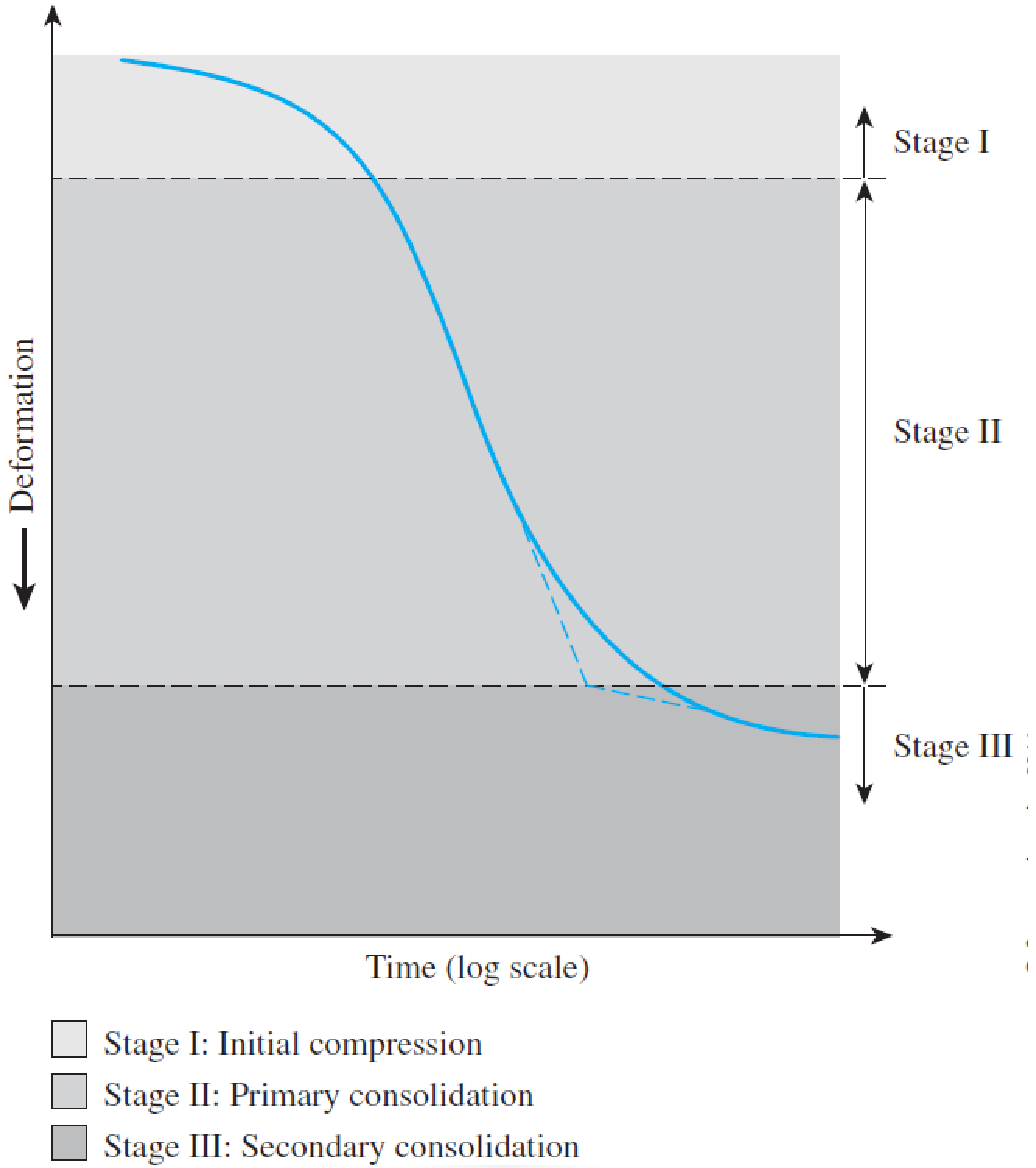
Principles of Geotechnical Engineering, Textbook by Das,2010

Fundamentals-of-Geotechnical-Engineering-Third-Edition, Textbook by Das

Secondary Consolidation Settlement (Secondary Compression)

at the end of **primary consolidation** (that is, after complete dissipation of excess pore water pressure) some settlement is observed because of **the plastic adjustment of soil fabrics**. This stage of consolidation is called ***secondary consolidation.***

During secondary consolidation the **plot of deformation against the log of time** is practically **linear**



Variation of e with log t under a given load increment and definition of secondary consolidation index

