C H A P T E R ONE (ELECTRIC FIELDS)

PROPERTIES OF ELECTRIC CHARGES:

The electric charge has the following important properties (according to Franklin's model of electricity):

•**Two kinds of electric charges occur in nature, with the property that unlike charges attract one another and like charges repel one another.**

• **Electric charge is conserved**(this means that, charge is not created in the process. The electric field state is due to a ***transfer*** of charge from one object to the other. One object gains some amount of negative charge while the other gains an equal amount of positive charge. For example ,when a glass rod is rubbed with silk, the silk obtains a negative charge that is equal in magnitude to the positive charge on the glass rod.

• **Electric charge is quantized (**this means that the net charge in a closed region remains the same).

Q:If you rub an inflated balloon against your hair, the two materials attract each other. Is the amount of charge present in the balloon and your hair after rubbing (a) less than, (b) the same as, or (c) more than the amount of charge present before rubbing?

INSULATORS AND CONDUCTORS:

It is convenient to classify substances in terms of their ability to conduct electric charge:

1-Electrical conductors(materials in which electric charges move freely such as copper, aluminum, and silver) When such materials are charged in some small region, the charge readily distributes itself over the entire surface of the material.

2-Electrical insulators(materials in which electric charges cannot move freely such as glass, rubber, and wood)When such materials are charged by rubbing, only the area rubbed becomes charged, and the charge is unable to move to other regions of the material.

3- Semiconductors are a third class of materials, and their electrical properties are somewhere between those of insulators and those of conductors such as silicon and germanium.

(Note: When a conductor is connected to the Earth by means of a conducting wire orpipe, it is said to be grounded.)

Charging by induction:

To understand induction, consider a neutral (uncharged) conducting sphere insulated from ground, as shown in Fig.1a. When a negatively charged rubber rod is brought near the sphere, the region of the sphere nearest the rod obtainsan excess of positive charge while the region farthest from the rod obtains anequal excess of negative charge, as shown in Fig.1b. (That is, electrons inthe region nearest the rod migrate to the opposite side of the sphere. This occurseven if the rod never actually touches the sphere.) If the same experiment is performedwith a conducting wire connected from the sphere to ground (Fig. 1c),some of the electrons in the conductor are so strongly repelled by the presence ofthe negative charge in the rod that they move out of the sphere through theground wire and into the Earth. If the wire to ground is then removed (Fig.1d), the conducting sphere contains an excess of *induced* positive charge. Whenthe rubber rod is removed from the vicinity of the sphere (Fig. 1e), this inducedpositive charge remains on the ungrounded sphere. Note that the chargeremaining on the sphere is uniformly distributed over its surface because of the repulsive forces among the like charges. Also note that the rubber rod loses none ofits negative charge during this process.

Charging an object by induction requires no contact with the body inducingthe charge. This is in contrast to charging an object by rubbing (that is, by *conduction*), which does require contact between the two objects.A process similar to induction in conductors takes place in insulators.

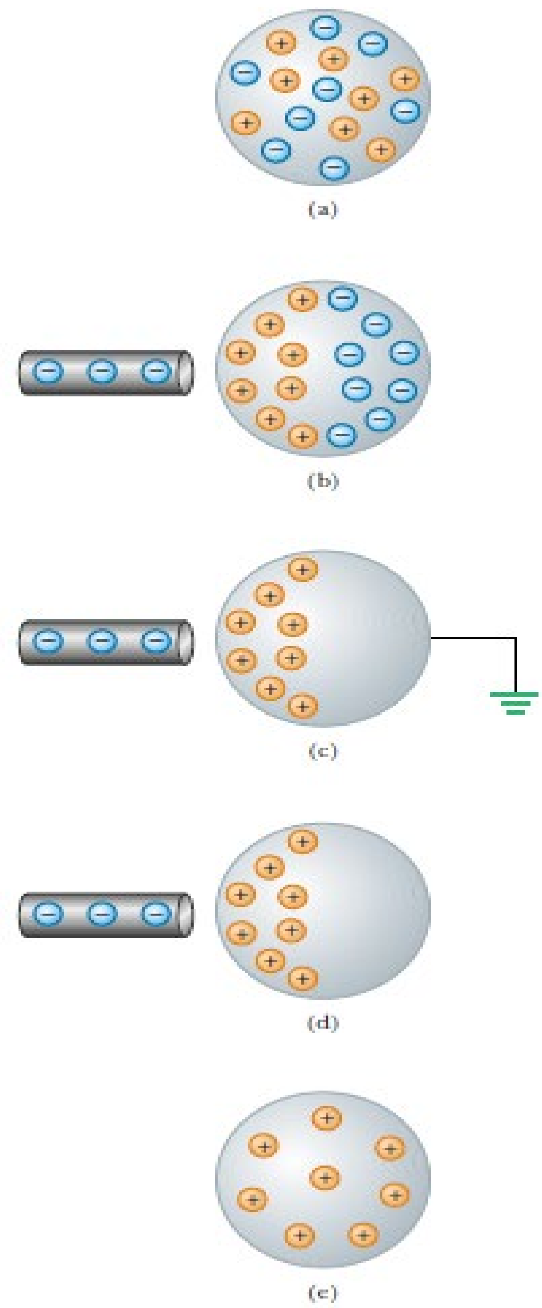


Figure 1 Charging a metallic object by *induction* (that is, the two objects never touch eachother). (a) A neutral metallic sphere, with equal numbers of positive and negative charges.(b) The charge on the neutral sphere is redistributed when a charged rubber rod is placed nearthe sphere. (c) When the sphere is grounded, some of its electrons leave through the groundwire. (d) When the ground connection is removed, the sphere has excess positive charge that isnonuniformly distributed. (e) When

the rod is removed, the excess positive charge becomes uniformly distributed over the surface of the sphere.

COULOMB'S LAW

Coulomb's experiments showed that the electric force between two stationary charged particles

- is inversely proportional to the square of the separation **r** between the particles and directed along the line joining them;
- is proportional to the product of the charges q_1 and q_2 on the two particles;
- is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

From these observations, we can express Coulomb's law as an equation giving the magnitude of the electric force (sometimes called the *Coulomb force*) betweentwo point charges:

$$F_e = k_e |q_1||q_2| / r^2$$

Where k_e is a constant called the Coulomb constant.The value of the Coulomb constant depends on the choice of units. The SIunit of charge is the coulomb (C). The Coulomb constant $k_e$in SI units has the value:

$$k_e = 8.987\,5 \times 10^9 \ \mathrm{N.m^2/C^2}$$

This constant is also written in the form

$$k_e = 1/4\pi€0$$

Where the constant **€0** is known as the *permittivity of free space* and has the value 8.854 2 $\times10^{-12}$ $C^2/N.m^2$.

The smallest unit of charge known in nature is the charge on an electron or proton, which has an absolute value of

$$|e| = 1.60219 \times 10^{-19}\ \mathrm{C}$$

Table 1 Charge and Mass of the Electron, Proton, and Neutron

Particle	Charge (C)	Mass (kg)
Electron (e)	-1.60219×10^{-19}	$9.109\,5 \times 10^{-31}$
Proton (p)	$+1.60219 \times 10^{-19}$	$1.672\,61 \times 10^{-27}$

Neutron (n)	**0**	**1.674 92 $x10^{-27}$**

EXAMPLE 1:-The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3 $x10^{-11}$ m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Solution: From Coulomb's law, we find that the attractive electric force has the magnitude

$$F_e = k_e|e^2|/r^2$$

$F_e = (8.99 \times 10^9\ N.m^2/C^2)\ (1.60 \times 10^{-19}\ C)^2/(5.3 \times 10^{-11}\ m)^2$

$F_e = 8.2 \times 10^{-8}\ N$

Using Newton's law of gravitation and Table 1 for theparticle masses, we find that the gravitational force has the magnitude

$$F_g = G\ m_e m_p/\ r^2$$

$F_g = (6.7 \times 10^{-11} N.m^2/kg^2) \times (9.11 \times 10^{-31} kg)(1.67 \times 10^{-27} kg)/(5.3 \times 10^{-11})^2$

$F_g = 3.6 \times 10^{-47}\ N$

The ratio Thus, the gravitational force between charged atomic particles is negligible when compared with the electric force. Note the similarity of form of Newton's law of gravitation and Coulomb's law of electric forces.

* * *

The force is a vector quantity thus, the law expressed in vector form

$$F_{12} = k_e q_1 q_2 \hat{r}/r^2 \qquad (1)$$

Where $\hat{r}$ is a unit vector directed from q_1 to q_2 , as shown in Figure (2a) from equation(1) , we see that if q_1 and q_2 have the same sign, as in Figure 2a, the product q_1q_2 is positive and the force is repulsive. If q_1 and q_2 are of opposite sign, as shown in Figure 2b, the product q_1q_2 is negative and the force is attractive. Noting the sign of the product q_1q_2 is an easy way of determining the direction of forces acting on the charges

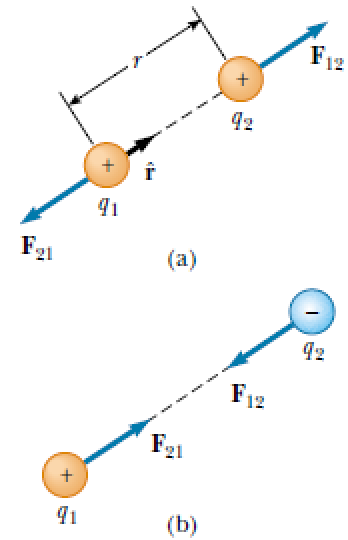


Figure 2*Two point charges separated by a distance r exert a force on each other that is given by Coulomb's law. The force F_{21} exerted by q_2 on q_1 is equal in magnitude and opposite in direction to the force F_{12} exerted by q_1 on q_2. (a) When the charges are of the same sign, the force is repulsive. (b) When the charges are of opposite signs, the force is attractive.*

Q:Object A has a charge of +2μC, and object B has a charge of+6 μC. Which statement is true?   a) $F_{AB}=3F_{BA}$ b) $F_{AB}=-F_{BA}$ c) $3F_{AB}=F_{BA}$

When more than two charges are present, the force between any pair of them is given by Equation (1). Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the various individual charges. Forexample, if four charges are present, then the resultant force exerted by particles2, 3, and 4 on particle 1 is:

$$F_1=F_{21}+F_{31}+F_{41}$$

EXAMPLE 2:-Consider three point charges located at the corners of a right triangle as shown in Figure 3, where $q_1=q_3=5\mu c$,$q_2=-2\ \mu c$, and a=0.1m. Find the resultant force exerted on q_3 .

Solution : First, note the direction of the individual forces exerted by q_1 and q_2 on q_3 . The force F_{23} exerted by q_2 on $q_3$is attractive because q_2 and q_3 have opposite signs. The forceF_{13} exerted by q_1 on q_3 is repulsive because both charges are positive.The magnitude of F_{23} is

$$F_{23}=k_e\ |q_2\ ||q_3|\ /\ a^2$$

F_{23}=(8.99 x10^9N.m^2/C^2)x(2.0 x10^{-6} C)(5.0 x10^{-6} C)/(0.10 m)^2
F_{23}= 9.0N

Note that because q_3 and q_2 have opposite signs, F_{23} is to the left, as shown in Figure 4.
The magnitude of the force exerted by q_1 on q_3 is

$$F_{13}= k_e\ |q_1|\ |q_3|\ /\ (\sqrt{2}a)^2$$

F_{13}=(8.99 x10^9 N.m^2/C^2) (5.0 x 10^{-6} C)(5.0 x10^{-6} C)/2(0.10 m)^2
F_{13}= 11N

The force F_{13} is repulsive and makes an angle of 45° with the x axis. Therefore, the x and y components of F_{13} are equal,with magnitude given by

$$F_{13}\cos 45° = 7.9\ N.$$

The force F_{23} is in the negative x direction. Hence, the xand y components of the resultant force acting on q_3 are

$$F_{3x}=F_{13x}+F_{23}=7.9\ N - 9.0\ N = -1.1\ N$$
$$F_{3y}=F_{13y}=7.9\ N$$

We can also express the resultant force acting on q_3 in unit vector form as

$$F_3=(-1.1i + 7.9j)\ N$$

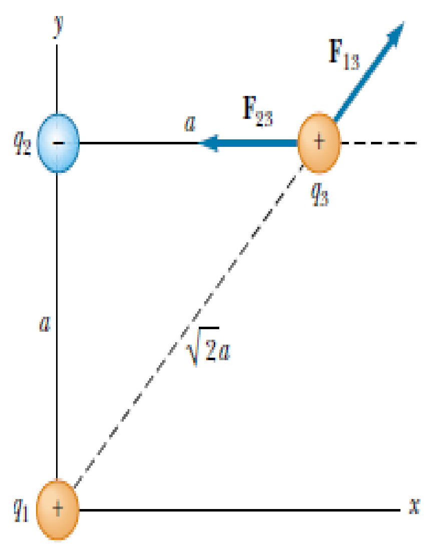


Figure 4:The force exerted by q_1 on q_3 is F_{13} The force exerted by q_2 on q_3 is F_{23} . The resultant force F3 exerted on q_3 is the vector sum $F_{13} + F_{23}$.

THE ELECTRIC FIELD

the electric field E at a point in space is defined as the electric force Fe acting on a positive test charge q_0 placed at that point divided by the magnitude of the test charge:

$$\mathbf{E \equiv F_e/q_0}$$

Note that E is the field produced by some charge external to the test charge—it is not the field produced by the test charge itself. For example, every electron comes with its own electric field. The vector E has the SI units of newtons per coulomb (N/C).

To determine the direction of an electric field, consider a point charge q located a distance r from a test charge q_0 located at a point P, as shown in Figure (5) According to Coulomb's law, the force exerted by q on the test charge is

$$\mathbf{F_e = k_e q\, q_0 \hat{r}/r^2}$$

where $\mathbf{\hat{r}}$ is a unit vector directed from q toward q_0. Because the electric field at P, the position of the test charge, is defined by $E \equiv F_e/q_0$,we find that at P, the electric field created by q is

$$\mathbf{E = k_e q \hat{r}/r^2}$$

If q is positive, as it is in Figure (5a), the electric field is directed radially out ward from it. If q is negative, as it is in Figure (5b), the field is directed toward it.

