مادة الميكانيك النظري

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قسم الفلك والفضاء

كلية العلوم

جامعة بغداد

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Lecture 1

Units and Vector

1.1. Units

Physics experiments involve the measurement of a variety of quantities.

These measurements should be accurate and reproducible.

The first step in ensuring accuracy and reproducibility is defining the units in which the measurements are made.

1.2. SI units

Meter (m): unit of length, kilogram (kg): unit of mass, second (s): unit of time.

Table 1.1 Units of Measurement

	System		
	SI	CGS	BE
Length	Meter (m)	Centimeter (cm)	Foot (ft)
Mass	Kilogram (kg)	Gram (g)	Slug (sl)
Time	Second (s)	Second (s)	Second (s)

The units for length, mass, and time (as well as a few others), are regarded as base SI units.

These units are used in combination to define additional units for other important physical quantities such as force and energy.

1.3. The Role of Units in Problem Solving

The conversion of units

1 ft = 0.3048 m

1 mi = 1.609 km

1 hp = 746 W

1 liter = 10^{-3} m^3

Example 1 The World's Highest Waterfall

The highest waterfall in the world is Angel Falls in Venezuela, with a total drop of 979.0 m. express this drop in feet.

Since 3.281 feet = 1 meter, it follows that

(3.281 feet)/ (1 meter) = 1

Length= (979.0 meters) (3.281 feet/1 meter) = 3212 feet.

Table 1.2 Standard Prefixes Used to Denote Multiples of Ten

Prefix	Symbol	Factor[a]
tera	T	10^{12}
giga[b]	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deka	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

1.4. Converting Between Units.

1. In all calculations, write down the units explicitly.

2. Treat all units as algebraic quantities. When identical units are divided, they are eliminated algebraically.

3. Use the conversion factors located on the page facing the inside cover. Be guided by the fact that multiplying or dividing an equation by a factor of **1** does not alter the equation.

Example 2 Interstate Speed Limit

Express the speed limit of 65 miles/hour in terms of meters/second.

Use 5280 feet = 1 mile and 3600 seconds = 1 hour and 3.281 feet = 1 meter.

$$\text{Speed} = \left(65\,\frac{\text{miles}}{\text{hour}}\right)(1)(1) = \left(65\,\frac{\text{miles}}{\text{hour}}\right)\left(\frac{5280\text{ feet}}{\text{mile}}\right)\left(\frac{1\text{ hour}}{3600\text{ s}}\right) = 95\,\frac{\text{feet}}{\text{second}}$$

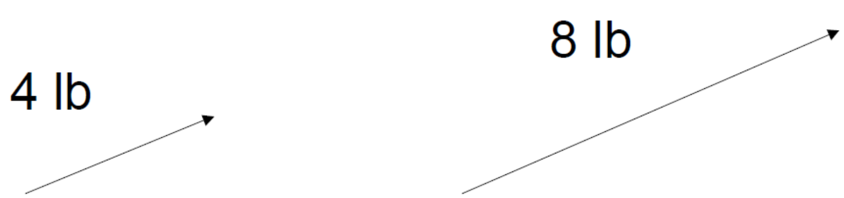
$$\text{Speed} = \left(95\,\frac{\text{feet}}{\text{second}}\right)(1) = \left(95\,\frac{\text{feet}}{\text{second}}\right)\left(\frac{1\text{ meter}}{3.281\text{ feet}}\right) = 29\,\frac{\text{meters}}{\text{second}}$$

1.5. Scalars and Vectors

A scalar quantity is one that can be described by a single number: temperature, speed, mass A vector quantity deals inherently with both magnitude and direction: velocity, force, displacement

Arrows are used to represent vectors. The direction of the arrow gives the direction of the vector.

By convention, the length of a vector arrow is proportional to the magnitude of the vector.



1.6 Vector Addition

When studying musculoskeletal biomechanics, it is common to have more than one force to consider. Therefore, it is important to understand how to work with more than one vector. When adding or subtracting two vectors, there are some important properties to consider. Vector addition is commutative:

A + B = B + A (1.1)

A + **B** =**A** + **(-B)** (1.2)

Vector addition is associative:

A + **(B** + **C)** = **(A** +**B)** +**C** (1.3)

Unlike scalars, which can just be added together, both the magnitude and orientation of a vector must be taken into account. The detailed procedure for adding two vectors **(A+ B = C)** is shown in Box 1.1 for the graphical, polar coordinate, and component representation of vectors. The graphical representation uses the "tip to tail" method. The first step is to draw the first vector, **A.** Then the second vector, **B,** is drawn so that its tail sits on the tip of the first vector. The vector representing the sum of these two vectors **(C)** is obtained by connecting the tail of vector **A** and the tip of vector **B.** Since vector addition is commutative, the same solution would have been obtained if vector **B** were the first vector. When using polar coordinates, the vectors are drawn as in the graphical method, and then the law of cosines is used to determine the magnitude of **C** and the law of sines is used to determine the direction of **C.** For the component resolution method, each vector is broken down into its respective *x* and *y* components. The components represent the magnitude of the vector in that direction. The *x* and *y* components are summed:

$C_X = A_X + B_X$ (1.4)

$C_\gamma = A_\gamma + B_\gamma$ (1.5)

The vector **C** can either be left in terms of its components, C_X and C_γ, or be converted into a magnitude, C, using the Pythagorean Theorem, and orientation, θ, using trigonometry. This method is the most efficient of the three presented and is used throughout the text.

1.6 Vector Multiplication

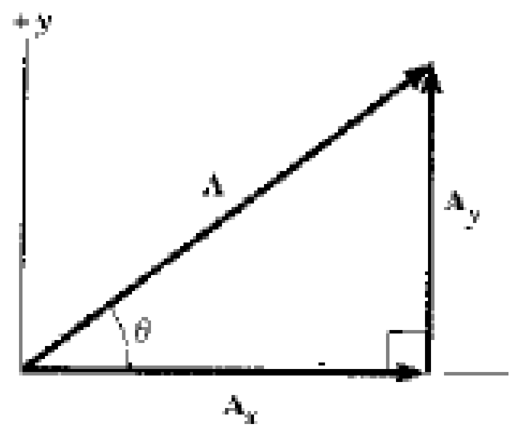
Multiplication of a vector by a scalar is relatively straight forward. Essentially, each component of the vector is individually multiplied by the scalar, resulting in another vector. For example, if the vector in Figure 1.2 is multiplied by 5, the result is A_X = 5x 4 N= 20 N and A_γ= 5 x 3 N = 15 N.