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log(\frac{t_2}{t_1})}$$

where C_α: secondary compression index

Δe :change of void ratio t: time

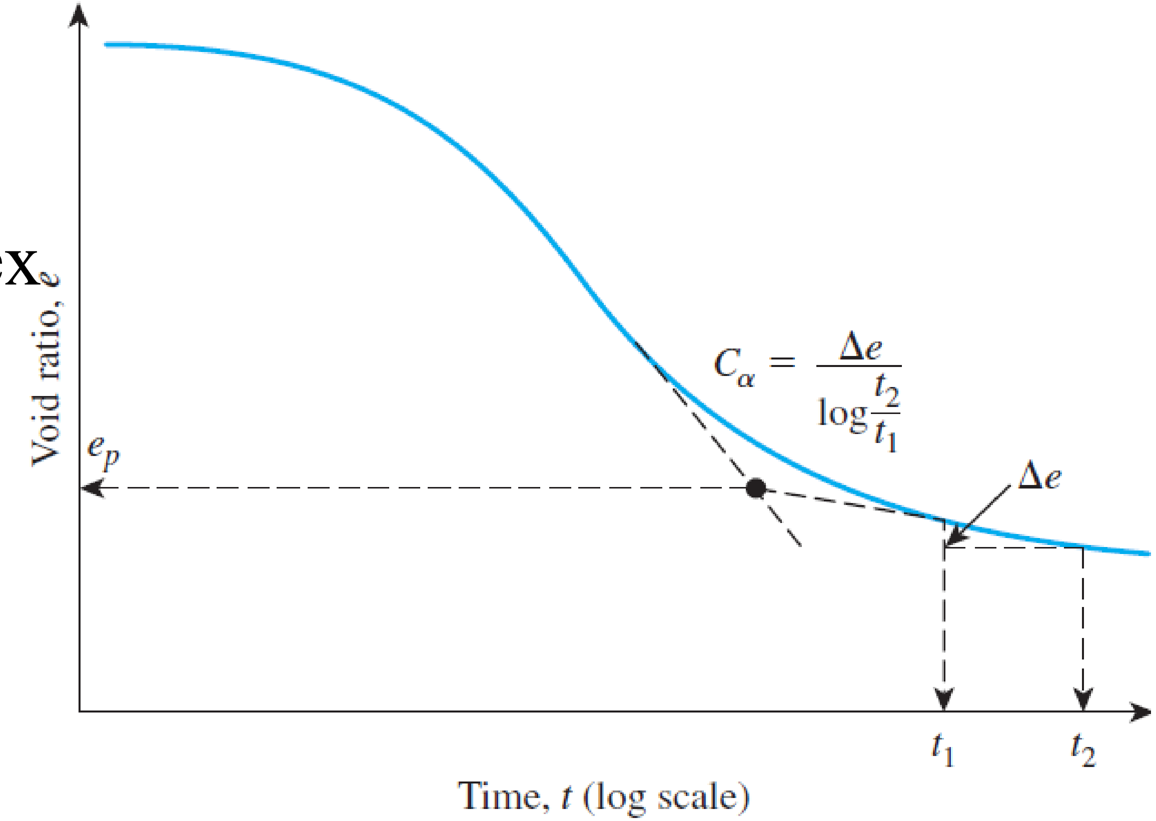
The secondary consolidation can be calculated as:

$$S_s = C_\alpha^{`} H \log(\frac{t_2}{t_1})$$

$$C_\alpha^{`} = \frac{C_\alpha}{1 + e_p}$$

where e_p: void ratio at the end of primary consolidation

H: thickness of clay layer



$C_{\alpha}^{`} \leq 0.001$ for OCC

$C_{\alpha}^{`} = 0.005\ to\ 0.03$ for NCC

$C_{\alpha}^{`} \geq 0.04$ for Organic soil

Mersri and Godlewski (1977) compiled the ratio

$C_{\alpha}^{`}/C_{\alpha} \approx 0.04 \pm 0.01\ for\ inorganic\ clays\ and\ silts$

$C_{\alpha}^{`}/C_{\alpha} \approx 0.05 \pm 0.01\ for\ organic\ clays\ and\ silts$

$C_{\alpha}^{`}/C_{\alpha} \approx 0.075 \pm 0.01\ for\ peat$

Important Note

Secondary consolidation settlement **is more important** than primary consolidation in **organic** and **highly compressible inorganic** soils. In overconsolidated inorganic clays, the secondary compression index is very small and of less practical significance.

Example

For a normally consolidated clay layer in the field, the following values are given: thickness of clay layer 2.6 m,void ratio (e_o) is 0.8, Compression index (c_c) is 0.28, Average effective pressure on the clay layer $\sigma_o^{`}$ =127 kN/m^2, $\Delta\sigma'$= 46.5 kN/m^2, Secondary compression index (C_α) is 0.02 What is the total consolidation settlement of the clay layer five years after the completion of primary consolidation settlement? (*Note:* Time for completion of primary settlement 1.5 years.)

$$C_\alpha^{`} = \frac{C_\alpha}{1+e_p} \qquad e_p = e_o - \Delta e \qquad \text{Compare } S_c = \frac{c_c H}{1+e_o}\log\frac{\sigma_o^{`}+\Delta\sigma^{`}}{\sigma_o^{`}} \text{ with } S_c = \frac{\Delta e}{1+e_o}H$$

$$\Delta e = c_c \log\frac{\sigma_o^{`}+\Delta\sigma^{`}}{\sigma_o^{`}} \qquad \Delta e = 0.28\log\left(\frac{127+46.5}{127}\right)=0.038 \qquad e_p = 0.8-0.038=0.762$$

$$C_\alpha^{`} = \frac{0.02}{1+0.762}=0.011 \qquad S_c = \frac{\Delta e}{1+e_o}H \qquad Sc = \frac{0.038}{1+0.8}(2.6*1000) = 54.9 \text{ mm}$$

$$S_s = C_\alpha^{`} H \log\left(\frac{t_2}{t_1}\right) \qquad S_s = 0.011(2.6*1000).\log\left(\frac{5}{1.5}\right) = 14.95 \text{ mm}$$

$$S_t = Sc+S_s = 54.9+14.95=69.85\text{mm}$$

H.W

For a normally consolidated clay layer in the field, the following values are given:

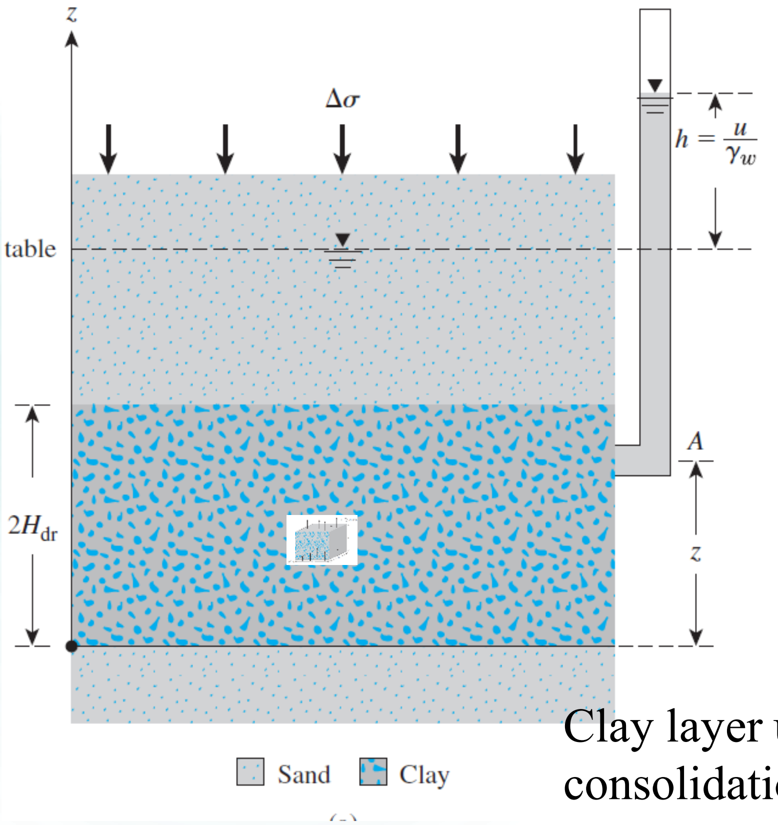
- Thickness of clay layer 3 m
- Void ratio e_o = 0.8
- Compression index Cc= 0.28
- Average effective pressure on the clay layer $\sigma_o^{`}$ = 130 kN/m2
- $\Delta\sigma'$ = 50 kN/m^2
- Secondary compression index C_α= 0.02

What is the total consolidation settlement of the clay layer five years after the completion of primary consolidation settlement? (Note: Time for completion of primary settlement = 1.5 years.)

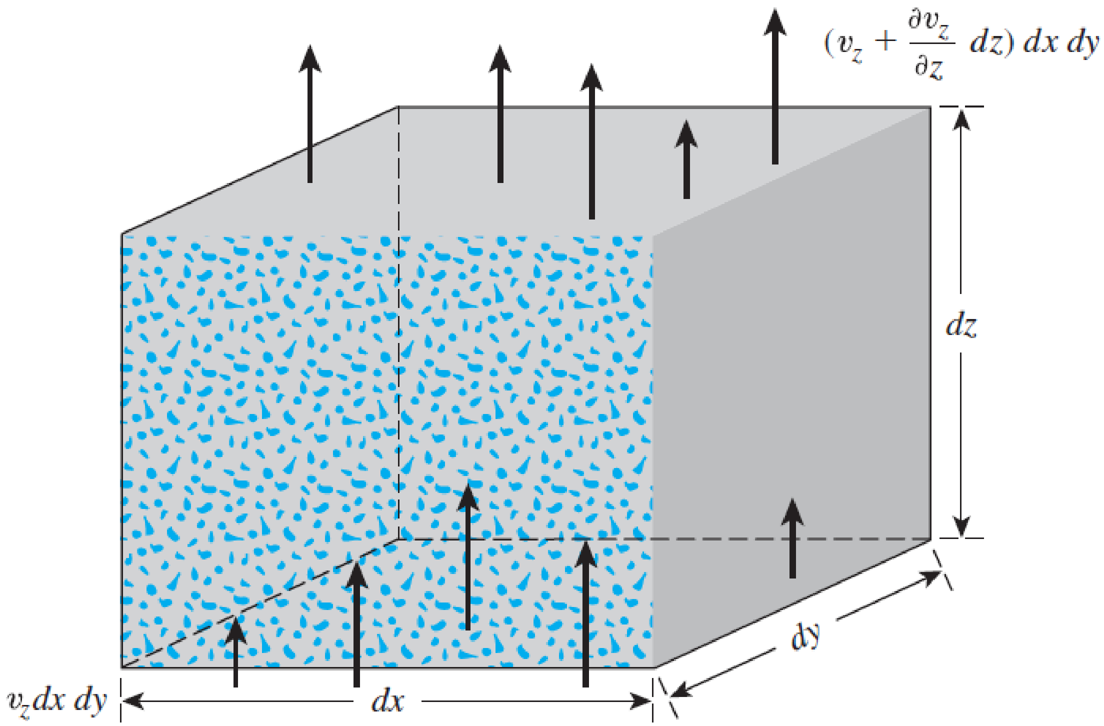
Ans. Total consolidation settlement = 84 mm

Time Rate of Consolidation

Terzaghi (1925) proposed the first theory to consider the rate of one-dimensional consolidation for saturated clay soils. The mathematical derivations are based on the assumptions which are discussed in consolidation section (lecture 2)(also see Taylor, 1948):



Clay layer undergoing consolidation



ion

Rate of outflow of water - Rate of inflow of water = Rate of volume change

$$\left(v_z+\frac{\partial v_z}{\partial z}dz\right)dxdy-v_z\,dxdy=\frac{\partial V}{\partial t}$$

$$\frac{\partial v_z}{\partial z}dzdxdy=\frac{\partial V}{\partial t}$$

where V volume of the soil element
v_z velocity of flow in z direction

$$\frac{\partial v_z}{\partial z}=\frac{1}{dxdydz}\frac{\partial V}{\partial t}$$