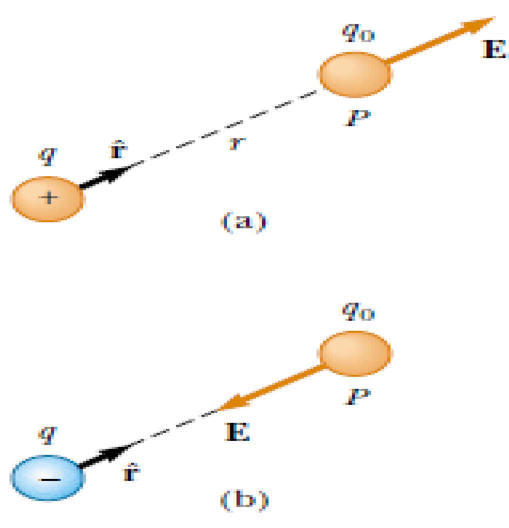


Figure 5: A test charge q0 at point P is a distance r from a point charge q. (a) If q is positive, then the electric field at P points radially outward from q. (b) If q is negative, then the electric field at P points radially inward toward q.

- at any point P, the total electric field due to a group of charges equals the vector sum of the electric fields of the individual charges

$$\mathbf{E} = k_e \Sigma_i q_i \hat{r}_i / r_i^2$$

where r_i is the distance from the ith charge q_i to the point P (the location of the test charge) and $\hat{\mathbf{r}}_i$ is a unit vector directed from q_i toward P.

Q:A charge of +3µC is at a point P where the electric field is directed to the right and has a magnitude of 4 X10^6 N/C. If the charge is replaced with a -3 µC charge, what happens to the electric field at P ?

EXAMPLE 3:-A charge q_1=7.0 µC is located at the origin, and a second charge q_2=5.0 µC is located on the **x**-axis, 0.30 m from the origin (Fig.6). zFind the electric field at the point **P**, which has coordinates (0, 0.40) m.

Solution First, let us find the magnitude of the electric field at P due to each charge. The fields E_1 due to the 7.0 µC charge and E_2 due to the -5.0 µC charge are shown in Figure(6)their magnitudes are:

$E_1 = k_e|q_1| / r_1^2 = (8.99 \times 10^9\ N.m^2/C^2)\ (7.0 \times 10^{-6}\ C)/\ (0.40\ m)^2$
$E_1 = 3.9 \times 10^5 N/C$

$E_2 = k_e|q_2| / r_2^2 = (8.99 \times 10^9\ N.m^2/C^2)\ (5.0 \times 10^{-6}\ C)/\ (0.50\ m)^2$
$E_2 = 1.8 \times 10^5 N/C$

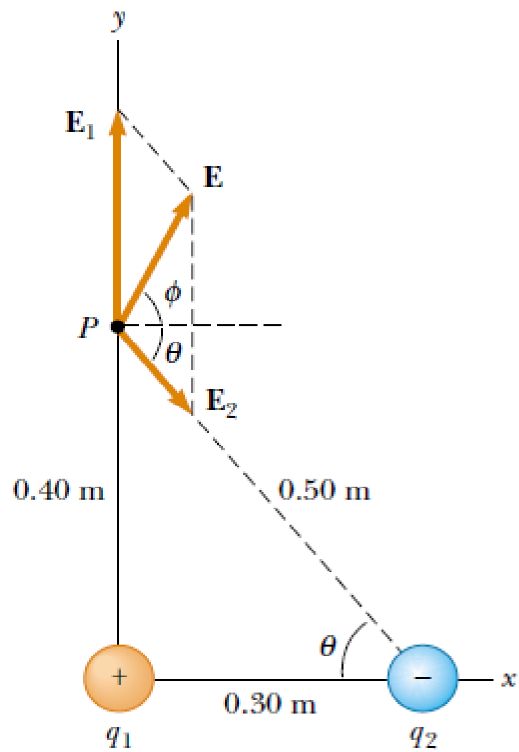


Figure 6 The total electric field E atP equals the vector sum E_1+E_2 where E_1 is the field due to the positive charge q $_1$ and E_2 is the field due to the negative charge q_2

The vector E_1 has only a **y** component. The vector E_2 has an **x** component given by $E_2 \cos\theta=3/5E_2$ and a negative **y** component given by
$-E_2 \sin\theta=-4/5E_2$Hence, we can express the vectors as:
$E_1=3.9\ x10^5 j$ N/C
$E_2=(1.1x10^5 i-1.4\ x10^5 j)$ N/C
The resultant field E at P is the superposition of E_1 and E_2 :
$E = E_1+E_2=(1.1\ x10^5 i+2.5\ x10^5 j)$ N/C

EXAMPLE 4:- An **electric dipole**is defined as a positive charge q and a negative charge -q separated by some distance. For the dipole shown in Figure 7, find the electric field E at P due to the charges, where P is a distance y>>a from the origin.

Solution : At P, the fields E_1 and E_2 due to the two charges are equal in magnitude because P is equidistant from the charges. The total field is
$E= E_1+ E_2$,where

$$E_1 = E_2 = k_e q/r^2 = k_e q/(y^2 + a^2)$$

The y components of E_1 and E_2 cancel each other, and the **x** components add because they are both in the positive **x** -direction. Therefore, E is

Fig. 7 we see that $\cos\theta = a/r = a/(y^2 + a^2)^{1/2}$ Therefore,

$$E = 2E_1\cos\theta = 2k_e qa/(y^2 + a^2)\,(y^2 + a^2)^{1/2}$$

$$E = 2k_e qa/(y^2 + a^2)^{3/2}$$

Because y>>awe can neglect a^2 and write

$$E \approx 2k_e qa/y^3$$

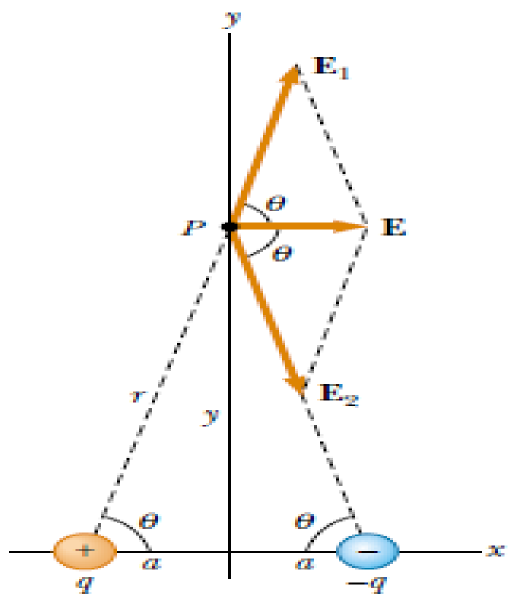


Figure 7 The total electric field E atP due to two charges of equal magnitude and opposite sign (an electric dipole) equals the vector sum $E_1 + E_2$.The field E_1 is due to the positive charge q, and E_2 is the field due to the negative charge -q.

ELECTRIC FIELD OF A CONTINUOUS CHARGE DISTRIBUTION

To evaluate the electric field created by a continuous charge distribution, we use the following procedure: First, we divide the charge distribution into small elements, each of which contains a small charge Δ*q*, as shown in Figure 8.

The electric field at *P* due to one element carrying charge Δ*q*is

$$\Delta E = k_e \Delta q \hat{r}/r^2$$

where r is the distance from the element to point P and $\hat{\mathbf{r}}$ is a unitvector directedfrom the charge element toward P. The total electric field at P due to all elementsin the charge distribution is approximately

$$\Delta \mathbf{E} \approx k_e \sum_i \Delta q_i \hat{\mathbf{r}}_i / r_i^2$$

where the index $i$refers to the ith element in the distribution. Because the chargedistribution is approximately continuous so:

$$E = k_e \int dq\hat{r}/r^2 \quad \text{(Electric field of a continuous charge distribution)}$$

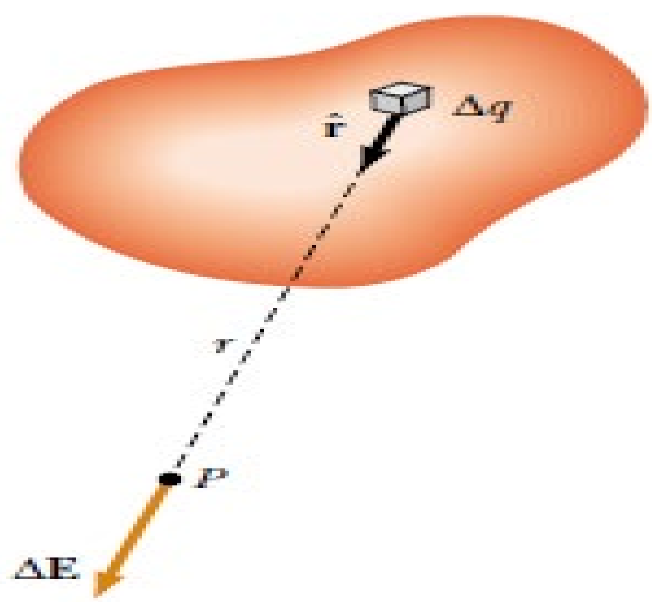


Figure 8: **The electric field at *P* due to a continuous charge distributionis the vector sum of the fieldsΔEdue to all the elements Δqof the charge distribution.**

Note

• If a charge *Q* is uniformly distributed throughout a volume *V*, the volume charge density ρ is defined by

$$\rho \equiv Q/V$$

where ρ has units of coulombs per cubic meter (C/m^3).

• If a charge *Q* is uniformly distributed on a surface of area *A*, the surface charge density σ is defined by

$$\sigma \equiv Q/A$$

where σ has units of coulombs per square meter (C/m^2).

• If a charge *Q* is uniformly distributed along a line of length L, the linear charge density λ is defined by

$$\lambda \equiv Q/L$$

where λ has units of coulombs per meter (C/m).

• If the charge is no uniformly distributed over a volume, surface, or line, we have to express the charge densities as

$$\rho=dQ/dV \quad \sigma=dQ/dA \quad \lambda=dQ/dL$$

where *dQ* is the amount of charge in a small volume, surface, or length element.

EXAMPLE 5:- A rod of length**L**has a uniform positive charge per unit Length **λ** and a total charge ***Q***. Calculate the electric field at apoint ***P*** that is located along the long axis of the rod and adistance ***a*** from one end (Fig. 9).

Solution: Let us assume that the rod is lying along the *x* axis, that *dx* is the length of one small segment, and that *dq* is the charge on that segment. Because the rod has a charge per unit length **λ**, the charge *dq* on the small segment is **dq=λdx** The field *d*Edue to this segment at *P* is in the negative *x*direction (because the source of the field carries a positivecharge *Q*), and its magnitude is

$$dE= k_e dq/x^2=k_e\lambda dx/x^2$$

The total field at *P*due to all segments of the rod, which are at different distancesfrom *P*, is given by

$$E= \int k_e\lambda dx/x^2$$

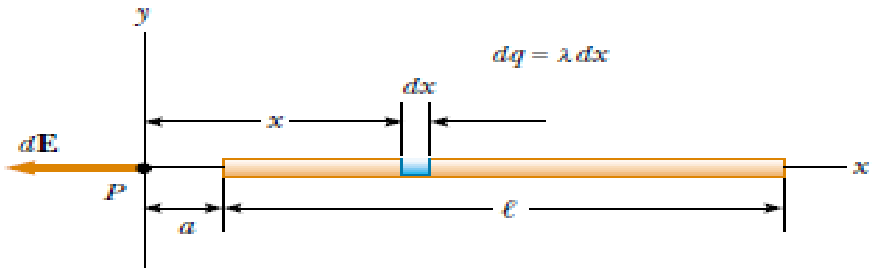


Figure 9The electric field at *P* due to a uniformly chargedrod lying along the *x* axis. The magnitude of the field at *P* due to thesegment of charge *dq*is $k_e dq/x^2$. The total field at *P* is the vector sum over all segments of the rod.

where the limits on the integral extend from one end of the rod (x=a) to the other(x=L+a). The constants $\mathbf{k_e}$ and **λ** can be removed from the integral to yield

$$E=k_e\lambda\int dx/x^2=k_e\lambda[-1/x]$$

$$E=\ k_e\lambda\{(1/a)-(1/(L+a)\}$$

$$E=k_e\ Q/a(L+a)$$

Where we have used the fact that the total charge *Q* =**λL.** If *P* is far from the rod **(a>>L),** then the l in the denominator can be neglected, and
$\mathbf{E=k_e\ Q/a^2}$**.**This is just the form you would expect for a point charge.

EXAMPLE 6:

A ring of radius ***a*** carries a uniformly distributed positive total charge *Q*. Calculate the electric field due to the ring at a point *P* lying a distance *x* from its center along the central axis perpendicular to the plane of the ring (Fig. 10).

Solution The magnitude of the electric field at *P* due to the segment of charge *dq*is

$$dE=\ k_e dq/r^2$$

This field has an *x* component dE_x=dEcosθ along the axis and a component $dE_\perp$ perpendicular to the axis. As we see in Figure 10b, however, the resultant field at *P* must lie along the *x* axis because the perpendicular components of all the various charge segments sum to zero. That is, the perpendicular component of the field created by any charge element is canceled by the perpendicular component created by an element on the opposite side of the ring. Because and $r=(x^2+a^2)^{1/2}$ and cosθ= x/r, we find that

$$dE_x=dE\cos\theta=(k_e dq/r^2)x/r=k_e x dq/(x^2+a^2)^{3/2}$$

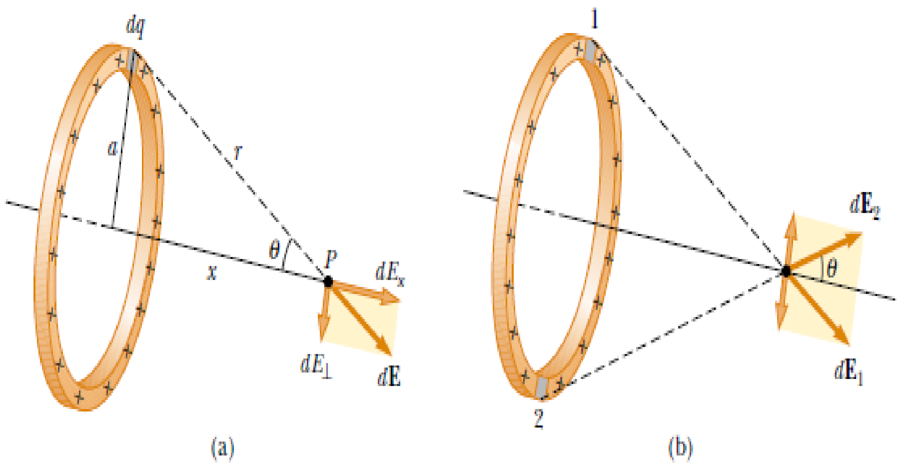


Figure 10 A uniformly charged ring of radius *a*. (a) The field at *P* on the *x* axis due to an elementof charge *dq*. (b) The total electric field at *P* is along the *x* axis. The perpendicular component ofthe field at *P* due to segment 1 is canceled by the perpendicular component due to segment 2.