Another form of vector multiplication is the **cross product,** in which two vectors are multiplied together, resulting in another vector (**C** = **A** x **B**). The orientation of **C** is such that it is mutually perpendicular to **A** and **B.** The magnitude of **C** is calculated as C = A.B sin (θ), where θ represents the angle between A and B, and (**.**) denotes scalar multiplication. These relationships are illustrated in Figure 1.3. The cross product is used for calculating joint torques below in this chapter.

Lecture 2

1.7 Unit Vectors

The basic idea behind vector components are any vector can be composed (put together) from component vectors. That is, it is always possible to think of a vector as the vector addition of component vectors, and the simplest component vectors would be a pair of mutually perpendicular vectors, pointing along the coordinate axes, that form the sides of a right triangle for which the desired vector is the hypotenuse.



In this document all vectors will be written in bold text. The vector addition can be written as:

$$A = A_x + A_y$$

The terms Ax and A_y are themselves vectors, in this case vectors pointing along the x and y axes, respectively. There is an alternative, completely equivalent approach that uses the discussion on pages 14 and 15. The alternative approach is to express a vector quantity in terms of unit vectors. **A unit vector is a dimensionless vector one unit in length used only to specify a given direction.** Unit vectors have no other physical significance. In Physics 2110 and 2120 we will use the symbols i, j, and k (if there is a third dimension, i.e. a “z” direction), although in many texts the symbols x^, y^, and z^ are often used. In class I will write these vectors with arrows above them to indicate they are unit vectors, i.e. i, j, k.

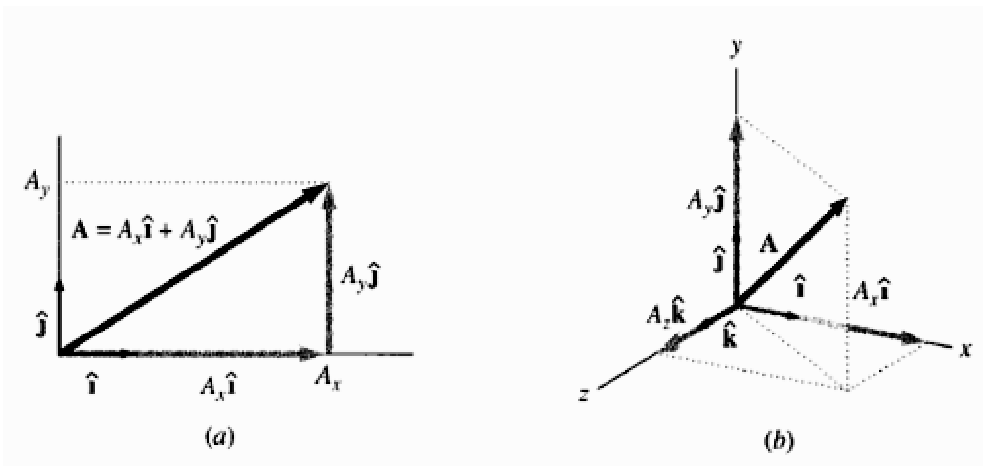
The component vectors can now be written in unit vector notations as:

$$A_x = A_x i \text{ and } A_y = A_y j$$

where the terms A_x and A_y are the scalar components of the vector A, respectively. Recall

$$A_x = A\cos\theta \text{ and } Ay = A\sin\theta.$$

The following figures show how unit vectors relate to the above figure.



As the figures to the right show the vector A can be written in unit vector notation as

$$A = A_x i + A_y j$$

where A_x and A_y are the scalar components of A, respectively.

This is certainly equivalent to specifying a vector in the "magnitude" and "direction" form, where

$$|A| = \sqrt{A_x^2 + A_y^2}$$

$$A_x = A\cos\theta$$

$$A_y = A\sin\theta$$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

The advantage of the unit vector approach to writing out vectors is that it saves space and computational effort. It is especially useful in vector addition and multiplication (which we will get to later in the course).

Examples:

1. The northwest corner of Liberty Park in Salt Lake City has coordinates 9^{th} South and 5^{th} East. Using the origin of the coordinate system at 0 South and 0 East with South corresponding to the +x- axis and East corresponding to the +y- axis, write out the position vector for the northwest corner of Liberty Park in both magnitude and direction form and unit vector form.

Solution:

a. Magnitude and Direction form:

Call this position vector r. The statement 9^{th} South and 5^{th} East are the actual statements of the x and y components of the vector r, which could be called r_x and r_y, respectively.

That is, r_x = 9 blocks and r_y = 5 blocks. Therefore,

$$|\mathrm{r}| = \mathrm{r} = \sqrt{\mathrm{r}_x^2 + \mathrm{r}_y^2} = \sqrt{(9\mathrm{blocks})^2 + (5\mathrm{blocks})^2} = 10.3\ \mathrm{blocks}$$

$$\theta = \tan^{-1}\frac{\mathrm{r}_y}{\mathrm{r}_x} = \tan^{-1}\frac{5\mathrm{blocks}}{9\mathrm{blocks}} = 29.1^{\mathrm{o}}$$

b. Unit Vector form:

Since the vector components, r_x and r_y are already known there is nothing more to do here than write down the result.

$$\mathrm{r} = \mathrm{r}_x\mathrm{i} + \mathrm{r}_y\mathrm{j} = (9\mathrm{blocks})\mathrm{i} + (5\mathrm{blocks})\mathrm{j}$$

Practice Problems

1. Look at #63 on page 25 in the text. Express each vector, A, B, and C in this question in unit vector notation. Then write A + B + C in unit vector notation. Finally, in unit vector notation what is the answer to the actual question asked?

2. In unit vector notation what would be the displacement vector that goes from the northwest corner of Liberty Park to the southeast corner of Liberty Park at 1300 south and 700 East?