Using Darcy's law, we have

$$v_z=ki=-k\frac{\partial h}{\partial z}=-\frac{k}{\gamma_w}\frac{\partial u}{\partial z}$$

where u excess pore water pressure caused by the increase of stress.

$$\frac{\partial v_z}{\partial z}=-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2}$$

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2}=\frac{1}{dxdydz}\frac{\partial V}{\partial t}$$

During consolidation, the rate of change in the volume of the soil element is equal to the rate of change in the volume of voids. Thus,

$$\frac{\partial V}{\partial t} = \frac{\partial V_V}{\partial t} = \frac{\partial (V_S + eV_S)}{\partial t} = \frac{\partial V_s}{\partial t} + e\frac{\partial V_s}{\partial t} + V_S\frac{\partial e}{\partial t}$$

where Vs volume of soil solids
Vv volume of voids
But (assuming that soil solids are incompressible)

$$\frac{\partial V_s}{\partial t} = 0$$

$$V_s = \frac{V}{1+e_0} = \frac{dxdydz}{1+e_0}$$

$$\frac{\partial V}{\partial t} = \frac{dxdydz}{1+e_0}\frac{\partial e}{\partial t}$$

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{dxdydz}\frac{\partial V}{\partial t}$$

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{dxdydz}\frac{dxdydz}{1+e_0}\frac{\partial e}{\partial t}$$

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{1+e_0}\frac{\partial e}{\partial t}$$

The change in the void ratio is caused by the increase of effective stress (i.e., a decrease of excess pore water pressure). Assuming that they are related linearly, we have

$$\partial e = a_v \partial \Delta\sigma^{`} = -a_v \partial u$$

where $\Delta\sigma^{`}$ change in effective pressure av coefficient of compressibility (av can be considered constant for a narrow range of pressure increase)

$$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{1+e_0}\frac{\partial e}{\partial t} \qquad -\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = -\frac{a_v}{1+e_0}\frac{\partial u}{\partial t} \qquad \frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{a_v}{1+e_0}\frac{\partial u}{\partial t}$$

mv coefficient of volume compressibility

$$\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = m_v \frac{\partial u}{\partial t}$$

$$or\ \frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} \qquad eq1$$

cv cofficient of consolidation

$$C_v = \frac{k}{\gamma_w m_v} = \frac{k}{\gamma_w\left(\frac{a_v}{1+e_0}\right)}$$

Equation (1) is the basic differential equation of Terzaghi's consolidation theory and can be solved with the following boundary conditions:

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} \qquad eq1$$

Equation (1) is the basic differential equation of Terzaghi's consolidation theory and can be solved with the following boundary conditions:

$z = 0, u = 0$

$z = 2H_{dr}, u = 0$

$t = 0, u = u_0$

u_0: initial excess pore water pressure

$$u = \sum_{m=0}^{m=\infty} \left[\frac{2u_o}{M} sin\left(\frac{Mz}{H_{dr}}\right)\right] e^{-M^2 T_v} \qquad \text{eq2}$$

$M = (\pi/2)(2m + 1)$

$m = an\ integer$

The solution yields

$$T_v = \frac{C_v t}{H_{dr}^2} = time\ factor$$

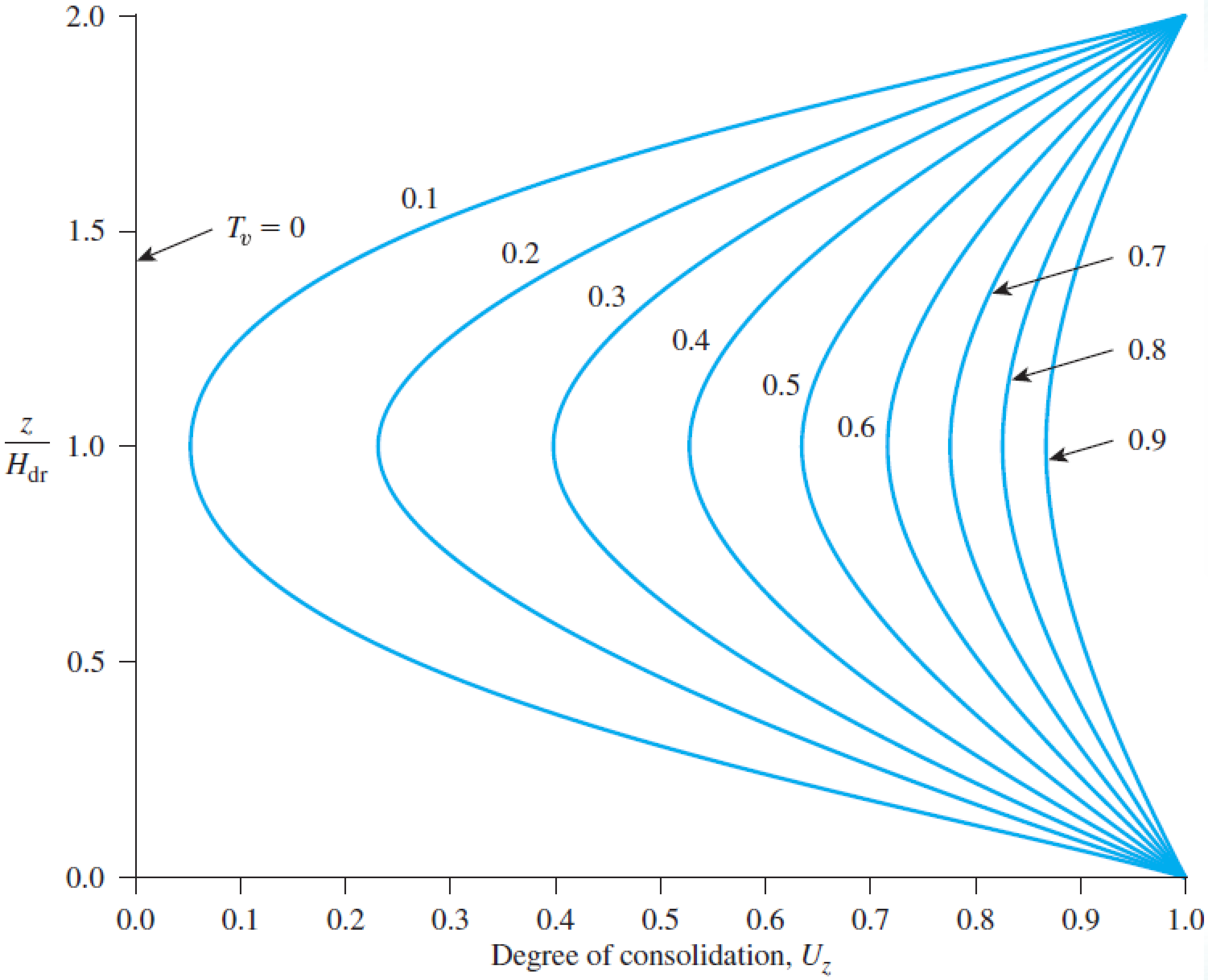
The time factor is a nondimensional number.