All segments of the ring make the same contribution to the field at *P* because they are all equidistant from this point. Thus, we can integrate to obtain the total field at *P* :

$$E_x=\int k_e x dq / (x^2+a^2)^{3/2} = \{k_e x / (x^2+a^2)^{3/2}\}\int dq$$

$$E_x=k_e x Q / (x^2+a^2)^{3/2}$$

This result shows that the field is zero at $x=0$

HOME WORKE:- **A disk of radius *R* has a uniform surface charge ﻻ density σ .Calculate the electric field at a point *P* that lies along the central perpendicular axis of the disk and a distance *x* from the center of the disk .**

ELECTRIC FIELD LINES

Rules for drawing electric field lines

- The lines must begin on a positive charge and terminate on a negative charge.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.

MOTION OF CHARGED PARTICLES IN AUNIFORM ELECTRIC FIELD

When a particle of charge *q* and mass *m* is placed in an electric field E, the electric force exerted on the charge is *q*E. If this is the only force exerted on the particle, it must be the net force and so must cause the particle to accelerate. In this case, Newton's second law applied to the particle gives

$$F_e=qE=ma$$

The acceleration of the particle is therefore

$$\mathbf{a=qE\ /m}$$

If E is uniform (that is, constant in magnitude and direction), then the accelerationis constant. If the particle has a positive charge, then its acceleration is in thedirection of the electric field. If the particle has a negative charge, then its accelerationis in the direction opposite the electric field.

EXAMPLE 23.11:-An electron enters the region of a uniform electric field (electric field in the positive y direction and the acceleration of the electron is in the negative direction), with v_i =3x10^{6}m/s and E=200N/C. The horizontal length of the plates is L=0.100 m. (a) Find the acceleration of the electron while it isin the electric field.

Solution : The charge on the electron has an absolutevalue of 1.60 x10^{-19}C, and m=9.11x10 $^{-31}$kg.Therefore,

$$\mathbf{a= -eEj\ /m}=-(1.60\times10^{-19}C)(200N/C)j/(9.11\times10^{-31}kg)$$

$$a=-3.51\times10^{13}j\ m/s^2$$

(b) Find the time it takes the electron to travel through the field.

Solution: The horizontal distance across the field is l= 0.100 m. we find that thetime spent in the electric field is

$\boldsymbol{t}$ **=l/v$_i$**= 0.100 m/3.00x10^6 m/s=3.33 x10^{-8}s

****** ****** *******

SUMMARY

Electric charges have the following important properties:

- **Unlike charges attract one another, and like charges repel one another.**
- **Charge is conserved.**
- **Charge is quantized.**

Conductorsare materials in which charges move freely. Insulators are materials in which charges do not move freely.

Coulomb's lawstates that the electric force exerted by a charge q_1 on a second chargeq_2 is:

$$F_{12}= k_e q_1 q_2 \hat{r}/r^2$$

where$r$ is the distance between the two charges and$\hat{r}$ is a unit vector directed fromq_1 to q_2 . The constant k_e, called the Coulomb constant, has the value

$$k_e= 8.987\,5 \times 10^9\ N.m^2/C^2$$

The smallest unit of charge known to exist in nature is the charge on an electron or proton:

$$|e|=1.60219 \times 10^{-19}\ C$$

The electric field E at some point in space is defined as the electric force F_e that acts on a small positive test charge placed at that point divided by the magnitudeof the test charge q_0 :

$$E \equiv F_e/q_0$$

At a distance r from a point charge q, the electric field due to the charge is given by:

$$E= k_e q \hat{r}/r^2$$

where$\hat{r}$ is a unit vector directed from the charge to the point in question. The electric field is directed radially outward from a positive charge and radially inward toward a negative charge

The electric field due to a group of point charges can be obtained by using the superposition principle. That is, the total electric field at some point equals the vector sum of the electric fields of all the charges:

$$E= k_e \Sigma_i q_i \hat{r}_i/r_i^2$$

The electric field at some point of a continuous charge distribution is

$$E= k_e \int dq \hat{r}/r^2$$

where$dq$is the charge on one element of the charge distribution and r is the distancefrom the element to the point in question.

Electric field lines describe an electric field in any region of space. The numberof lines per unit area through a surface perpendicular to the lines is proportionalto the magnitude of E in that region.

A charged particle of mass *m* and charge *q* moving in an electric field E has acceleration

$$a = qE / m$$

CHAPTER TWO (GAUSS'S LAW)

In the preceding chapter we showed how to use Coulomb's law to calculate the electric field generated by a given charge distribution. In this chapter, we describe Gauss's law and an alternative procedure for calculating electric fields.

ELECTRIC FLUX

Consider an electric field that is uniform in both magnitude and direction, as shown in Figure 2.1.The product of the magnitude of the electric field E and surface area A perpendicular to the field is called the **electric flux**Φ_E

$$\Phi_E = EA \quad (2.1)$$

From the SI units of E and A, we see that Φ_E has units of ($N.m^2/C$).**Electric flux is proportional to the number of electricfield lines penetrating some surface.**

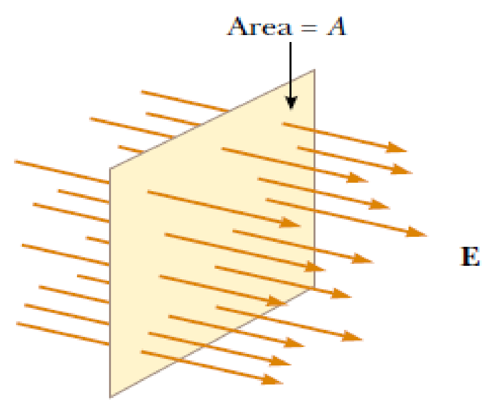


Figure 2.1 Field lines representing a uniform electric field penetrating a plane of area A perpendicular to the field. The electric flux Φ_E through this area is equal toEA.

EXAMPLE1:

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of +1.00 µC at its center?

Solution The magnitude of the electric field 1.00 m from this charge is given by equation

$$E=\frac{ke\ |q|}{r^2}= (8.99\times10^9\ N.m^2/C^2)\ (1.0\times10^{-6}\ C)/\ (1.00\ m)^2$$

$E_1=8.9\times10^3 N/C$

The field points radially outward and is therefore everywhereperpendicular to the surface of the sphere. The flux through the sphere (whose surface areaA=$4\pi r^2$=($12.6m^2$) is thus

$$\Phi_E = EA = (8.9\times10^3 N/C)\ (12.6m^2)$$
$$=1.13\times10^5\ N.m2/C$$

Home work: What would be the (a)electric field and (b) flux through the sphere if it had a radius of 0.500 m?

*** *** ***

If the surface under consideration is not perpendicular to the field, the flux through it must be less than that given by Equation 2.1.From Figure 2.2 we see that the two areas are related by Å= A cosθ. Because the flux through A equals the flux through Å, weconclude that the flux through A is

Φ_E=E Å =EA cosθ

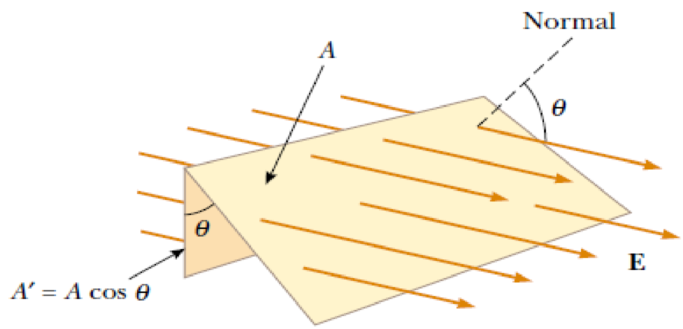


Figure 2.2 Field lines representing auniform electric field penetrating an area Athat is at an angle θ to the field. Because the number of lines that go through the area Å is the same as the number that go through A, the flux through Åis equal to the flux through A and is given by
Φ_E = EA cosθ

From this result, we see that the flux through a surface of fixed area A has a maximum value EA when the surface is perpendicular to the field (in other words, when the normal to the surface is parallel to the field, that is, θ=0 in Figure 2.2); the flux is zero when the surface is parallel to the field (in other words, when the normal to the surface is perpendicular to the field, that is, θ=90. In more general consider a surface divided up into a large number of small elements (Figure 2.3)

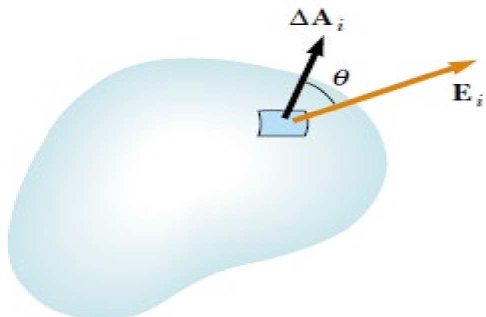
$$\Phi_E = \int_{surface} E.dA \qquad \text{(Definition of electric flux)}$$

The net flux through the surface is proportional to the net number of lines leaving the surface.If more lines are leaving than entering, the net flux is positive. If more lines are entering than leaving, the net flux is negative.
we can write the net flux $\Phi_E$through a closed surface as

$$\mathbf{\Phi_E = \int E.dA = \int E_n dA} \qquad \mathbf{(2.2)}$$

Where $E_n$represents the component of the electric field normal to the surface.

Figure 2.3 a small element of surface area ΔA_i. The electric field makes an angle θwith the vector ΔA_i , defined as being normal to the surface element, and the flux through the element is equal to $E_i \Delta A_i$ cos .



EXAMPLE 2:-Consider a uniform electric field E oriented in the x direction.Find the net electric flux through the surface of a cubeof edgesL, oriented as shown in Figure 2.4.

Solution : The net flux is the sum of the fluxes through allfaces of the cube. First, note that the flux through four of the faces (3),(4), and the unnumbered ones) is zero because E is perpendicular to dAon these faces.

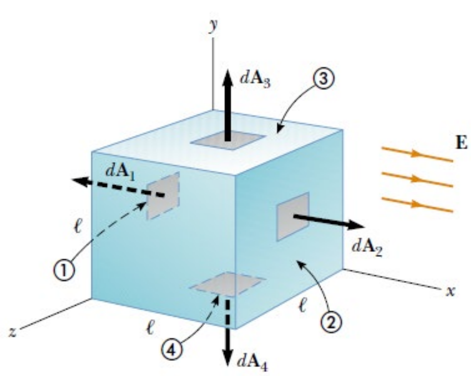


Figure 2.4 A closed surface inthe shape of a cube in a uniform electric field oriented parallel to the x axis.

The net flux through faces 1and2 is

$$\Phi_E = \int_1 E.dA + \int_2 E.dA$$

For face(1),E is constant and directed inward but dA_1 is directed Outward(θ=180); thus, the flux through this face is

$$\int_1 E.dA = \int_1 E.(\cos 180)dA = -E\int_1 dA = -EA = -EL^2$$

Because $\mathbf{L^2}$ the area of each face is$A=L^2$

For face(2) ,E is constant and outward and in the same direction asdA_2(θ=0); hence, the flux through this face is

$$\int_2 E.dA = \int_2 E.(\cos 0)dA = -E\int_2 dA = +EA = EL^2$$

Therefore, the net flux over all six faces is

$$\Phi_E = -EL^2 + EL^2 + 0 + 0 + 0 + 0 = 0$$

GAUSS'S LAW

Consider a positive point charge q located at the center of a sphere of radius r, as shown in Figure 2.5. From Equation ($E = k_e|q|/r^2$) we know that themagnitude of the electric field everywhere on the surface of the sphere

As noted in Example 2.1, the field lines are directed radially outward and hence perpendicular to the surface at every point on the surface. That is, at each surface point, E is parallel to the vector ΔA_irepresenting a local element of area ΔA_isurrounding the surface point. Therefore,

$$E_i.\Delta A_i = E_i \Delta A_i$$

and from Equation 2.2 we find that the net flux through the gaussian surface is

$$\Phi_E = \int E.dA = \int E dA = E\int dA$$

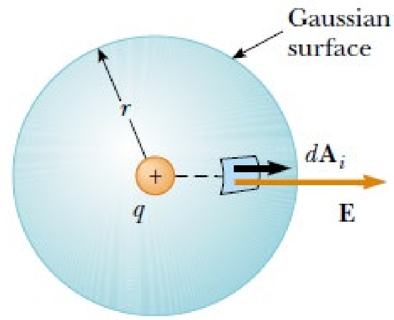


*Figure 2.5*A spherical Gaussian surface of radius *r* surrounding apoint charge *q*. When the charge is at the center of the sphere, theelectric field is everywhere normalto the surface and constant in magnitude

where we have moved E outside of the integral because, by symmetry, E is constant over the surface and given by $E= k_e q/r^2$.Furthermore, because the surface is spherical, Hence, the net flux through the gaussian surface is

$$\Phi_E = \frac{ke\ q\ (4\pi r^2)}{r^2} = 4\ \pi\ k_e q = \frac{q}{€0} \rightarrow (\text{because } k_e = 1/4\pi€0)$$

We can verify that this expression for the net flux gives the same result as Example 1:

<u>Home work:</u>

Suppose that the charge in Example 24.1 is just outside the sphere, 1.01 m from its center. What is the total flux through the sphere?