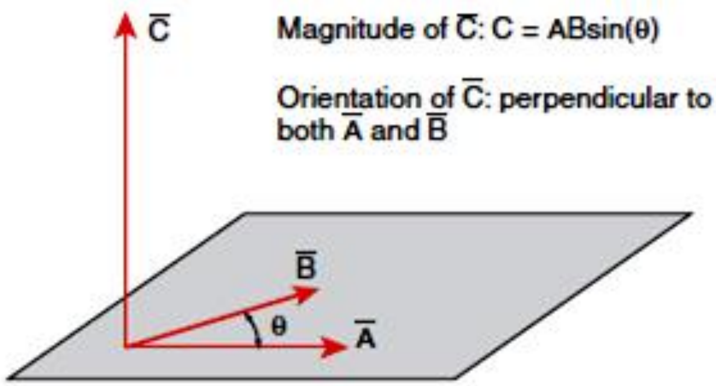


Figure 1.3: Vector cross product. **C** is shown as the cross product of **A** and **B**. Note that **A** and **B** could be any two vectors in the indicated plane and **C** would still have the same orientation.

Lecture 3

Chapter 2

Kinematics in One Dimension

2.1 Mechanics

Mechanics is the branch of physics which deals with the motion of material objects and their interaction.

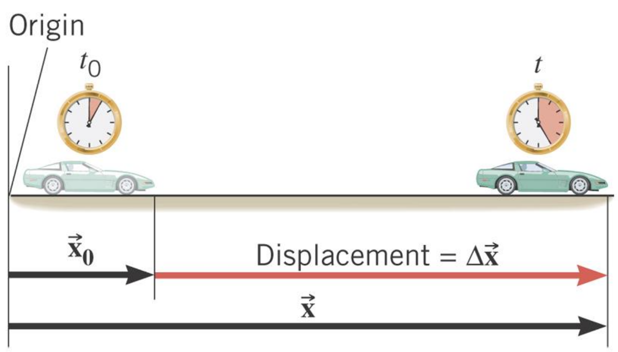
Kinematics

Kinematics deals with the concepts that are needed to describe motion.

Dynamics deals with the effect that forces have on motion.

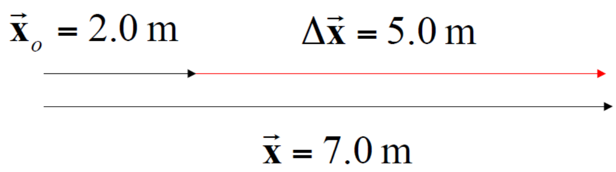
Together, kinematics and dynamics form the branch of physics known as Mechanics.

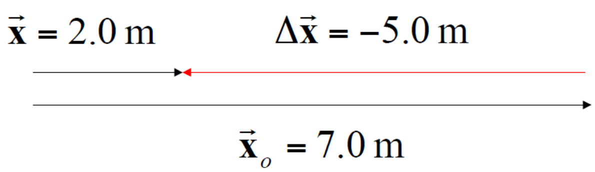
2.2 Displacement



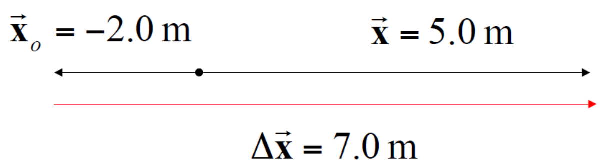
$\vec{\mathbf{x}}_o$ = initial position $\qquad$ $\vec{\mathbf{x}}$ = final position

$$\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_o = \text{displacement}$$





$$\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_o = 2.0\text{ m} - 7.0\text{ m} = -5.0\text{ m}$$



$$\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_o = 5.0\text{ m} - (-2.0)\text{m} = 7.0\text{ m}$$

2.3 Speed and Velocity

2.3.1Average speed is the distance traveled divided by the time required to cover the distance.

$$\text{Average speed} = \frac{\text{Distance}}{\text{Elapsed time}}$$

SI units for speed: meters per second (m/s)

Example 1 Distance Run by a Jogger How far does a jogger run in 1.5 hours (5400) if his average speed is 2.22 m/s?

$$\text{Average speed} = \frac{\text{Distance}}{\text{Elapsed time}}$$

Distance = (Average speed) (Elapsed time) = (2.22 m /s) (5400 s) =12000 m

2.3.2 Average Velocity

The average velocity (<v>) of a particle during the time interval Δt is defined as the ratio of its displacement ($\Delta\vec{r}$) to the time interval for this displacement:

$$\langle\vec{\mathbf{v}}\rangle = \frac{\Delta\vec{\mathbf{x}}}{\Delta \mathbf{t}}$$

The unit of velocity measurement is m/s.

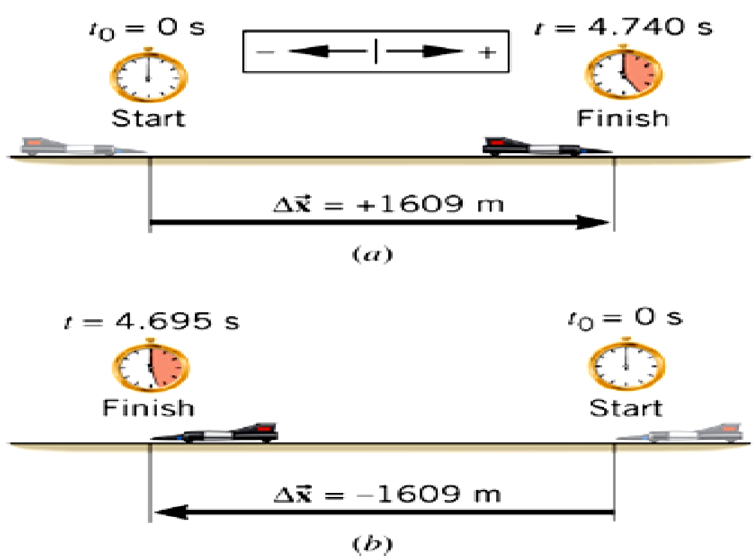
Example 1. A particle moving along the x axis is located at x_i (10 m) at t_i (1 s) and at x_f (6 m) at t_f (3 s). Find its displacement and the average velocity during this time interval.