Because consolidation progresses by the dissipation of excess pore water pressure, the degree of consolidation at a distance *z* at any time *t* is

$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o} \qquad eq3$$

where u_z excess pore water pressure at time *t*.

Equations (2) and (3) can be combined to obtain the degree of consolidation at any depth *z*.


2.0
1.5
1.0
0.5
0.0
z / H_dr
T_v = 0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Degree of consolidation, U_z

The average degree of consolidation for the entire depth of the clay layer at any time *t* can be written from Eq. (3) as

$$\mathrm{U} = \frac{S_{c(t)}}{S_c} = 1 - \frac{\left(\frac{1}{2H_{dr}}\right)\int_0^{2H_{dr}} u_z dz}{u_o}$$

U: average degree of consolidation

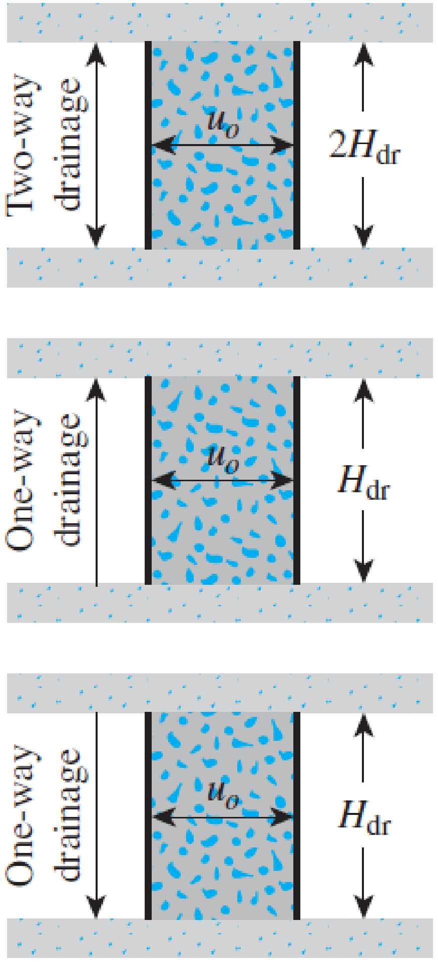
$S_{c(t)}$: settlement of the layer at time t

S_c:ultimate settlement of the layer from primary consolidation

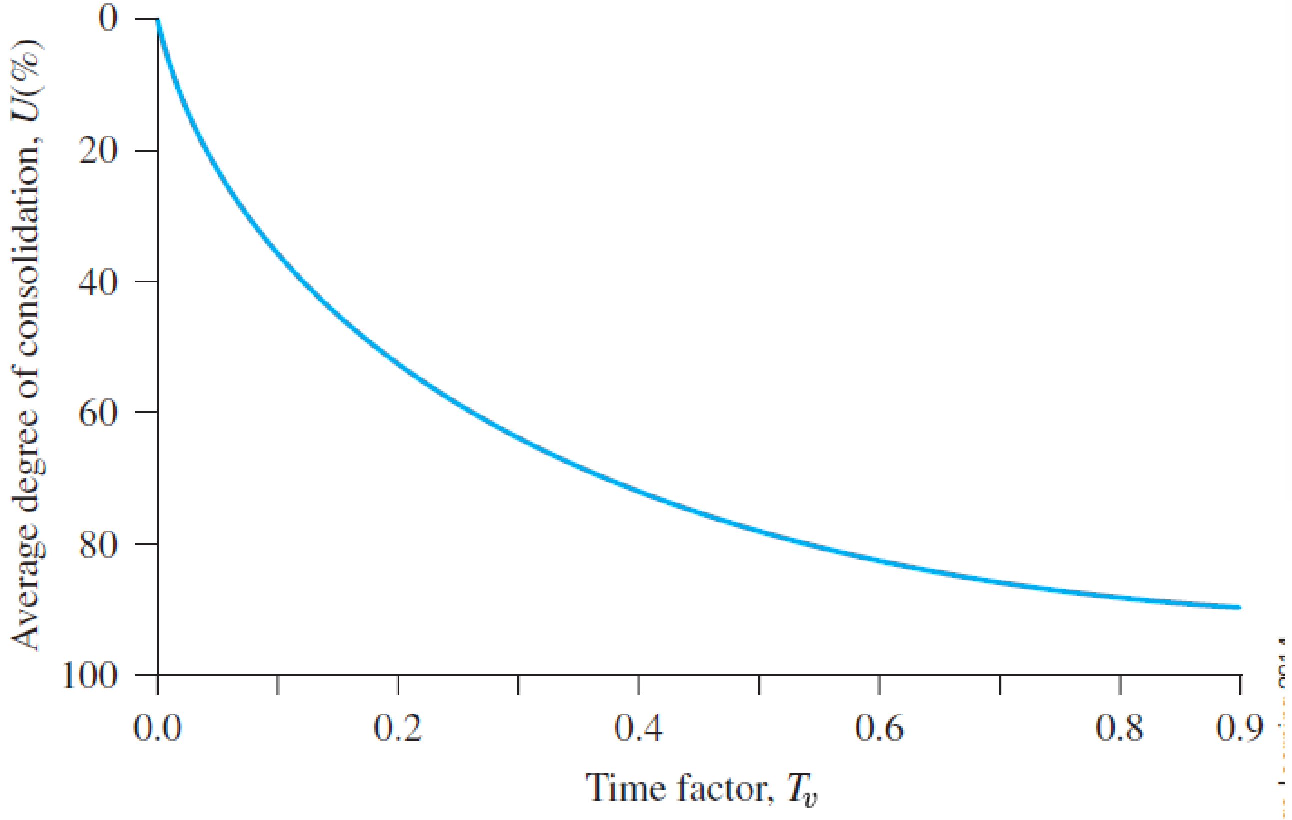
Substitution of the expression for excess pore water pressure u_z