Notes:-

1- the net flux through any closed surface is

$$\Phi_E = \int E.dA = = \frac{q}{€0} \qquad \text{(Gauss's law)}$$

2-the net electric flux through a closed surface thatsurrounds no charge is zero.

3-the electric field due to many charges is the vector sum of the electric fields produced by the individual charges

$$\int E.dA = \int (E_1 + E_2 + \ldots\ldots\ldots).dA$$

5-Gauss's law is useful for evaluating E when the charge distribution has high symmetry

6-The net electric fluxthrough any closed surface depends only on the charge insidethat surface(see figure 2.6).

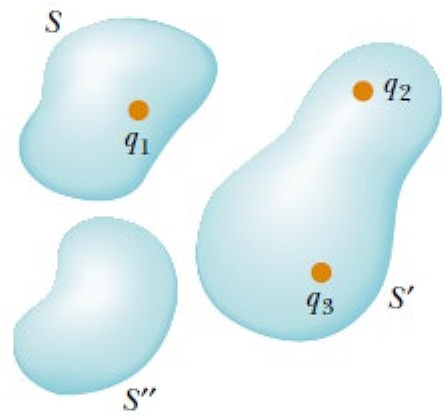


Figure 2.6 The net flux throughsurface S is **q_1/ €0** ,
the net fluxthrough surface $S^{\backslash}$is**q_1+q_2/ €0** and the net flux through surface$S^{\backslash\backslash}$is zero.

Home work:

For a gaussian surface through which the net flux is zero, the following four statements could be true. Which of the statements must be true? (a) There are no charges inside the surface. (b) The net charge inside the surface is zero. (c) The electric field is zero everywhere on the surface. (d) The number of electric field lines entering the surface equals the numberleaving the surface

EXAMPLE 2.3 :A spherical gaussian surface surrounds a point charge *q*. Describewhat happens to the total flux through the surface if(a) the charge is tripled, (b) the radius of the sphere is doubled,(c) the surface is changed to a cube, and (d) the chargeis moved to another location inside the surface.

Solution (a) The flux through the surface is tripledbecause flux is proportional to the amount of charge insidethe surface.

(b) The flux does not change because all electric field lines from the charge pass through the sphere, regardless ofits radius.

(c) The flux does not change when the shape of the Gaussian surface changes because all electric field lines from thecharge pass through the surface, regardless of its shape.

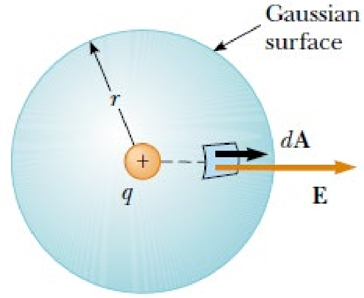
(d) The flux does not change when the charge is moved to another location inside that surface because Gauss's law refers to the total charge enclosed, regardless of where the charge is located inside the surface.

EXAMPLE 2.4:- Starting with Gauss's law, calculate the electric field due to anisolated point charge *q*.

Solution : We choose aspherical gaussian surface of radius *r* centered on the pointcharge, as shown in Figure 2.7 .The electric field due to apositive point charge is directed radially outward by symmetryand is therefore normal to the surface at every point(E is parallel to *d*Aat each point). Therefore,E.dA= EAand Gauss's law gives

$$\Phi_E = \int E.dA = \int EdA = \frac{q}{€0}$$

*Figure 2.7*The point charge *q* is at the center of the sphericalgaussian surface, and E is parallel to *d*Aat every point on thesurface.



By symmetry, *E* is constant everywhere on the surface, so it can be removed from the integral.Therefore,

$$\int EdA = E\int dA = E(4\pi r^2) = \frac{q}{€0}$$

$$E = \frac{q}{(4\pi r^2)\,€0}$$

$$E = k_e q/r^2$$

This is the familiar electric field due to a point charge that wedeveloped from Coulomb's law in Chapter 1.

EXAMPLE 2.5:-An insulating solid sphere of radius ***a*** has a uniform volume charge density ρ and carries a total positive charge *Q* (Fig. 2.8).

(a) Calculate the magnitude of the electric field at a point outside the sphere.

Solution : Because the charge distribution is spherically symmetric, we again select a spherical gaussian surface of radius *r*, concentric with the sphere, as shown in Figure 2.7a.for the point charge in Example 2.4. we find that

$$E = k_e Q/r^2 \rightarrow \text{(for } r>a\text{)}$$

Note that this result is identical to the one we obtained for apoint charge. Therefore, we conclude that, for a uniformly charged sphere, the field in the region external to the sphereis *equivalent* to that of a point charge located at the center ofthe sphere.

(b) Find the magnitude of the electric field at a point inside the sphere.

Solution: In this case we select a spherical gaussian surface having radius (*r <a)*, concentric with the insulated sphere(Fig. 2.8b). Let us denote the volume of this smaller sphere by $V^\backslash$. To apply Gauss's law in this situation, it is importantto recognize that the charge q_{in} within the Gaussian surface of volume $V^\backslash$is less than *Q*. To calculate q_{in} , we usethe fact that$q_{in}=\rho V^\backslash$

$$q_{in}=\rho\ V^\backslash= \rho\ (\frac{4}{3}\pi\ r^{3})$$

By symmetry, the magnitude of the electric field is constant everywhere on the spherical gaussian surface and is normalto the surface at each point. Therefore, Gauss's law in the region *r <a* gives

$$\int EdA = E\int dA = E(4\pi\ r^{2}) = \frac{q\text{in}}{€0}$$

Solving for *E* gives

$$E = \frac{q\text{in}}{4\pi r^2 €0} = \frac{\rho\ (\frac{4}{3}\pi r^3)}{4\pi r^2 €0} = \frac{\rho\ r}{3€0}$$

Because $\rho=Q/\ (\frac{4}{3}\pi a^{3})$by definition and since $k_e=1/4\pi€0$, this expression for *E* can be written as

$$E = \frac{Qr}{4\pi a^3 €0} = \frac{k\ Q\ r}{a^3} \rightarrow \text{(for r<a)}$$

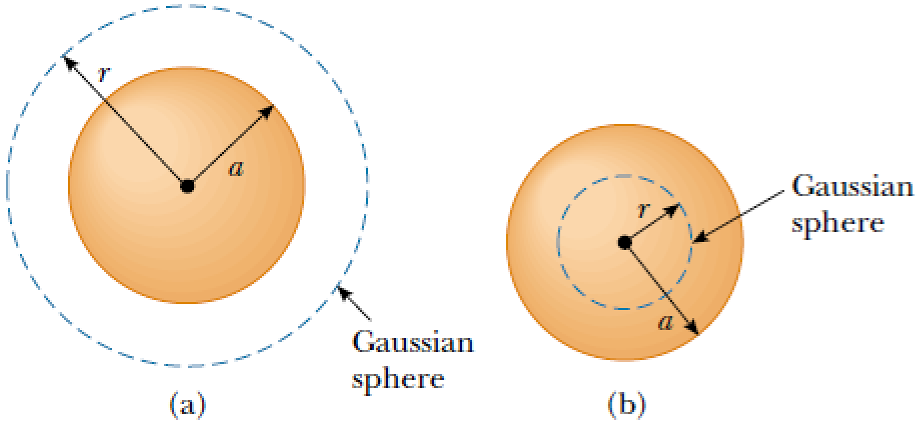


Figure 2.8A uniformly charged insulating sphere of radius *a*and total charge *Q*. (a) The magnitude of the electric field at a pointexterior to the sphere is k_eQ/r^2 (b) The magnitude of the electricfield inside the insulating sphere is due only to the charge *within* thegaussian sphere is k_eQr/a^3.

EXAMPLE 2.6:- A thin spherical shell of radius ***a*** has a total charge ***Q*** distributed uniformly over its surface (Fig. 2.9). Find the electric field at points (a) outside and (b) inside the shell.

Solution:(a) The calculation for the field outside the shellis identical to

that for the solid sphere shown in Example2.5a. Therefore,

$$E=k_eQ/r^2 \rightarrow (\text{for } r>a)$$

(b) The electric field inside the spherical shell is zero.Because of the spherical symmetry of the charge distribution and because the net charge inside the surface is zero.

Shows that E =0 in the region r <a. We obtain the same results using Equation ($E= k_e\int dq/r^2$) from chapter one and integrating over the charge distribution(Gauss's law allows us to determine these results in a much simpler way).

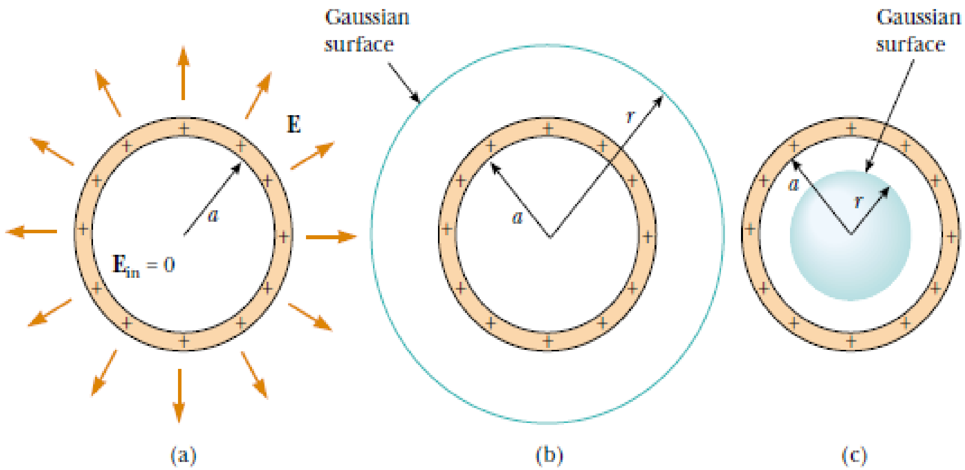


Figure 2.9(a) The electric field inside a uniformly charged spherical shell is zero. The fieldoutside is the same as that due to a point charge *Q* located at the center of the shell. (b) Gaussiansurface for $r > a$. (c) Gaussian surface for $r < a$.

EXAMPLE 2.7:- Find the electric field a distance *r* from a line of positive charge of infinite length and constant charge per unit lengthλ(Fig. 2.10).

Solution :we select a cylindrical gaussian surface of radius *r* and length Lthat is coaxialwith the line charge. For the curved part of this surface, E is constant in magnitude and perpendicular to the surface at each point. Furthermore,the flux through the ends of the gaussian cylinder is zero because E is parallel to these surfaces.
The total charge inside our gaussian surface is λL. ApplyingGauss's law we find that forthe curved surface

$$\Phi_E = \int E.dA = E\int dA = EA = q_{in}/€0 = \frac{\lambda L}{€0}$$

The area of the curved surface A=2 πrL; is therefore,

$$E = \frac{\lambda}{2\ \pi €0 r} = 2k\frac{\lambda}{r}$$

Thus, we see that the electric field due to a cylindrically symmetriccharge distribution varies as 1/*r*, whereas the field externalto a spherically symmetric charge distribution varies as$1/r^2$.

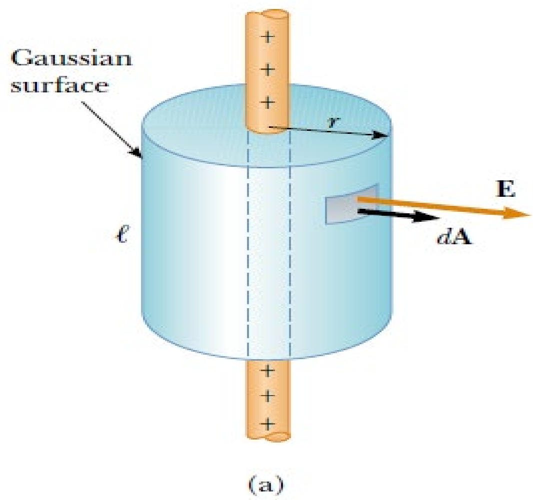


Figure 2.10An infinite line of charge surrounded by a cylindricalgaussian surface concentric with the line.

EXAMPLE 2.8:- Find the electric field due to a nonconducting, infinite plane of positive charge with uniform surface charge density σ.

Solution : By symmetry, E must be perpendicular to theplane and must have the same magnitude at all pointsequidistant from the plane. The fact that the direction of E isaway from positive charges indicates that the direction of Eon one side of the plane must be opposite its direction on the other side, as shown in Figure 2.11. The flux through each end of the cylinder is *EA*;hence, the total flux through the entire gaussian surface isjust that through the ends,Φ_E=2EA.

Noting that the total charge inside the surface is $q_{in}=\sigma A$,we use Gauss's law and find that

$\Phi_E = 2EA = q_{in}/€0 = \sigma A/€0$

$E = \sigma / 2€0$ →*(*)*

Because the distance from each flat end of the cylinder tothe plane does not appear in Equation (*), we conclude that$E=\sigma / 2€0$ at any distance from the plane. That is, the field isuniform everywhere

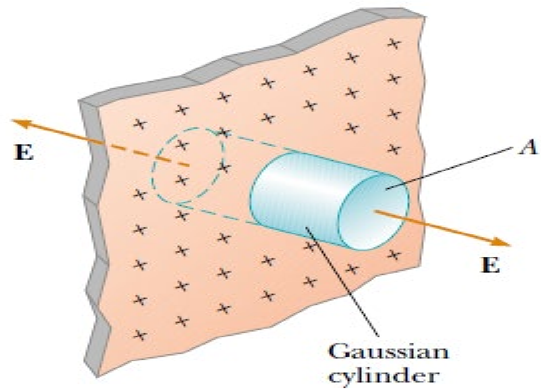


Figure 2.11 A cylindrical gaussian surface penetrating an infiniteplane of charge. The flux is *EA* through each end of the gaussiansurface and zero through its curved surface.

EXAMPLE 2.9:-Explain why Gauss's law cannot be used to calculate the electric field near an electric dipole,a charged disk, or a triangle with a point charge at each corner.

Solution The charge distributions of all these configurations do not have sufficient symmetry to make the use of Gauss's law practical. We cannot find a closed surface surroundingany of these distributions.