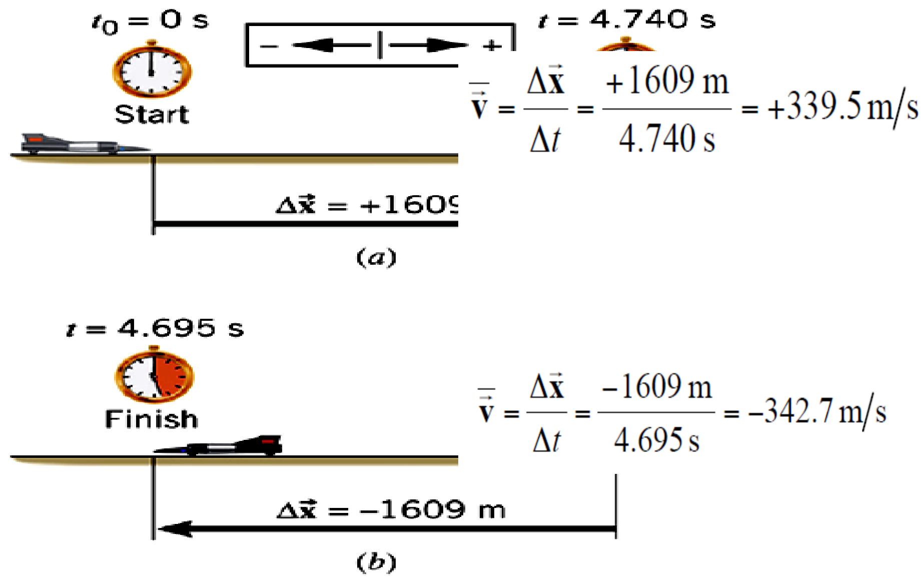
Solution. The displacement is given by:

$\Delta x = x_f - x_i = 6\text{ m} - 10\text{ m} = -4\text{ m}$

Example 2 The World's Fastest Jet-Engine Car

Andy Green in the car Thrust SSC set a world record of 341.1 m/s in 1997. To establish such a record, the driver makes two runs through the course, one in each direction, to nullify wind effects. From the data, determine the average velocity for each run.





2.3.3 Instantaneous Velocity

The instantaneous velocity ($\vec{v}$) equals the limit of the average velocity, $\frac{\Delta\vec{r}}{\Delta t}$ as Δt approaches zero:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{x}}{\Delta t} = \frac{d\vec{x}}{dt}$$

Example 3. The position of a particle moving along the x axis varies in time according to the expression $x = 4t^2$, where x is in m, and t is in s. Find the instantaneous velocity at any time.

Solution. We can compute the velocity at any time (t) by using the definition of the instantaneous velocity. If the initial coordinate of the particle at time t is $x_i = 4t^2$, then the coordinate at a later time $(t + \Delta t)$ is:

$x_f = 4(t + \Delta t)^2 = 4[t^2 + 2t\Delta t + (\Delta t)^2] = 4t^2 + 8t\Delta t + 4(\Delta t)^2$

Therefore, the displacement in the time interval is:

$\Delta_x = x_f - x_i = 4t^2 + 8t\Delta t + 4(\Delta t)^2 - 4t^2 = 8t\Delta t + 4(\Delta t)^2$

The average velocity in the time interval is:

$$\langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = 8t + 4\Delta t$$

To find the instantaneous velocity, we take the limit of this expression as Δt approaches zero. In doing so, we see that the term 4Δt goes to zero, therefore:

$$\boldsymbol{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = 8t\ m/s$$

Lecture 4

2.4. Average Acceleration

The average acceleration of a particle in the time interval $\Delta t = t_f - t_i$ is defined as the ratio Δv/Δt, where $\Delta\upsilon = \upsilon_f - \upsilon_i$ is the change in velocity during the time interval:

$$\langle\vec{a}\rangle = \frac{\Delta\vec{\mathbf{v}}}{\Delta \mathbf{t}} = \frac{\mathbf{v_f} - \mathbf{v_i}}{\mathbf{t_f} - \mathbf{t_i}}$$

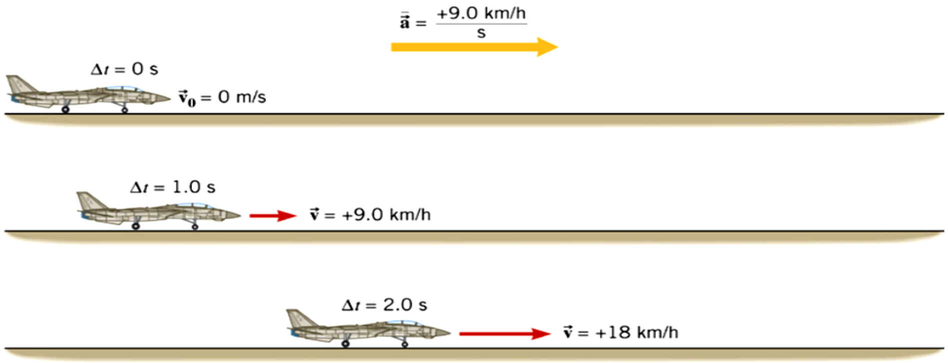
2.5. Instantaneous Acceleration

It is useful therefore to define the instantaneous acceleration ($\vec{a}$) as the limit of the average acceleration as Δt approaches zero:

Example 4 Acceleration and Increasing Velocity Determine the average acceleration of the plane.