$$U = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} e^{-M^2 T_v} \qquad \text{eq2}$$

The variation in the average degree of consolidation with the non dimensional time factor, *T*v ,



Different types of drainage with u_o constant



The values of the time factor and their corresponding average degrees of consolidation may also be approximated by the following simple relationship:

$$For\ U = 0\ to\ 60\%, T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$$

$$For\ U > 60\%, T_v = 1.781 - 0.933\log(100 - U\%$$

Sivaram and Swamee (1977

$$\frac{U\%}{100} = \frac{(4T_v/\pi)^{0.5}}{[1-(4T_v/\pi)^{2.8}]^{0.179}}$$

$$T_v = \frac{(\pi/4)(U\%/100)^2}{[1-(U\%/100)^{5.6}]^{0.357}}$$

Equations above give an error in T_v of less than 1% for U =0% -90% and less than 3% for U =90% -100%.

U (%)	T_v	U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	26	0.0531	52	0.212	78	0.529
1	0.00008	27	0.0572	53	0.221	79	0.547
2	0.0003	28	0.0615	54	0.230	80	0.567
3	0.00071	29	0.0660	55	0.239	81	0.588
4	0.00126	30	0.0707	56	0.248	82	0.610
5	0.00196	31	0.0754	57	0.257	83	0.633
6	0.00283	32	0.0803	58	0.267	84	0.658
7	0.00385	33	0.0855	59	0.276	85	0.684
8	0.00502	34	0.0907	60	0.286	86	0.712
9	0.00636	35	0.0962	61	0.297	87	0.742
10	0.00785	36	0.102	62	0.307	88	0.774
11	0.0095	37	0.107	63	0.318	89	0.809
12	0.0113	38	0.113	64	0.329	90	0.848
13	0.0133	39	0.119	65	0.340	91	0.891
14	0.0154	40	0.126	66	0.352	92	0.938
15	0.0177	41	0.132	67	0.364	93	0.993
16	0.0201	42	0.138	68	0.377	94	1.055
17	0.0227	43	0.145	69	0.390	95	1.129
18	0.0254	44	0.152	70	0.403	96	1.219
19	0.0283	45	0.159	71	0.417	97	1.336
20	0.0314	46	0.166	72	0.431	98	1.500
21	0.0346	47	0.173	73	0.446	99	1.781
22	0.0380	48	0.181	74	0.461	100	∞
23	0.0415	49	0.188	75	0.477		
24	0.0452	50	0.197	76	0.493		
25	0.0491	51	0.204	77	0.511		

Example

The time required for 50% consolidation of a 25 mm thick clay layer (drained at both top and bottom) in the laboratory is 3 min 15 sec. How long (in days) will it take for a 2m thick clay layer of the same clay in the field under the same pressure increment to reach 50% consolidation? In the field, sand layers are present at the top and bottom of the clay layer.

Solution

$$T_v = \frac{C_v t}{H_{dr}^2} \qquad T_{50} = \frac{C_v t_{lab}}{H_{dr(lab)}^2} = \frac{C_v t_{field}}{H_{dr(field)}^2} \qquad \frac{t_{lab}}{H_{dr(lab)}^2} = \frac{t_{field}}{H_{dr(field)}^2}$$

$$\frac{195\ sec}{\left(\frac{0.025\text{ m}}{2}\right)^2} = \frac{t_{field}}{\left(\frac{2\text{ m}}{2}\right)^2}$$

t_{field}=1,248,000 sec= **14.44 days**

Example

Refer to previous Example. How long (in days) will it take in the field for 30% primary consolidation to occur? Use Eq.