CONDUCTORS IN ELECTROSTATIC EQUILIBRIUM

When there is no net motion of charge within a conductor, the conductor is in electrostatic equilibrium. As we shall see, a conductor in electrostatic equilibrium has the following properties:

1. The electric field is zero everywhere inside the conductor.
2. If an isolated conductor carries a charge, the charge resides on its surface.
3. The electric field just outside a charged conductor is perpendicular to the surfaceof the conductor and has a magnitude σ/ϵ_0 , whereσis the surface chargedensity at that point($\Phi_E = \int E.dA = EA = q_{in}/\epsilon_0 = \sigma A/\epsilon_0$where we have used the fact that $q_{in} = \sigma A$. Solving for *E* gives $E = \sigma/\epsilon_0$*).*

4. On an irregularly shaped conductor, the surface charge density is greatest at locationswhere the radius of curvature of the surface is smallest.

EXAMPLE 2.10:-A solid conducting sphere of radius ***a*** carries a net positivecharge 2*Q*. A conducting spherical shell of inner radius ***b***and outer radius

c is concentric with the solid sphere and carriesa net charge -*Q*. Using Gauss's law, find the electricfield in the regions labeled (1),(2),(3) and(4) in Figure 2.12and the charge distribution on the shell when the entire systemis in electrostatic equilibrium.

Solution : To determine the electricfield at various distances *r* from this center, we construct aspherical gaussian surface for each of the four regions of interest.

To find *E* inside the solid sphere (region (1)), consider a gaussian surface of radius ***r <a***. Because there can be nocharge inside a conductor in electrostatic equilibrium, we seethat $q_{in}=0$; thus, on the basis of Gauss's law and symmetry $E_1=0$,for***r<a***.
In region (2)between the surface of the solid sphere andthe inner surface of the shell—we construct a sphericalgaussian surface of radius *r* where
a <r <b and note that thecharge inside this surface is +2*Q*. Because of the spherical symmetry, the electric field lines must be directed radially outward and be constant inmagnitude on the gaussian surface. Following Example 2.4and using Gauss's law, we find that:

$$E_2A=E_2(4\pi r^2)=q_{in}/\epsilon_0=2Q/\epsilon_0 \quad (\text{for } a<r<b)$$

(In region(4), where *r >c*, the spherical gaussian surfacewe construct surrounds a total charge of$q_{in}=2Q+(-Q)=Q$. Therefore, application of Gauss's law tothis surface gives

$$E_4=kQ/r^2 \quad (\text{for } r>c)$$

In region (3), the electric field must be zero because thespherical shell is also a conductor in equilibrium. If we constructa gaussian surface of radius *r* where *b<r <c*, we seethat q_{in} must be zero because $E_3=0$ From this argument, weconclude that the charge on the inner surface of the sphericalshell must be -2*Q* to cancel the charge +2*Q* on the solid
sphere. Because the net charge on the shell is -*Q*, we concludethat its outer surface must carry a charge+*Q*.

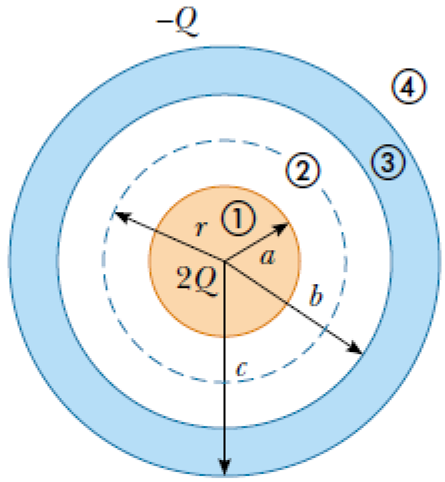


Figure 2.12A solid conducting sphere of radius *a* and carrying acharge 2*Q* surrounded by a conducting spherical shell carrying acharge -*Q*.

SUMMARY

Electric flux is proportional to the number of electric field lines that penetrate a surface. If the electric field is uniform and makes an angle θwith the normal to a surface of area *A*, the electric flux through the surface is

$$\Phi_E = EA\cos\theta \quad (1)$$

In general, the electric flux through a surface is

$$\Phi_E = \int E.dA \quad (2)$$

You need to be able to apply Equations 1 and 2 in a variety of situations, particularlythose in which symmetry simplifies the calculation.

Gauss's law says that the net electric flux $\Phi_E$through any closed gaussian surfaceis equal to the *net* charge inside the surface divided by €0:

$$\Phi_E = \int E.dA = q_{in}/€0$$

Using Gauss's law, you can calculate the electric field due to various symmetric charge distributions. Table 24.1 lists some typical results.

TABLE 24.1 Typical Electric Field Calculations Using Gauss's Law

Charge Distribution	Electric Field	Location
Insulating sphere of radius R, uniform charge density, and total charge Q	$k_e \frac{Q}{r^2}$	$r > R$
	$k_e \frac{Q}{R^3} r$	$r < R$
Thin spherical shell of radius R and total charge Q	$k_e \frac{Q}{r^2}$	$r > R$
	0	$r < R$
Line charge of infinite length and charge per unit length λ	$2k_e \frac{\lambda}{r}$	Outside the line
Nonconducting, infinite charged plane having surface charge density σ	$\frac{\sigma}{2\epsilon_0}$	Everywhere outside the plane
Conductor having surface charge density σ	$\frac{\sigma}{\epsilon_0}$	Just outside the conductor
	0	Inside the conductor

C H A P T E R THREE (Electric Potential)

POTENTIAL DIFFERENCE AND ELECTRIC POTENTIAL

The potential energy of the charge–field system is decreased by an amount **(*dU* = -*q*$_0$E.ds).** For a finite displacement of the charge from a point *A* to a point *B*, the change in potential energy of the system $\Delta U = U_B - U_A$ is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad \textbf{(3.1)}$$

Because the force q_0E is conservative, this line integral does not depend on the path taken from *A* to *B*.

The potential energy per unit charge U/q_0 is independent of the value of q_0 and has a unique value at every point in an electric field. This quantity U/q_0 is called the electric potential (or simply the potential) *V*. Thus, the electric potential at any point in an electric field is

$$V = U/q_0 \qquad \textbf{(3.2)}$$

The potential difference $\Delta V = V_B - V_A$ between any two points *A* and *B* in an electric field is defined as the change in potential energy of the system divided by the test charge q_0 :

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad \textbf{(3.3)}$$

the potential difference is proportional to the change in potential energy, and we see from Equation 3.3

$$\Delta U = q_0 \Delta V$$

-Electric potential is a scalar characteristic of an electric field, independent of the charges that may be placed in the field.

-The electric potential at an arbitrary point in an electric field equals the work required per unit charge to bring a positive test charge from infinity to that point.

Thus, if we take point *A* in Equation 3.3 to be at infinity, the electric potential at any point *P* is

$$V_p = -\int \mathbf{E} . d\mathbf{s} \qquad (3.4)$$

In reality, V_p represents the potential difference ΔV between the point *P* and a point at infinity. Because electric potential is a measure of potential energy per unit charge, the SI unit of both electric potential and potential difference is joules per coulomb, which is defined as a volt (V):
That is, 1 J of work must be done to move a 1-C charge through a potential difference of 1 V.

$$1\ \text{V} = 1\ \frac{\text{J}}{\text{C}}$$

Equation 3.3 shows that potential difference also has units of electric field times distance. From this, it follows that the SI unit of electric field (N/C) can also be expressed in volts per meter:

$$1\ \frac{\text{N}}{\text{C}} = 1\ \frac{\text{V}}{\text{m}}$$

Because 1 V =1 J/C and because the fundamental charge is approximately 1.6×10^{-19} C the electron volt is related to the joule as follows:

$$\mathbf{1\ eV = 1.60 \times 10^{-19}\ C.V = 1.60 \times 10^{-19}\ J}$$

POTENTIAL DIFFERENCES IN A UNIFORM ELECTRIC FIELD

Consider a uniform electric field directed along the negative *y* axis, as shown in Figure 3.1. Let us calculate the potential difference between two points *A* and *B* separated by a distance *d*, where *d* is measured parallel to the field lines. Equation 3.3 gives

$$V_B - V_A = \Delta V = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = -\int_A^B E \cos 0^\circ\, ds = -\int_A^B E\, ds$$

Because *E* is constant, we can remove it from the integral sign; this gives

$$\Delta V = -E\int_A^B ds = -Ed$$

→*(3.6)* **(Potential difference in a uniform electric field)**

The minus sign indicates that point *B* is at a lower electric potential than point *A*; that is $V_B < V_A$.

Electric field lines always point in the direction of decreasing electric potential, as shown in Figure 3.1a.

Now suppose that a test charge q_0 moves from *A* to *B*. We can calculate the change in its potential energy from Equations 3.3 and 3.6:

$$\Delta U = q_0 \Delta V = -q_0 E d \quad \rightarrow (3.7)$$

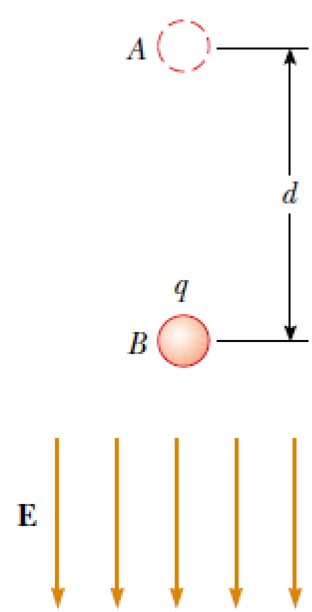


Figure 3.1 When the electric field E is directed downward, point *B* is at a lower electric potential than point *A*. Apositive test charge that moves from point *A* to point *B* loses electric potential energy.

Note

1- A positive charge loses electric potential energy when it moves in the direction of the electric field

2- As the charged particle gains kinetic energy, it loses an equal amount of potential energy.
3- A negative charge gains electric potential energy when it moves in the direction of the electric field
4- The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.

***EXAMPLE 3.1*:-** A 12-V battery is connected between two parallel plates, as shown in Figure 3.2. The separation between the plates is *d* = 0.30 cm, and we assume the electric field between the plates to be uniform. Find the magnitude of the electric field between the plates.

Solution: The magnitude of the electric field between the plates is, from Equation 3.6,

$$E= \frac{|VB - VA|}{\mathrm{d}} = \frac{12\mathrm{V}}{0.3x10^{-2}} = 4.0\mathrm{x}10^{3}\ \mathrm{V/m}$$

This configuration, which is shown in Figure 3.2 and called a *parallel-plate capacitor.*

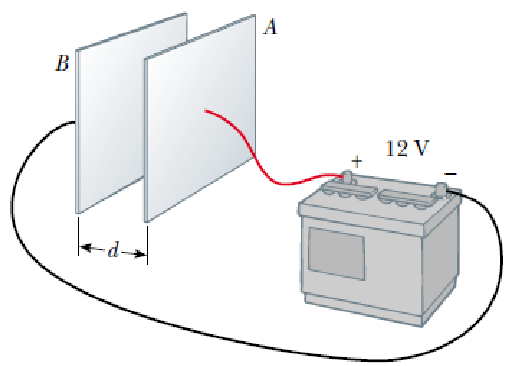


Figure 3.2 A 12-V battery connected to two parallel plates. The electric field between the plates has a magnitude given by the potential difference Δ*V* divided by the plate separation *d*.

***EXAMPLE 3.2*:-** A proton is released from rest in a uniform electric field that has a magnitude of 8.0 X10^4 V/m and is directed along the positive *x* axis (Fig. 3.3). The proton undergoes a displacement of 0.50 m in the direction of E. (a) Find the change in electric potential between points *A* and *B.*

Solution Because the proton (which, as you remember, carries a positive charge) moves in the direction of the field, we expect it to move to a position of lower electric potential. From Equation 3.6, we have

$\Delta V = -Ed = -(8.0 \times 10^4\ V/m)(0.50\ m)$
$= -4 \times 10^4 V$

(b) Find the change in potential energy of the proton for this displacement.

Solution:-

$\Delta U = q_0 \Delta V = e\Delta V$
$= (1.6 \times 10^{-19} C)(-4 \times 10^4 V)$
$= -6.4 \times 10^{-15} J$

The negative sign means the potential energy of the proton decreases as it moves in the direction of the electric field.

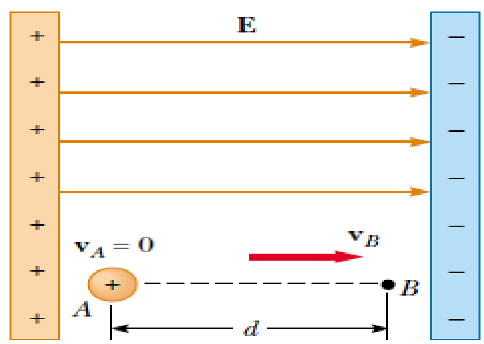


Figure 3.3 A proton accelerates from *A* to *B* in the direction of the electric field.

ELECTRIC POTENTIAL AND POTENTIAL ENERGY DUE TO POINT CHARGES

Consider an isolated positive point charge *q*. To find the electric potential at a point located a distance *r* from the charge

$$V_B - V_A = -\int \mathbf{E} . d\mathbf{s}$$

where *A* and *B* are the two arbitrary points .At any field point, the electric field due to the point charge is $E = k_e q\hat{r}/r^2$ where $\hat{r}$ is a unit vector directed from the charge toward the field point. The quantity E.ds can be expressed as

$$E.ds = k_e \frac{q}{r^2} \hat{r}.ds$$

Because the magnitude of r̂ is 1, the dot product r̂.ds=ds cosθ where θ is the angle between r̂ and *d*s. thus, *ds* cosθ =*dr*. That is, any displacement *d*s along the path from point *A* to point *B* produces a change *dr* in the magnitude of r. Making these substitutions, we find that hence, the expression for the potential difference becomes

$$V_B - V_A = -\int E_r\,dr = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r}\Bigg]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \qquad \rightarrow (3.10)$$

The integral of E.ds is *independent* of the path between points *A* and *B*—as it must be because the electric field of a point charge is conservative.