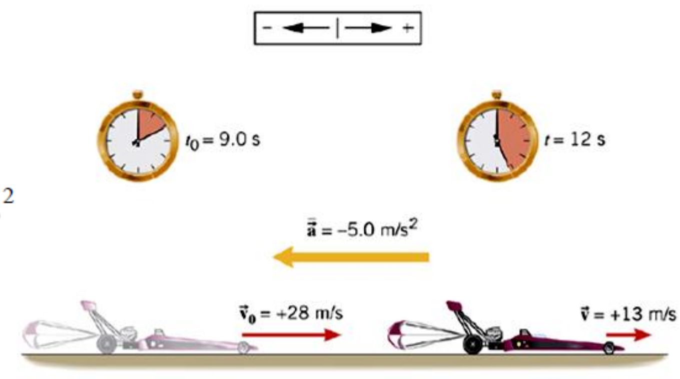
$\vec{\mathbf{v}}_o = 0\,\mathrm{m/s}$ $\quad\vec{\mathbf{v}} = 260\,\mathrm{km/h}$ $\quad t_o = 0\,\mathrm{s}$ $\quad t = 29\,\mathrm{s}$

$$\bar{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{260\,\mathrm{km/h} - 0\,\mathrm{km/h}}{29\,\mathrm{s} - 0\,\mathrm{s}} = +9.0\frac{\mathrm{km/h}}{\mathrm{s}}$$



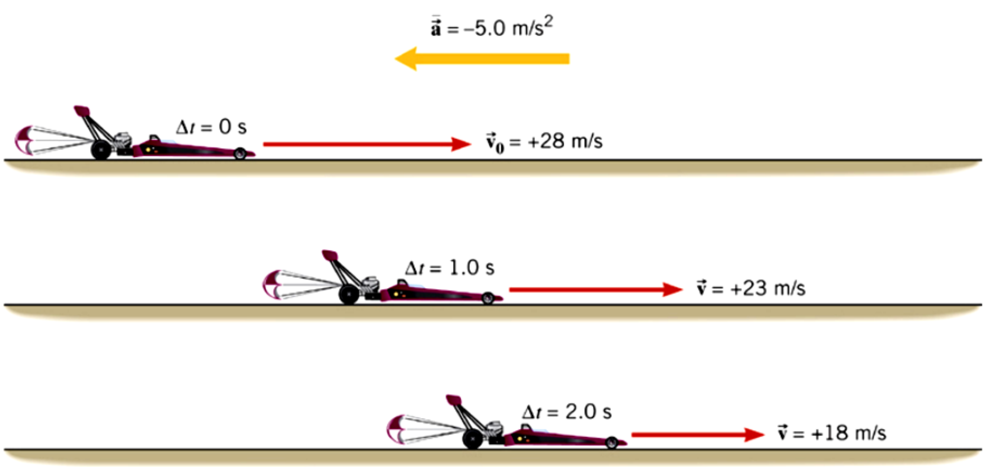
Example 5

Acceleration and Decreasing Velocity



(b)

$$\bar{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{13\,\mathrm{m/s} - 28\,\mathrm{m/s}}{12\,\mathrm{s} - 9\,\mathrm{s}} = -5.0\,\mathrm{m/s}^2$$



Lecture 5

2.6. Gravitational Acceleration and Free Fall

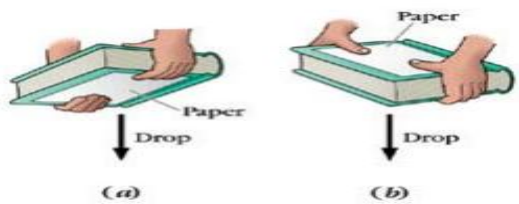
When an object's fall to Earth is not helped or opposed by anything else, not even air resistance, we say the object is freely falling or in free fall.

If you drop a dense, heavy object from rest and monitor its fall with a sonic range finder, you will find that the object has uniform acceleration. Moreover, you will find the value of the acceleration to be very nearly 9.8 (m/s)/s or 9.8 m/s^2. This value is called its gravitational acceleration.

If you drop this textbook, the acceleration will have approximately that value. If you drop a single sheet of paper, in contrast, you will find that the acceleration is not uniform and is on average much less than 9.8 m/s^2, so that the sheet of paper takes much longer to reach the ground. If you crumple the sheet of paper into a ball before dropping it, you reduce the effect of air resistance, and the paper falls more nearly like the textbook.

On-The-Spot Activity 2-1

Take a smooth sheet of paper, small enough so that it doesn't extend beyond the edges of the cover of this book, and hold it to the underside of the book, as in Figure 2-20a. Hold the book and paper horizontally and release them together. The book and the sheet of the paper should hit the ground together. Under these circumstances, their accelerations are the same. “Big deal!” you say. The book is making the paper move with it. But what if you hold the sheet of paper flat on top of the book, as in Figure 2-20b, and drop it again? First decide what you think will happen to the paper, then try it. Does the paper do what you expected? If not, why not?



When air resistance is prevented from affecting the sheet of paper, the paper falls with the same acceleration as the book, roughly 9.8 m/s^2.

Dropped simultaneously from the same height, bodies in free fall will hit level ground at the same time. A hammer and a feather did exactly that when they were dropped in the airless conditions on the moon by a member of the 1971 Apollo 15 mission. In another dramatic demonstration in place for many years at the Boston Museum of Science, feathers released inside a two-story glass column would drop to the bottom like the proverbial ton of bricks when the air was pumped out. Over short distances above or below Earth's surface (a few hundred meters or less), the acceleration of a freely falling body will vary with height by less than one part in a thousand. To two-place accuracy, it remains 9.8 m/s^2. We use g as a symbol for this special value.

Important: The symbol g stands for this positive number, no matter which direction we call positive. When we take the positive direction to be upward, the acceleration $a = -g$; that is, $a = -9.8$ m/s^2.

Applying the Constant Acceleration Equations of Motion to Free Fall

1. If you choose the positive direction to be upward, let a = −g in all the equations (let a = +g if you let downward be positive).

2. Because we tend to label the vertical axis y, it is common though not necessary to replace x with y in all the equations to indicate vertical position.

By these two steps, you should be able to show whenever needed (rather than memorize more equations)

$$v = v_0 - gt$$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

Lecture 6

Chapter 3

Dynamics

Dynamics is study of the motion of an object in connection with the cause(s) of the motion.

3.1. Newton’s Laws

Newton’s First Law: An object at rest will remain at rest and an object in motion will continue in motion with a constant velocity unless it experiences a net external force (or resultant force).

Newton’s Second Law: The acceleration of an object is directly proportional to the resultant force acting on it and inversely proportional to its mass:

$$\sum \vec{\mathbf{F}} = \mathbf{m}\vec{\mathbf{a}}$$

where $\vec{\boldsymbol{F}}$ is the resultant force,$\vec{\boldsymbol{a}}$ is the acceleration.