Solution

$$For\ U = 0\ to\ 60\%, T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$$

$U = 50$ $\quad t_{field}$=**14.44 days**=t_1

$U = 30$ $\quad t_{field}$=**?**=t_2

$$T_v = \frac{C_v t_{field}}{H^2_{dr(field)}} \qquad T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$$

$$t_{field} \propto U^2$$

$$\frac{C_v t_{field}}{H^2_{dr(field)}} = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$$

$$\frac{t_1}{t_2} = \frac{U_1^2}{U_2^2} \qquad \frac{14.44}{t_2} = \frac{50^2}{30^2} \qquad \boldsymbol{t_2 = 5.2\ \mathbf{days}}$$

H.W For a particular loading Condition unsaturated clay layer undergoes 30% consolidation in a period of 180 days. What would be the **additional time** required for **further 20%** consolidation to occur

Example

A 3m thick layer (double drainage) of saturated clay under a surcharge loading underwent 90% primary consolidation in 75 days. Find the coefficient of consolidation of clay for the pressure range.

Solution

$$T_v = \frac{C_v t_{field}}{H^2_{dr(field)}} \qquad T_{90} = \frac{C_v t_{90}}{H^2_{dr(field)}}$$

T_{90}= 0.848 from the table

Because the clay layer has two-way drainage H_{dr}=3/2=1.5 m

$$0.848 = \frac{C_v(75 * 24 * 60 * 60)}{150^2}$$

C_v=**0.00294 cm²/sec**

Example

For a normally consolidated laboratory clay specimen drained on both sides, the following are given: $\sigma_o^{`} = 150$ kN/m$^2$ with $e = e_o$=1.1, $\sigma_o^{`} + \Delta\sigma^{`}$= 300 kN/m^2 with e=0.9, Thickness of clay specimen =25 mm, Time for 50% consolidation = 2 min

a. Determine the hydraulic conductivity (cm/min) of the clay for the loading range.
b. How long (in days) will it take for a 1.8 m clay layer in the field (drained on one side) to reach 60% consolidation?

Solution

Part a

$m_v = \frac{a_v}{1+e_{av.}}$ $where\ m_v$= coefficient of volume compressibility; a_v= coefficient of compressibility of soil a_v= $\frac{\Delta e}{\Delta \sigma^{`}}$ $\Delta e$=1.1-0.9=0.2, $\Delta\sigma^{`}$=300-150= 150 kN/m^2

$$e_{av.} = \frac{1.1 + 0.9}{2} = 1.0 \qquad m_v = \frac{\frac{0.2}{150}}{1 + 1.0} = \mathbf{6.67 * 10^{-4}}\, m^2/kN$$

$$T_v = \frac{C_v t}{H_{dr}^2}$$

From Table, for U = 50%, T_v =0.197

$$C_v = \frac{T_v H_{dr}^2}{t} = \frac{0.197\left(\frac{25}{2*1000}\right)^2}{2} = 1.54*10^{-5}\, m^2/min$$

$$k = m_v C_v \gamma_w$$

$$k = \mathbf{6.67*10^{-4}}\frac{m^2}{kN} * 1.54*10^{-5}\frac{m^2}{min} * 9.81\frac{kN}{m^3}$$

$$k = \mathbf{100.77*10^{-9}}\, m/\text{min}\ \mathbf{100.77*10^{-7}}\, cm/\text{min}$$

Part b

$$T_v = \frac{C_v t}{H_{dr}^2} \qquad t = \frac{T_v H_{dr}^2}{C_v}$$

Because the clay layer has one-way drainage H_{dr}=1.8 m

From Table, for U = 60%, T_v =0.286

$$t = \frac{0.286(1.8)^2}{1.54*10^{-5}} = 60{,}171 \text{ min} = 1003 \text{ hrs} = 41.77 \text{ days}$$

H.W

A laboratory consolidation test on a soil specimen (drained on both sides) determined the following results:

- thickness of the clay specimen 25 mm
- $\sigma_1^{`} = 50$ kN/m^2 , e_1=0.92
- $\sigma_2^{`} = 120$ kN/m^2 , e_2=0.78

time for 50% consolidation= 2.5 min

Determine the hydraulic conductivity, *k*, of the clay for the loading range

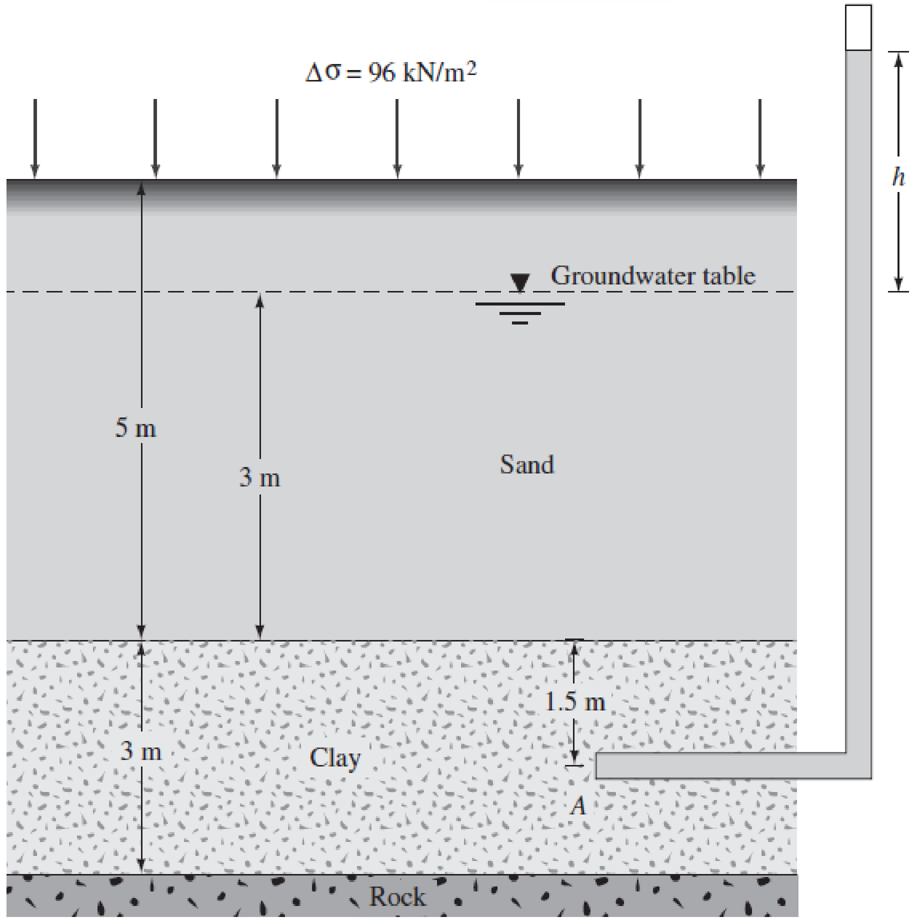
$Answer\ k = 1.303 * 10^{-7}$ m/min Answer