-The electric potential created by a point charge at any distance *r* from the charge is

$$V = k_e q/r \quad \rightarrow (3.11)$$

For a group of point charges, we can write the total electric potential at *P* in the form (Electric potential due to several point charges)

$$V = k_e \sum_i \frac{q_i}{r_i} \qquad \rightarrow (3.12)$$

where the potential is again taken to be zero at infinity and r_i is the distance from the point *P* to the charge q_i .

we can express the potential energy or Electric potential energy due to two charges as

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

the total potential energy of the system of three charges shown in Figure 3.4 is

$$U = k_e\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) \qquad \rightarrow (3.13)$$

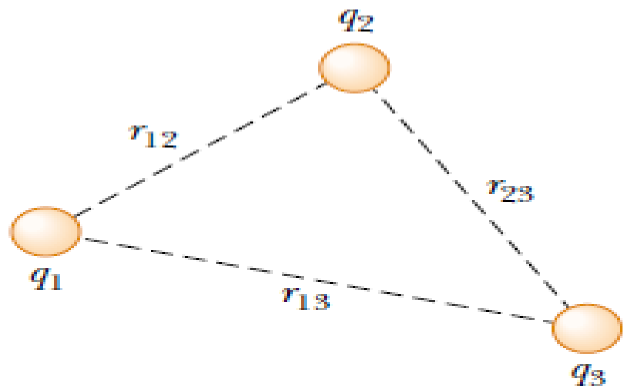


Figure 3.4Three point charges are fixed at the positions shown. The potential energy of this system of charges is given by Equation 3.13.

***EXAMPLE 3.3*:-**A charge q_1=2.00 µC is located at the origin, and a charge q_2=-6.00 C is located at (0, 3.00) m, as shown in Figure 3.5a. (a) Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m.

Solution For two charges, the sum in Equation 3.12 gives

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= 8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}\left(\frac{2.00 \times 10^{-6}\ \mathrm{C}}{4.00\ \mathrm{m}} + \frac{-6.00 \times 10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right)$$

$$= \boxed{-6.29 \times 10^3\ \mathrm{V}}$$

(b) Find the change in potential energy of a 3.00µC charge as it moves from infinity to point *P* (Fig. 3.6b).

Solution When the charge is at infinity,U_i=0 , and when the charge is at *P*, U_f=q_3V_p; therefore,

$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \mathrm{C})(-6.29 \times 10^3\ \mathrm{V})$$

$$= \boxed{-18.9 \times 10^{-3}\ \mathrm{J}}$$

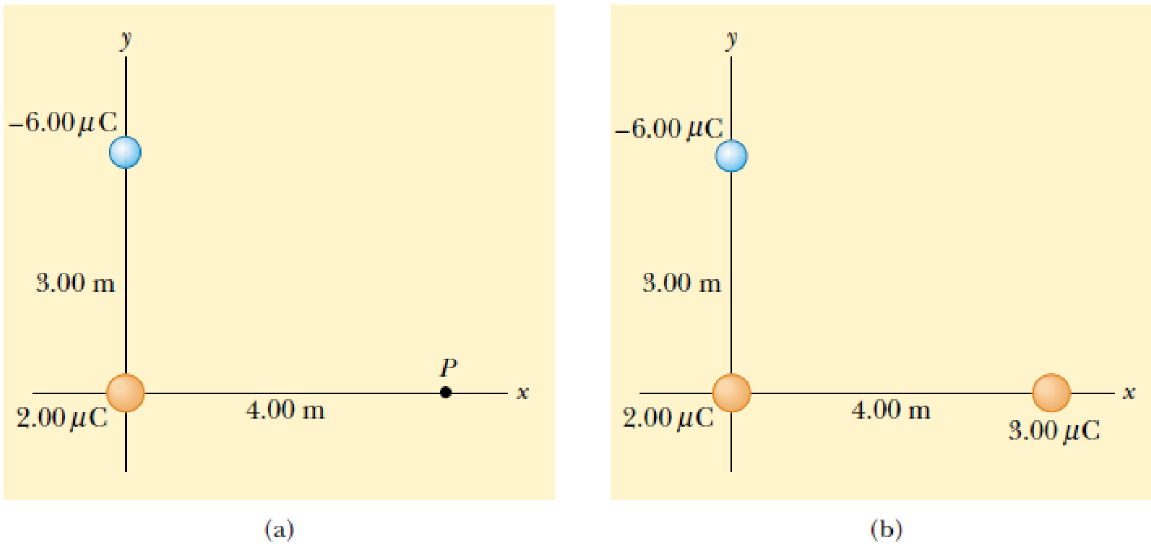


Figure 3.5 (a) The electric potential at *P* due to the two charges is the algebraic sum of the potentials due to the individual charges. (b) What is the potential energy of the three-charge system?

OBTAINING THE VALUE OF THE ELECTRIC FIELD FROM THE ELECTRIC POTENTIAL

From Equation 3.3 we can express the potential difference *dV* between two points a distance *ds* apart as

$$dV = -\mathbf{E}\cdot d\mathbf{s} \qquad \rightarrow (3.14)$$

If the electric field has only one component E_x , then E. *d*s = E_x *d*x

Therefore, Equation 3.14 becomes *dV*= - E_x *d*x or

$$E_x = -\frac{dV}{dx} \qquad \text{\_ \_ (3.15)}$$

If the charge distribution creating an electric field has spherical symmetry such that the volume charge density depends only on the radial distance *r*, then the electric field is radial. In this case, E. *d*s = E_r *dr* ,and thus we can express *dV* in the form *dV*=- E_r *dr* ,Therefore,

$$E_r = -\frac{dV}{dr}$$

_ _ **(3.16)**

(Note:- equipotential surfaces are perpendicular to field lines)
When a test charge undergoes a displacement ds along an equipotential surface, then dv=0 because the potential is constant along an equipotential surface. From Equation 3.14, then, dV= - E. ds ; thus, E must be perpendicular to the displacement along the equipotential surface .This shows that the equipotential surfaces must always be perpendicular to the electric field lines.

In general, the electric potential is a function of all three spatial coordinates. If $V(r)$ is given in terms of the cartesian coordinates, the electric field components E_x , E_y , and E_z can readily be found from $V(x, y, z)$ as the partial derivatives

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

For example, if $V = 3x^2y + y^2 + yz$, then

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(3x^2y + y^2 + yz) = \frac{\partial}{\partial x}(3x^2y) = 3y\frac{d}{dx}(x^2) = 6xy$$

EXAMPLE 3.4:- An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$, as shown in Figure 3.6. The dipole is along the x axis and is centered at the origin. (a) Calculate the electric potential at point P.

Solution:- For point P in Figure 3.7

$$V = k_e \sum \frac{q_i}{r_i} = k_e\left(\frac{q}{x-a} - \frac{q}{x+a}\right) = \frac{2k_e qa}{x^2 - a^2}$$

(b) Calculate V and Ex at a point far from the dipole.

Solution If point P is far from the dipole, such that x>>a then a^2 can be neglected in the term $x^2 - a^2$ and V becomes

$$V \approx \frac{2k_e qa}{x^2} \qquad (x \gg a)$$

Using Equation 3.15 and this result, we can calculate the electric field at a point far from the dipole:

$$E_x = -\frac{dV}{dx} = \frac{4k_e qa}{x^3} \qquad (x \gg a)$$

(c) Calculate V and E_x if point P is located anywhere between the two charges.

Solution:-

$$V = k_e \sum \frac{q_i}{r_i} = k_e \left(\frac{q}{a - x} - \frac{q}{x + a} \right) = -\frac{2k_e qx}{x^2 - a^2}$$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qx}{x^2 - a^2} \right) = 2k_e q \left(\frac{-x^2 - a^2}{(x^2 - a^2)^2} \right)$$

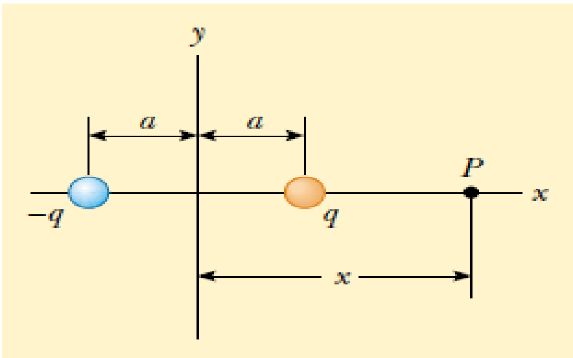


Figure 3.6 An electric dipole located on the *x* axis

ELECTRIC POTENTIAL DUE TO CONTINUOUS CHARGE DISTRIBUTIONS

The electric potential dV at some point P due to the charge element dq is

$$dV = k_e \frac{dq}{r}$$

where r is the distance from the charge element to point P. in general, a different distance from point P and because ke is constant, we can express V as

$$V = k_e \int \frac{dq}{r}$$

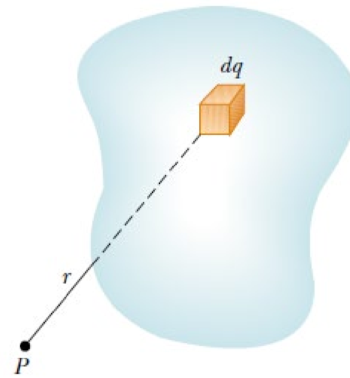


Figure 3.7 The electric potential at the point *P* due to a continuous charge distribution can be calculated by dividing the charged body into segments of charge *dq* and summing the electric potential contributions over all segments.

EXAMPLE 3.5:- (a) Find an expression for the electric potential at a point *P* located on the perpendicular central axis of a uniformly charged ring of radius *a* and total charge *Q*.

Solution Let us orient the ring so that its plane is perpendicular to an *x* axis and its center is at the origin. We can then take point *P* to be at a distance *x* from the center of the ring, as shown in Figure 3.8. The charge element *dq* is at a distance $\sqrt{x^2 + a^2}$ from point *P*. Hence, we can express *V as:*

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

Because each element *dq* is at the same distance from point *P,* we can remove $\sqrt{x^2 + a^2}$ from the integral, and *V* reduces to

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \frac{k_e Q}{\sqrt{x^2 + a^2}} \qquad (3.17)$$

The only variable in this expression for *V* is *x*. This is not surprising because our calculation is valid only for points along the *x* axis, where *y* and *z* are both zero.

(b) Find an expression for the magnitude of the electric field at point *P*.

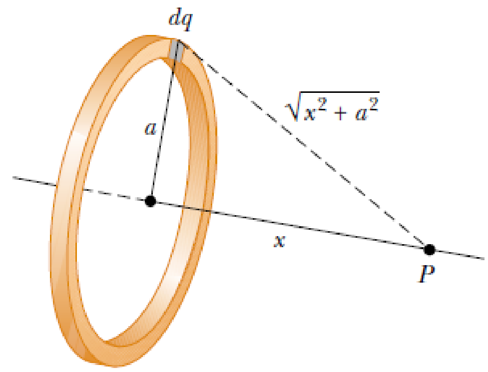
Solution From symmetry, we see that along the *x* axis E can have only an *x* component. Therefore, we can use Equation 3.15:

$$E_x = -\frac{dV}{dx} = -k_eQ\frac{d}{dx}(x^2 + a^2)^{-1/2}$$

$$= -k_eQ(-\tfrac{1}{2})(x^2 + a^2)^{-3/2}(2x)$$

$$= \frac{k_eQx}{(x^2 + a^2)^{3/2}}$$

Figure 3.8 A uniformly charged ring of radius *a* lies in a plane perpendicular to the *x* axis. All segments *dq* of the ring are the same distance from any point *P* lying on the *x* axis



EXAMPLE 3.6:- Find (a) the electric potential and (b) the magnitude of the electric field along the perpendicular central axis of a uniformly charged disk of radius *a* and surface charge density σ.

Solution (a) Again, we choose the point *P* to be at a distance *x* from the center of the disk and take the plane of the disk to be perpendicular to the *x* axis. We can simplify the problem by dividing the disk into a series of charged rings. The electric potential of each ring is given by Equation 3.17. Consider one such ring of radius *r* and width *dr*, .The surface area of the ring is $dA = 2\pi r\,dr$;and $dq = \sigma dA = \sigma 2\pi r\,dr$

the potential at the point *P* due to this ring is

$$dV = \frac{k_e\,dq}{\sqrt{r^2 + x^2}} = \frac{k_e\,\sigma 2\pi r\,dr}{\sqrt{r^2 + x^2}}$$

To find the *total* electric potential at *P*, we sum over all rings making up the disk. That is, we integrate *dV* from *r* = 0 to *r* = *a*

$$V = \pi k_e \sigma \int_0^a \frac{2r\,dr}{\sqrt{r^2 + x^2}} = \pi k_e \sigma \int_0^a (r^2 + x^2)^{-1/2}\, 2r\,dr$$

This integral is of the form $u^n\,du$ and has the value $u^{n+1}/(n+1)$, where n=-1/2 and $u=r^2+x^2$ .This gives

$$V = 2\pi k_e \sigma[(x^2 + a^2)^{1/2} - x]$$

(b) As in Example 3.5, we can find the electric field at any axial point from

$$E_x = -\frac{dV}{dx} = 2\pi k_e \sigma \left(1 - \frac{x}{\sqrt{x^2 + a^2}}\right)$$

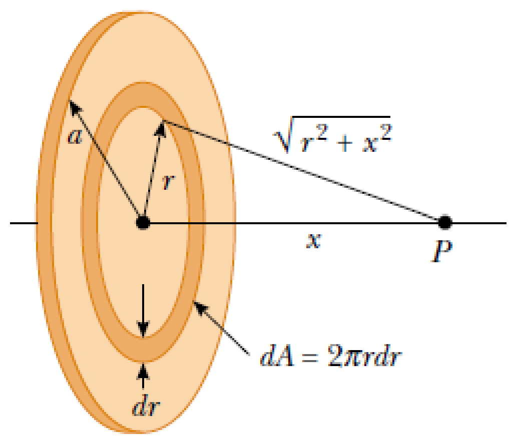


Figure 3.9 A uniformly charged disk of radius *a* lies in a plane perpendicular to the *x* axis. The calculation of the electric potential at any point *P* on the *x* axis is simplified by dividing the disk into many rings each of area 2π*r dr*.

EXAMPLE 3.7:- A rod of length L located along the *x* axis has a total charge *Q* and a uniform linear charge density λ= *Q*/L . Find the electric potential at a point *P* located on the *y* axis a distance *a* from the origin
(Fig. 3.10).