Newton's Third Law: If two bodies interact, the force exerted on body 1 by body 2 is equal to and opposite the force exerted on body 2 by body 1

$$\vec{\mathbf{F}}_{\mathbf{12}} = \vec{\mathbf{F}}_{\mathbf{21}}$$

3.2. Newton's Universal Law of Gravity

Newton's Law of Gravitation: Every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. If the particles have masses m1 and m2 and are separated by a distance r, the magnitude of this gravitational force is:

$$F = G\frac{m_1 m_2}{r^2}$$

where G is a universal constant called the gravitational constant, which has been measured experimentally. Its value in SI units is:

$$\mathrm{G} = 6.672 \times 10^{-11}\frac{\mathrm{N.m^2}}{\mathrm{Kg^2}}$$

3.3. Weight

The force exerted by the earth on a body is called the *weight* of the body ($\overrightarrow{\mathrm{W}}$). A freely falling body experiences an acceleration ($\vec{\mathrm{g}}$) acting toward the center of the earth. Applying Newton's second law to a free falling body, with $\vec{\mathrm{a}} = \vec{\mathrm{g}}$ and $\vec{\mathrm{F}} = \vec{\mathrm{w}}$ gives:

$$\overrightarrow{\mathrm{W}} = \mathrm{m}\vec{\mathrm{g}}$$

Lecture 7

3.4. Elasticity – Hooke's Law

Deformation means the change of size, shape and mass distribution of an object under load conditions. Deformation is an important consideration in understanding the mechanics of materials and biomechanics of living organisms. Elastic means that when the deforming forces are removed, the object returns to its original shape.

Consider a body on a horizontal, smooth surface which is connected to a helical spring. If the spring is stretched or compressed a small distance from its equilibrium configuration, the spring will exert a force on the body given by Hooke's Law: The force (F) required stretching (compress) body is directly proportional to the extension (x):

$$F = kx$$

where k is the stiffness constant (N/m); it is assumed that the stretching or compressing of the body occurs along the axis OX.

The force F per unit cross-sectional area S on an elastic body fixed at one end is called the stress (σ) which is equal to F/S and has units N/m2.

The term strain refers to the relative change in dimensions or shape of a body through the application of external force. The strain (ε) is a measure of the degree of deformation; it is defined by the ratio x/l, where l is original length of the body (i.e., ε = x/l).

For sufficiently small stresses, the stress is proportional to the strain; the constant of proportionality depends on the material being deformed and on the nature of the deformation. This proportionality constant is called the elastic modulus or Young's modulus if it measures the resistance of a solid to change in its length:

$$Y = \frac{F/S}{x/l} = \frac{\sigma}{\epsilon}$$

Hooke's law can be written as:

$$\sigma = E.\varepsilon$$

The stress-strain curve is called the tensile diagram (fig. 3.1).

Example. A strip of tissue 6 cm long with a cross-sectional area (S) of 0.12 cm2 has a Young's modulus (Y) of approximately 105 N·m-2. What mass must be suspended from the strip hung vertically to cause a 0.6 cm elongation?

Solution. Force (F), which is applied to the tissue, can be defined as:

$$F=m.g$$

The last equation can be written as:

$$\frac{m.g}{S} = E.\frac{\Delta l}{l}$$

The mass is:

$$m = \frac{E \cdot \Delta l \cdot S}{g \cdot l} = \frac{(10^5 \frac{\text{N}}{\text{m}^2}) \cdot (0.6 \cdot 10^{-2}\,\text{m}) \cdot (0.12 \cdot 10^{-4}\ \text{m}^2)}{(9.8 \frac{\text{m}}{\text{c}^2}) \cdot (6 \cdot 10^{-2}\,\text{m})} = 1.22 \cdot 10^{-2}\,\text{kg}.$$

3.5. Work and Energy

The work done by a constant force is defined as the product of the component of the force in the direction of the displacement and the magnitude of the displacement:

$$\mathrm{A} = \vec{\mathrm{F}}.\vec{\mathrm{s}} = (\mathrm{F}\cos\theta)\mathrm{s}$$

where (Fcosθ) is the component of $\vec{F}$ in the direction of $\vec{s}$. Work is a scalar quantity and equation 1.22 is the scalar product of the two vectors $\overrightarrow{F}$ and $\vec{s}$. The SI unit of work is the joule (J = N·m).

The product of one half the mass (m) and the square of the speed (V) is defined as the kinetic energy of a particle:

$$E_k = \frac{1}{2}mv^2$$

Potential energy is accumulated in a system as a result of previous work being done. For instances, the gravitational potential energy near the Earth's surface is:

$$E_p = mgh$$

where m is the mass of the particle and h the displacement.

The elastic potential energy, such as that stored in a spring, is:

$$E_p = \frac{1}{2}kx^2$$

where k is the force constant of the spring and x the displacement.

Control Questions and Problems

1. What studies mechanics?
2. What is the material point?
3. Define average and instantaneous velocity.
4. Define average and instantaneous acceleration.
5. Formulate the first, second and third laws of Newton.
6. Formulate the Universal Law of Gravity.
7. What is the elastic deformation?
8. Formulate Hooke's law.
9. What is the work and Energy?

Lecture 8

Chapter 4

Linear Momentum and Collisions

4.1. Linear Momentum

The linear momentum of a particle with mass m moving with velocity v is defined as:

$$p = mv$$

Linear momentum is a vector. When giving the linear momentum of a particle you must specify its magnitude and direction. We can see from the definition that its units must be (kg·m/ s). The momentum of a particle is related to the net force on that particle in a simple way; since the mass of a particle remains constant, if we take the time derivative of a particle's momentum we find:

$$\frac{d\mathbf{p}}{dt} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a} = \mathbf{F}_{\text{net}}$$

$$\mathbf{F}_{\text{net}} = \frac{d\mathbf{p}}{dt} \qquad (4.1)$$

4.2. Impulse

Average Force When a particle moves freely then interacts with another system for a (brief) period and then moves freely again, it has a definite change in momentum; we define this change as the impulse I of the interaction forces:

$$I = p_f - p_i = \Delta p$$

Impulse is a vector and has the same units as momentum. When we integrate Eq.4.2 we can show:

$$\mathbf{I} = \int_{t_i}^{t_f} \mathbf{F}\, dt = \Delta \mathbf{p}$$

We can now define the average force which acts on a particle during a time interval Δt. It is:

$$\bar{\mathbf{F}} = \frac{\Delta p}{\Delta t} = \frac{\mathbf{I}}{\Delta t}$$

The value of the average force depends on the time interval chosen.