Example

A soil profile is shown. A surcharge load of 96 kN/m2 is applied on the ground surface. Determine the following:

a. How high the water will rise in the piezometer immediately after the application of load.

b. After 104 days of the load application, h= 4 m.
Determine (C_v) of the clay soil.



Solution

Part a $u_o = \Delta\sigma' = 96 \text{ kN/m}^2$

$$h = \frac{u}{\gamma_w} = \frac{96}{9.81} = \mathbf{9.79\ m}$$

Part b

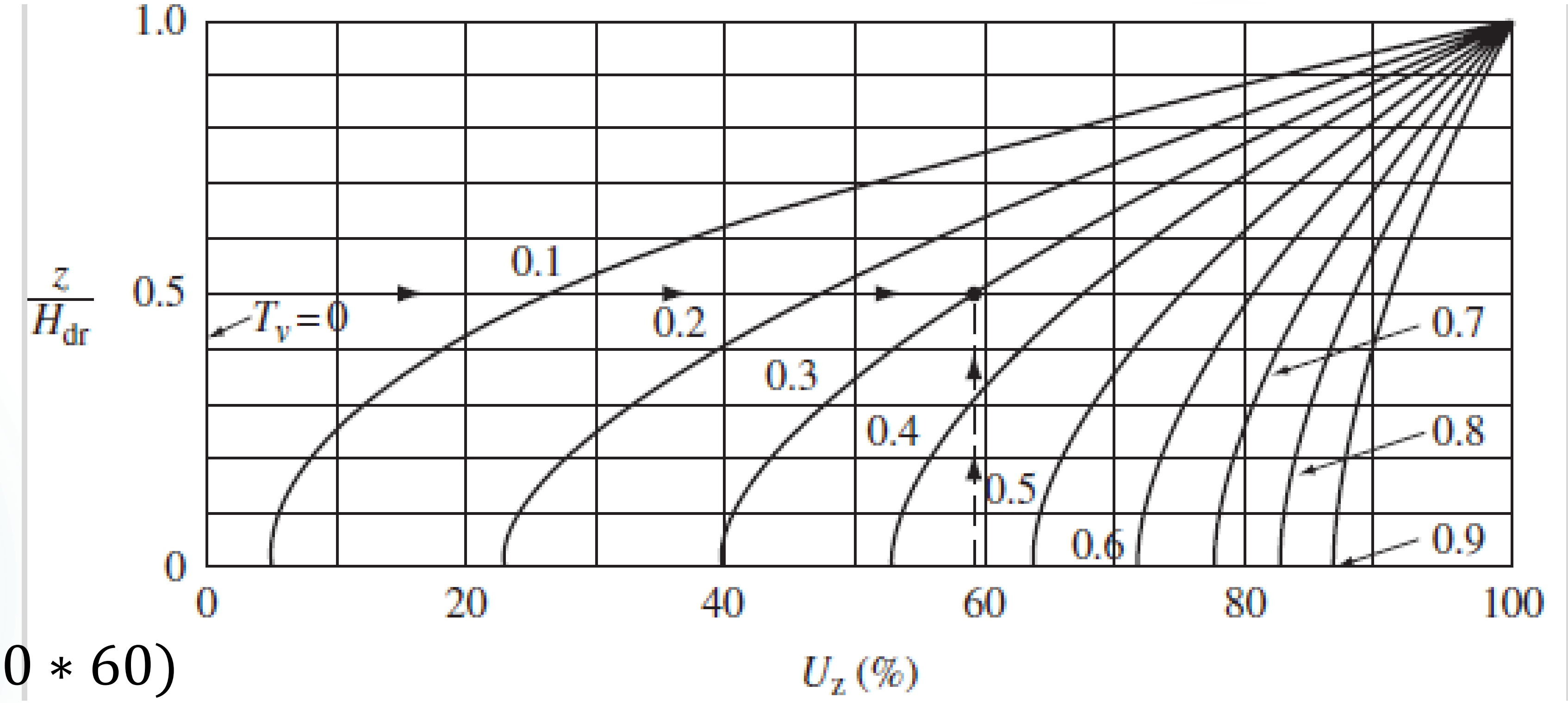
$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o} = \left(1 - \frac{4*9.81}{96}\right)*100=59\%$$

$$from\ z/H_{dr}=1.5/3=0.5 \text{ and } Uz = 59\%$$

$from\ z/H_{dr}$=1.5/3=0.5 and Uz =59%

$$T_v \approx 0.3$$

$$T_v = \frac{C_v t}{H_{dr}^2}$$



$$0.3 = \frac{C_v(104 * 24 * 60 * 60)}{(300)^2}$$

C_v= 0.003 cm²/sec

Thank you