Solution The length element *dx* has a charge *dq* = λ *dx*. Because this element is a distance r =$\sqrt{x^2 + a^2}$ from point *P*, we can express the potential at point *P* due to this element

$$dV = k_e \frac{dq}{r} = k_e \frac{\lambda\, dx}{\sqrt{x^2 + a^2}}$$

To obtain the total potential at *P*, we integrate this expression over the limits *x* = 0 to *x*=L . Noting that k_e and λ are constants, we find that

$$V = k_e \lambda \int_0^{\ell} \frac{dx}{\sqrt{x^2 + a^2}} = k_e \frac{Q}{\ell} \int_0^{\ell} \frac{dx}{\sqrt{x^2 + a^2}}$$

This integral has the following value

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2})$$

Evaluating *V*, we find that

$$V = \frac{k_e Q}{\ell} \ln\left(\frac{\ell + \sqrt{\ell^2 + a^2}}{a}\right)$$

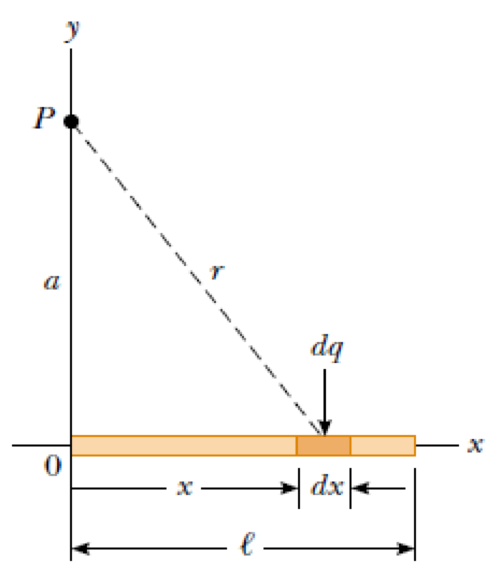


Figure 3.10 A uniform line charge of length located along the *x* axis. To calculate the electric potential at *P*, the line charge is divided into segments each of length *dx* and each carrying a charge
dq =λ *dx*.

<u>***EXAMPLE 3.7:-***</u> An insulating solid sphere of radius *R* has a uniform positive volume charge density and total charge *Q*. (a) Find the electric potential at a point outside the sphere, that is, for r >R.Take the potential to be zero at r =∞

Solution:- In Example 2.5, we found that the magnitude of the electric field outside a uniformly charged sphere of radius *R* is

$$E_r = k_e \frac{Q}{r^2} \qquad (\text{for } r > R)$$

where the field is directed radially outward when *Q* is positive. In this case, to obtain the electric potential at an exterior point, such as *B* in Figure 3.11, we use Equation 3.4 and the expression for E_r given above:

$$V_B = -\int_{\infty}^{r} E_r \, dr = -k_e Q \int_{\infty}^{r} \frac{dr}{r^2}$$

$$V_B = k_e \frac{Q}{r} \qquad (\text{for } r > R)$$

Because the potential must be continuous at *r* = *R*, we can use this expression to obtain the potential at the surface of the sphere. That is, the potential at a point such as *C* shown in Figure 3.12 is

$$V_C = k_e \frac{Q}{R} \qquad (\text{for } r = R)$$

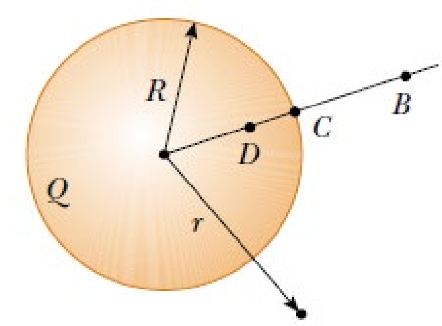


Figure 25.11A uniformly charged insulating sphere of radius *R* and total charge *Q*. The electric potentials at points *B* and *C* are equivalent to those produced by a point charge *Q* located at the center of the sphere

ELECTRIC POTENTIAL DUE TO A CHARGED CONDUCTOR

every point on the surface of a charged conductor in equilibrium is at the same electric potential. Consider two points *A* and *B* on the surface of a charged conductor, as shown in Figure 3.12. Along a surface path connecting these points, E is always perpendicular to the displacement *d*s; therefore Using this result and Equation 3.3, we conclude that the potential difference between *A* and *B* is necessarily zero:

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s} = 0$$

This result applies to any two points on the surface. Therefore, *V* is constant everywhere on the surface of a charged conductor in equilibrium.

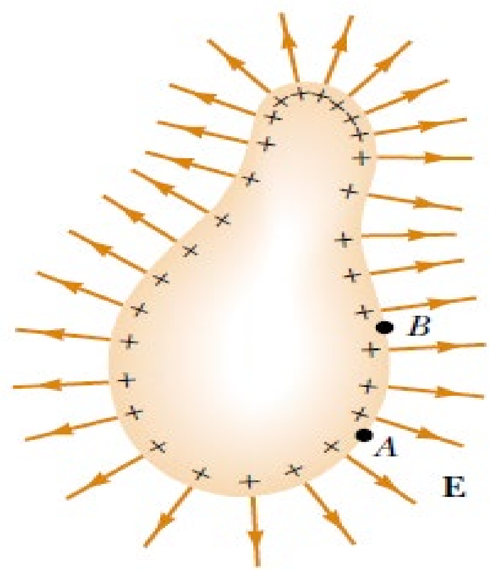


Figure 3.12 An arbitrarily shaped conductor carrying a positive charge.

Note:-

1-The surface of any charged conductor in electrostatic equilibrium is an equipotential surface. Furthermore, because the electric field is zero inside the conductor, we conclude from the relationship that the electric potential is constant everywhere inside the conductor and equal to its value at the surface.

2- the electric field is large near convex points having small radii of curvature and reaches very high values at sharp points.

SUMMARY

When a positive test charge q_0 is moved between points *A* and *B* in an electric field E, the change in the potential energy is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s}$$

The electric potential $V=U/q_0$ is a scalar quantity and has units of joules per coulomb (J/C), where 1 J/C=1 V.

The potential difference ΔV between points *A* and *B* in an electric field E is defined as

$$\Delta V = \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

The potential difference between two points *A* and *B* in a uniform electric field E is

$$\Delta V = -Ed$$

where *d* is the magnitude of the displacement in the direction parallel to E.

An equipotential surface is one on which all points are at the same electric potential. Equipotential surfaces are perpendicular to electric field lines.

If we define V=0 at $r_A=\infty$ the electric potential due to a point charge at any distance *r* from the charge is

$$V = k_e \frac{q}{r}$$

We can obtain the electric potential associated with a group of point charges by summing the potentials due to the individual charges.

The potential energy associated with a pair of point charges separated by a distance r_{12} is

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

This energy represents the work required to bring the charges from an infinite separation to the separation r_{12}. We obtain the potential energy of a distribution of point charges by summing terms like Equation 3.12 over all pairs of particles.

TABLE 25.1 Electric Potential Due to Various Charge Distributions

Charge Distribution	Electric Potential	Location
Uniformly charged ring of radius a	$V = k_e \frac{Q}{\sqrt{x^2 + a^2}}$	Along perpendicular central axis of ring, distance x from ring center
Uniformly charged disk of radius a	$V = 2\pi k_e \sigma[(x^2 + a^2)^{1/2} - x]$	Along perpendicular central axis of disk, distance x from disk center
Uniformly charged, *insulating* solid sphere of radius R and total charge Q	$V = k_e \frac{Q}{r}$	$r \geq R$
	$V = \frac{k_e Q}{2R}\left(3 - \frac{r^2}{R^2}\right)$	$r < R$
Isolated *conducting* sphere of radius R and total charge Q	$V = k_e \frac{Q}{r}$	$r > R$
	$V = k_e \frac{Q}{R}$	$r \leq R$

If we know the electric potential as a function of coordinates *x*, *y*, *z*, we can obtain the components of the electric field by taking the negative derivative of the electric potential with respect to the coordinates. For example, the *x* component of the electric field is

$$E_x = -\frac{dV}{dx}$$

The electric potential due to a continuous charge distribution is

$$V = k_e \int \frac{dq}{r}$$

Every point on the surface of a charged conductor in electrostatic equilibrium is at the same electric potential. The potential is constant everywhere inside the conductor and equal to its value at the surface.

C H A P T E R FOUR (CAPACITANCE AND DIELECTRICS)

DEFINITION OF CAPACITANCE

*Consider two conductor's carrying charges of equal magnitude but of opposite sign. Such a combination of two conductors is called a capacitor. The conductors are called *plates.*

*The capacitance *C* of a capacitor is the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between them:

$$C \equiv \frac{Q}{\Delta V}$$

Note that by definition *capacitance is always a positive quantity*

*we see that capacitance has SI units of coulombs per volt.The SI unit of capacitance is the farad (F), which was named in honor of MichaelFaraday:

$$1\ \mathrm{F} = 1\ \mathrm{C/V}$$

The farad is a very large unit of capacitance. In practice, typical devices have capacitances ranging from microfarads (10^{-6} F) to picofarads (10^{-12} F).

CALCULATING CAPACITANCE

*We can calculate the capacitance of a pair of oppositely charged conductors, We assume a charge of magnitude *Q* , and we calculate the potential difference, We then use the expression C=Q/ΔVto evaluate the capacitance

*We can calculate the capacitance of an isolated spherical conductor of radius*R* and charge *Q* if we assume that the second conductor making up the capacitor isa concentric hollow sphere of infinite radius. The electric potential of the sphereof radius *R* is simply k_eQ/R, and setting V=0 at infinity as usual, we have

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q/R} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

This expression shows that the capacitance of an isolated charged sphere is proportional to its radius and is independent of both the charge on the sphere and the potential difference.

*The capacitance of a pair of conductors depends on the geometry of the conductors. Let us illustrate this with three familiar geometries, namely, parallel plates, concentric cylinders, and concentric spheres. In these examples, we assume that the charged conductors are separated by a vacuum.

Parallel-Plate Capacitors

The value of the electric field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals *Ed*; therefore,

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

Therefore the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d} \quad \rightarrow (4.1)$$

The capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation

EXAMPLE 4.1:-A parallel-plate capacitor has an area $A = 2.00 \text{ X}10^{-4}$ $m^2$and a plate separation d=1.00 mm. Find its capacitance.

Solution from Equation 4.1, we find that

$$C = \epsilon_0 \frac{A}{d} = (8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})\left(\frac{2.00 \times 10^{-4}\ \mathrm{m^2}}{1.00 \times 10^{-3}\ \mathrm{m}}\right)$$

$$= 1.77 \times 10^{-12}\text{ F} = \boxed{1.77\text{ pF}}$$

Exercise **What is the capacitance for a plate separation of 3.00 mm?**

EXAMPLE 4.2:- A solid cylindrical conductor of radius *a* and charge *Q* is Coaxial with a cylindrical shell of negligible thickness, radius b>a and charge -*Q* (Fig. 4.1a). Find the capacitance of this cylindrical capacitor if its length is L.

Solution:- If we assume that Lis much greater than *a* and *b*, we can Neglect end effects. In this case, the electric field is perpendicular to the long axis of the cylinders and is confined to the region between them (Fig. 4.1b). We must first calculate the potential difference between the two cylinders, which is given in general by

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s}$$

where E is the electric field in the region $a < r < b$ In Chapter2, we showed using Gauss's law that the magnitude of theelectric field of a cylindrical charge distribution having linearcharge densityλ is $E_r = 2k_e\lambda/r$.

$$V_b - V_a = -\int_a^b E_r\,dr = -2k_e\lambda \int_a^b \frac{dr}{r} = -2k_e\lambda \ln\left(\frac{b}{a}\right)$$

using the factthat we obtainλ=Q\L

$$C = \frac{Q}{\Delta V} = \frac{Q}{\dfrac{2k_e Q}{\ell}\ln\left(\dfrac{b}{a}\right)} = \boxed{\frac{\ell}{2k_e \ln\left(\dfrac{b}{a}\right)}}$$

we see that the capacitance per unit length ofa combination of concentric cylindrical conductors is

$$\frac{C}{\ell} = \frac{1}{2k_e \ln\left(\frac{b}{a}\right)}$$

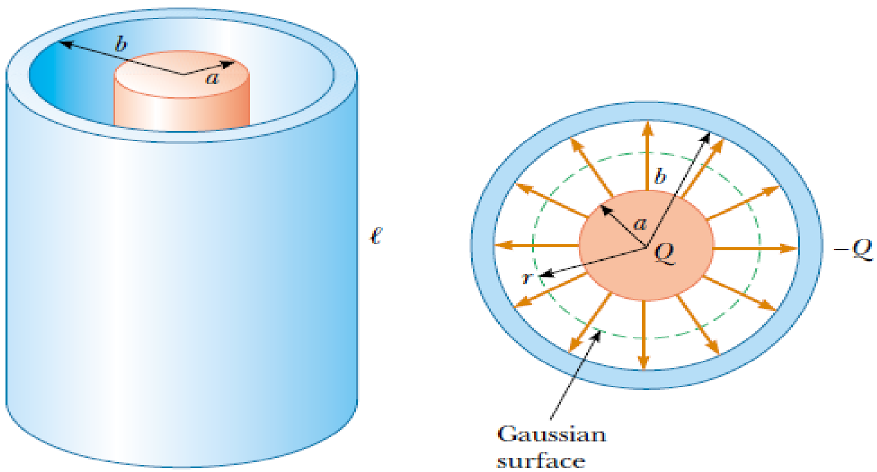


Figure 4.1(a) A cylindrical capacitor consists of a solid cylindricalconductor of radius *a* and length Lsurrounded by a coaxial cylindricalshell of radius *b*. (b) End view. The dashed line represents theend of the cylindrical Gaussian surface of radius *r* and length L.

EXAMPLE 4.3:-A spherical capacitor consists of a spherical conducting shell of radius *b* and charge-*Q* concentric with a smaller conducting Sphere of radius *a* and charge *Q* (Fig. 4.2). Find the capacitance of this device.