4.3. Conservation of Linear Momentum

Linear momentum is a useful quantity for cases where we have a few particles (objects) which interact with each other but not with the rest of the world. Such a system is called an isolated system. We often have reason to study systems where a few particles interact with each other very briefly, with forces that are strong compared to the other forces in the world that they may experience. In those situations, and for that brief period of time, we can treat the particles as if they were isolated.

We can show that when two particles interact only with each other (i.e. they are isolated) then their total momentum remains constant:

$$p_{1i} + p_{2i} = p_{1f} + p_{2f} \qquad (4.3)$$

or, in terms of the masses and velocities,

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \qquad (4.4)$$

Or, abbreviating $p_1 + p_2 = P$ (total momentum), this is: $P_i = P_f$. It is important to understand that Eq. 4.3 is a vector equation; it tells us that the total x component of the momentum is conserved, and the total y component of the momentum is conserved.

Lecture 9

4.4. Collisions

When we talk about a collision in physics (between two particles, say) we mean that two particles are moving freely through space until they get close to one another; then, for a short period of time they exert strong forces on each other until they move apart and are again moving freely. For such an event, the two particles have well-defined momenta p1i and p_{2i} before the collision event and p_{1f} and p_{2f} afterwards. But the sum of the momenta before and after the collision is conserved, as written in Eq. 4.3. While the total momentum is conserved for a system of isolated colliding particles, the mechanical energy may or may not be conserved. If the mechanical energy (usually meaning the total kinetic energy) is the same before and after a collision, we say that the collision is elastic. Otherwise we say the collision is inelastic. If two objects collide, stick together, and move off as a combined mass, we call this a perfectly inelastic collision. One can show that in such a collision more kinetic energy is lost than if the objects were to bounce off one another and move off separately.

When two particles undergo an elastic collision then we also know that:

$$\tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2 = \tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2$$

In the special case of a one-dimensional elastic collision between masses m_1 and m_2 we can relate the final velocities to the initial velocities. The result is:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i} \qquad (4.5)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i} \qquad (4.6)$$

This result can be useful in solving a problem where such a collision occurs, but it is not a fundamental equation.

4.5. Center of Mass

For a system of particles (that is, lots of ’em) there is a special point in space known as the center of mass which is of great importance in describing the overall motion of the system. This point is a weighted average of the positions of all the mass points. If the particles in the system have masses m_1, m_2, . . .m_N, with total mass:

$$\sum_i^N m_i = m_1 + m_2 + \cdots + m_N \equiv M$$

and respective positions r_1, r_2, . . . ,r_N, then the center of mass r_{CM} is:

$$\mathbf{r}_{\mathrm{CM}} = \frac{1}{M}\sum_i^N m_i\mathbf{r}_i \qquad (4.7)$$

which means that the x, y and z coordinates of the center of mass are:

$$x_{\rm CM} = \frac{1}{M}\sum_i^N m_i x_i \qquad y_{\rm CM} = \frac{1}{M}\sum_i^N m_i y_i \qquad z_{\rm CM} = \frac{1}{M}\sum_i^N m_i z_i \tag{4.8}$$

For an extended object (i.e. a continuous distribution of mass) the definition of r_{CM} is given by an integral over the mass elements of the object:

$$\mathbf{r}_{\rm CM} = \frac{1}{M}\int \mathbf{r}\, dm \tag{4.9}$$

which means that the x, y and z coordinates of the center of mass are now:

$$x_{\rm CM} = \frac{1}{M}\int x\, dm \qquad y_{\rm CM} = \frac{1}{M}\int y\, dm \qquad z_{\rm CM} = \frac{1}{M}\int z\, dm \tag{4.10}$$

When the particles of a system are in motion then in general their center of mass is also in motion. The velocity of the center of mass is a similar weighted average of the individual velocities:

$$\mathbf{v}_{\rm CM} = \frac{d\mathbf{r}_{\rm CM}}{dt} = \frac{1}{M}\sum_i^N m_i \mathbf{v}_i \tag{4.11}$$

In general the center of mass will accelerate; its acceleration is given by:

$$\mathbf{a}_{\rm CM} = \frac{d\mathbf{v}_{\rm CM}}{dt} = \frac{1}{M}\sum_i^N m_i \mathbf{a}_i \tag{4.12}$$

If P is the total momentum of the system and M is the total mass of the system, then the motion of the center of mass is related to P by:

$$\mathbf{v}_{\rm CM} = \frac{\mathbf{P}}{M} \qquad \text{and} \qquad \mathbf{a}_{\rm CM} = \frac{1}{M}\frac{d\mathbf{P}}{dt} \tag{4.13}$$

Lecture 10

4.6. Motion of a System of Particles

A system of many particles (or an extended object) in general has a motion for which the description is very complicated, but it is possible to make a simple statement about the motion of its center of mass. Each of the particles in the system may feel forces from the other particles in the system, but it may also experience a net force from the (external) environment; we will denote this force by F_{ext}. We find that when we add up all the external forces acting on all the particles in a system, it gives the acceleration of the center of mass according to:

$$\sum_{i}^{N} \mathbf{F}_{\mathrm{ext},\,i} = M\mathbf{a}_{\mathrm{CM}} = \frac{d\mathbf{P}}{dt}$$

Here, M is the total mass of the system; $F_{ext,\,i}$ is the external force acting on particle i.

In words, we can express this result in the following way: For a system of particles, the center of mass moves as if it were a single particle of mass M moving under the influence of the sum of the external forces.

4.7. Worked Examples (Linear Momentum)

1. A 3.00 kg particle has a velocity of (3.0i − 4.0j) m/s. Find its x and y components of momentum and the magnitude of its total momentum.

Using the definition of momentum and the given values of m and v we have:

p = mv = (3.00 kg) (3.0i − 4.0j) m/s = (9.0i − 12.j) kg·m/s

So the particle has momentum components:

p_x = +9.0 kg·m/s and p_y = −12. kg·m/s .

The magnitude of its momentum is:

$$p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.0)^2 + (-12.)^2}\,\frac{\mathrm{kg\cdot m}}{\mathrm{s}} = 15.\,\frac{\mathrm{kg\cdot m}}{\mathrm{s}}$$

2. A child bounces a superball on the sidewalk. The linear impulse delivered by the sidewalk is 2.00N· s during the 1/ 800 s of contact. What is the magnitude of the average force exerted on the ball by the sidewalk.

The magnitude of the change in momentum of (impulse delivered to) the ball is |Δp| =|I| = 2.00 N·s. (The direction of the impulse is upward, since the initial momentum of the ball was downward and the final momentum is upward.)

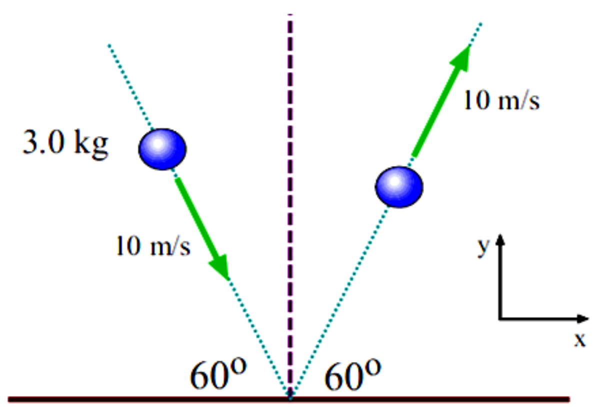
Since the time over which the force was acting was:

$$\Delta t = \frac{1}{800}\,\mathrm{s} = 1.25 \times 10^{-3}\,\mathrm{s}$$

then from the definition of average force we get:

$$|\overline{\mathbf{F}}| = \frac{|\mathbf{I}|}{\Delta t} = \frac{2.00\,\mathrm{N}\cdot\mathrm{s}}{1.25 \times 10^{-3}\,\mathrm{s}} = 1.60 \times 10^{3}\,\mathrm{N}$$

3. A 3.0 kg steel ball strikes a wall with a speed of 10 m/ s at an angle of 60_ with the surface. It bounces off with the same speed and angle, as shown in the Figure. If the ball is in contact with the wall for 0.20 s, what is the average force exerted on the wall by the ball?



The average force is defined as F = Δp / Δt, so first find the change in momentum of the ball. Since the ball has the same speed before and after bouncing from the wall, it is clear that its x velocity (see the coordinate system in the Figure.) stays the same and so the x momentum stays the same. But the y momentum does change. The initial y velocity is:

$$v_{iy} = -(10\,\tfrac{\text{m}}{\text{s}})\sin 60^\circ = -8.7\,\tfrac{\text{m}}{\text{s}}$$

and the final y velocity is

$$v_{fy} = +(10\,\tfrac{\text{m}}{\text{s}})\sin 60^\circ = +8.7\,\tfrac{\text{m}}{\text{s}}$$

so the change in y momentum is

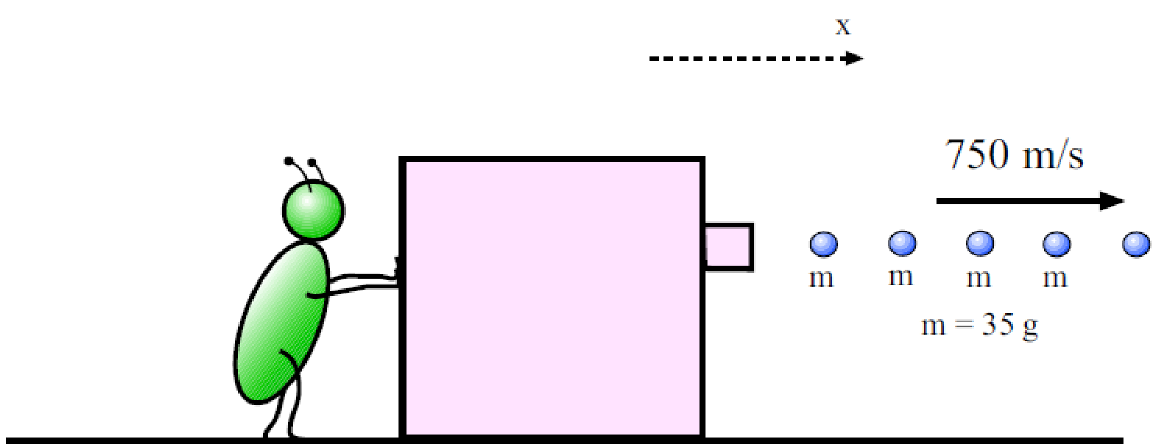
$$\Delta p_y = mv_{fy} - mv_{iy} = m(v_{fy} - v_{iy}) = (3.0\,\text{kg})(8.7\,\tfrac{\text{m}}{\text{s}} - (-8.7\,\tfrac{\text{m}}{\text{s}})) = 52\,\tfrac{\text{kg}\cdot\text{m}}{\text{s}}$$

The average y force on the ball is

$$\overline{F}_y = \frac{\Delta p_y}{\Delta t} = \frac{I_y}{\Delta t} = \frac{(52\,\frac{\text{kg}\cdot\text{m}}{\text{s}})}{(0.20\,\text{s})} = 2.6 \times 10^2\,\text{N}$$

Since F has no x component, the average force has magnitude 2.6×10^2 N and points in the y direction (away from the wall).

H.W 4. A machine gun fires 35.0 g bullets at a speed of 750.0 m/s. If the gun can fire 200 bullets/min, what is the average force the shooter must exert to keep the gun from moving?



5. A 10.0 g bullet is stopped in a block of wood (m = 5.00 kg). The speed of the bullet–plus–wood combination immediately after the collision is 0.600 m/ s. What was the original speed of the bullet?