Solution As we showed in Chapter 2, the field outside a spherically symmetric charge distribution is given by the expressionk_eQ/r^2. In this case, this result applies to the field between the spheres ($a < r < b$). From Gauss's law we see that only the inner sphere contributes to this field. Thus, the potential difference between the spheres is

$$V_b - V_a = -\int_a^b E_r\,dr = -k_eQ\int_a^b \frac{dr}{r^2} = k_eQ\left[\frac{1}{r}\right]_a^b$$
$$= k_eQ\left(\frac{1}{b} - \frac{1}{a}\right)$$

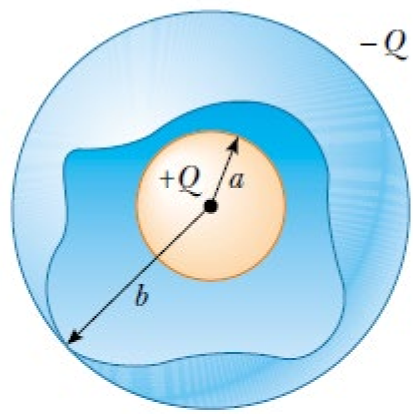
The magnitude of the potential difference is

$$\Delta V = |V_b - V_a| = k_eQ\frac{(b-a)}{ab}$$

Substituting this value for ΔV into Equation 4.1, we obtain

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e(b-a)}$$

Figure 4.2A spherical capacitor consists of an inner sphere of radius*a* surrounded by a concentric spherical shell of radius *b*. Theelectric field between the spheres is directed radially outward whenthe inner sphere is positively charged.



COMBINATIONS OF CAPACITORS:

-Parallel Combination

Two capacitors connected as shown in Figure 4.3a are known as a parallel combination of capacitors. **The individual potential differences across capacitors connected in parallel are all the same and are equal to the potential difference applied across the combination.**

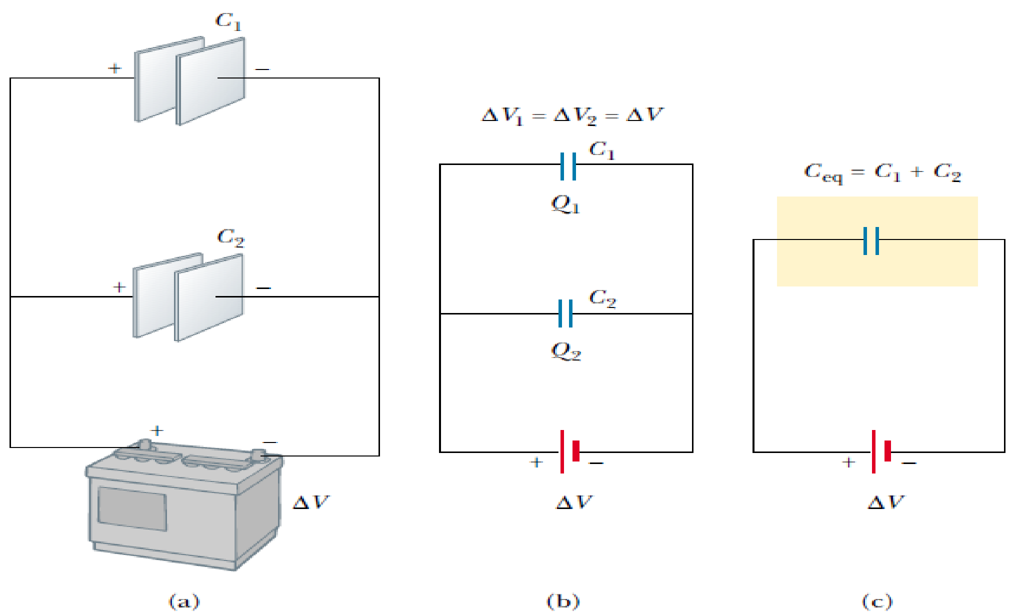


Figure 4.3(a) A parallel combination of two capacitors in an electric circuit in which the potentialdifference across the battery terminals is Δ*V*. (b) The circuit diagram for the parallel combination.(c) The equivalent capacitance is $C_{eq}=C_1+C_2$.

The *total charge Q* stored by the two capacitors is:

$$Q = Q_1 + Q_2 \qquad \textbf{(4.2)}$$

That is, **the total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors**. Because the voltages across the capacitors are the same, the charges that they carry are

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

Suppose that we wish to replace these two capacitors by one *equivalent capacitor*having a capacitance C_{eq} , as shown in Figure 4.3c. The effect this equivalent capacitorhas on the circuit must be exactly the same as the effect of the combinationof the two individual capacitors.

$$Q = C_{eq} \Delta V$$

Substituting these three relationships for charge into Equation 4.2, we have

$$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V$$

$$C_{eq} = C_1 + C_2 \quad \text{(parallel combination)}$$

If we extend this treatment to three or more capacitors connected in parallel,we find the equivalent capacitance to be

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

Thus, the equivalent capacitance of a parallel combination of capacitors is Greater than any of the individual capacitances.

-Series Combination:-

Two capacitors connected as shown in Figure 4.4a are known as a *series combination*of capacitors.

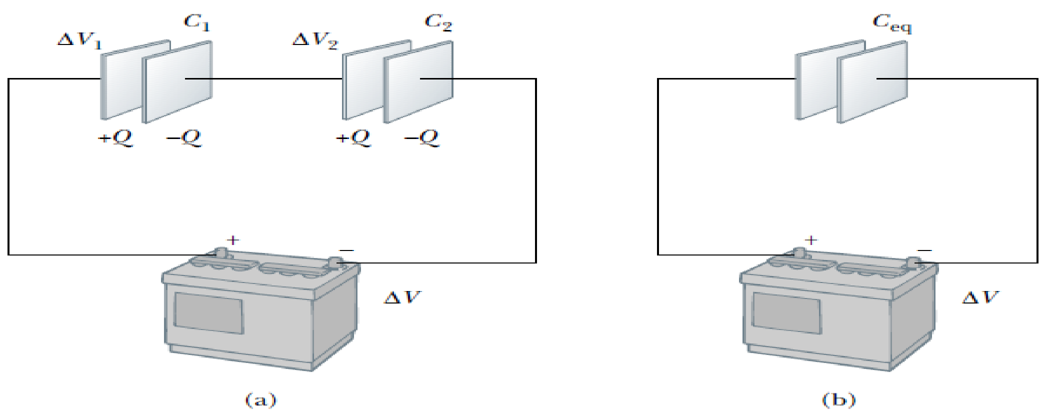


Figure 4.4 (a) A series combination of two capacitors. The charges on the two capacitors are the same. (b) The capacitors replaced by a single equivalent capacitor.

the charges on capacitors connected in series are the same.

From Figure 26.9a, we see that the voltage Δ*V* across the battery terminals issplit between the two capacitors:

$$\Delta V = \Delta V_1 + \Delta V_2$$

where ΔV_1 and ΔV_2 are the potential differences across capacitors C_1 and C_2, respectively.In general, **the total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.**

$$\Delta V = \frac{Q}{C_{eq}}$$

Because we can apply the expression $Q = C\,\Delta V$ to each capacitor shown in Figure4.4a, the potential difference across each is

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

Therefore $\Delta V = Q/C_{eq}$,

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Canceling Q , we arrive at the relationship

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

When this analysis is applied to three or more capacitors connected in series, therelationship for the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \begin{pmatrix}\text{series}\\ \text{combination}\end{pmatrix}$$

EXAMPLE 4.4:-Find the equivalent capacitance between *a* and *b* for the combination of capacitors shown in Figure 4.5a. All capacitances are in microfarads.

Solution:-The 1.0μF and 3.0μF capacitors are in parallel and combine according to the expression$C_{eq}= C_1+C_2$=4.0F. The 2.0μF and 6.0μF capacitors also are in parallel and have an equivalent capacitance of 8.0 μF. Thus, the upper branch in Figure 4.5b consists of two 4.0μF capacitors in series, which combine as follows:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\ \mu F} + \frac{1}{4.0\ \mu F} = \frac{1}{2.0\ \mu F}$$

$$C_{eq} = \frac{1}{1/2.0\ \mu F} = 2.0\ \mu F$$

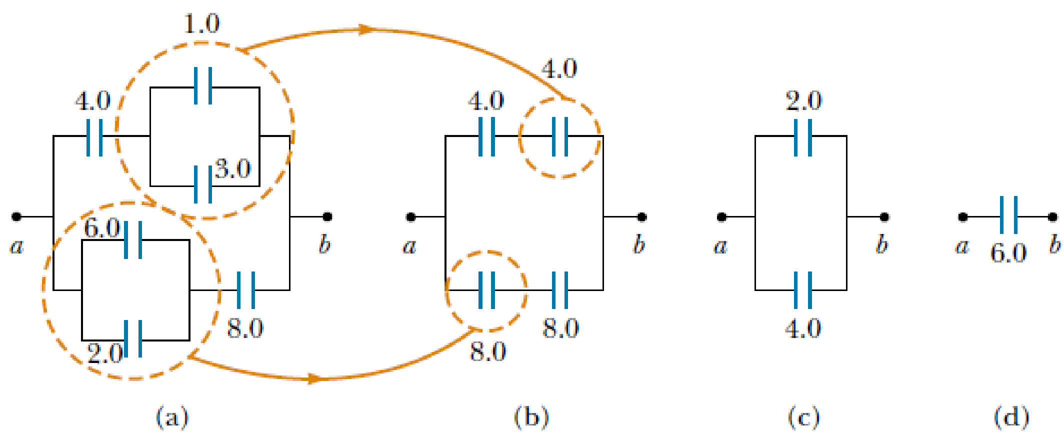


Figure 4.5To find the equivalent capacitance of the capacitors in part (a), wereduce the various combinations in steps as indicated in parts (b), (c), and (d), usingthe series and parallel rules described in the text.

The lower branch in Figure 4.5b consists of two 8.0 μF capacitors in series, which combine to yield an equivalent capacitance of 4. μF. Finally, the 2.0 μF and 4.0 μF capacitors in Figure 26.10c are in parallel and thus have an equivalent capacitance of 6.0 μF.

Note:-

1-Energy stored in a charged capacitor is

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

2-Energy stored in a parallel-plate capacitor is

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \frac{1}{2}(\epsilon_0 A d)E^2$$

3- Energy density in an electric field is

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

CAPACITORS WITH DIELECTRICS

A dielectric is a no conducting material, such as rubber and glass.When a dielectric is inserted between the plates of a capacitor, the capacitance increases .If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor (k), which is called the dielectric constant. The dielectric constant is a property of a material and varies from one material to another.

Consider a parallel-plate capacitor that without a dielectric has a charge Q_0 and a capacitance C_0 . The potential difference across the capacitor is
$\Delta V_0 = Q_0/C_0$

The voltages with and without the dielectric are related by the factor (k)as follows:

$$\Delta V = \frac{\Delta V_0}{\kappa}$$

Because $\Delta V < \Delta V_0$, we see that $\kappa > 1$.

Because the charge Q_0 on the capacitor does not change, we conclude that the capacitance must change to the value

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$C = \kappa C_0$$

That is, the capacitance *increases* by the factor (k)when the dielectric completely fillsthe region between the plates. For a parallel-plate capacitor, where$C_0 = \epsilon_0 A/d$we can express the capacitance when the capacitor is filled with a dielectric as

$$C = \kappa \frac{\epsilon_0 A}{d}$$

Types of Capacitors

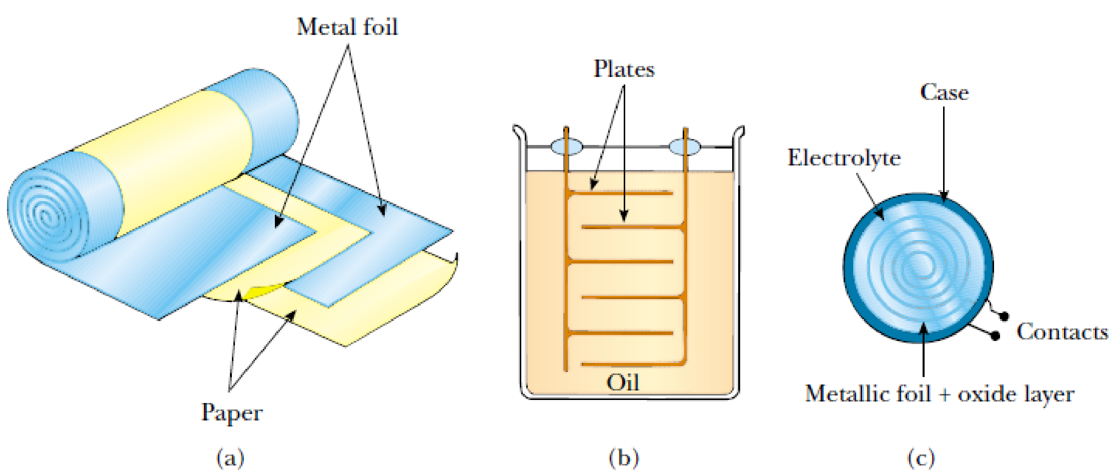


Figure 4.6Three commercial capacitor designs. (a) A tubular capacitor, whose plates are separated by paper and then rolled into a cylinder. (b) A high-voltage capacitor consisting of many parallel plates separated by insulating oil. (c) An electrolytic capacitor.

EXAMPLE 4.5:- A parallel-plate capacitor has plates of dimensions 2.0 cm by3.0 cm separated by a 1.0-mm thickness of paper. (a) Find its capacitance.

Solution:-Because k=3.7 for paper, we have

$$C = \kappa \frac{\epsilon_0 A}{d} = 3.7(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\left(\frac{6.0 \times 10^{-4}\ \text{m}^2}{1.0 \times 10^{-3}\ \text{m}}\right)$$

$$= 20 \times 10^{-12}\ \text{F} = \boxed{20\ \text{pF}}$$

(b) What is the maximum charge that can be placed on the capacitor?

Solution: -Because the thickness of the paper is 1.0 mm, the maximum voltage that can be applied before breakdown is

$$\Delta V_{\text{max}} = E_{\text{max}} d = (16 \times 10^6\ \text{V/m})(1.0 \times 10^{-3}\ \text{m})$$
$$= 16 \times 10^3\ \text{V}$$

Hence, the maximum charge is

$$Q_{\text{max}} = C\Delta V_{\text{max}} = (20 \times 10^{-12}\ \text{F})(16 \times 10^3\ \text{V}) = \boxed{0.32\ \mu\text{C}}$$

EXAMPLE 4.6*:-*A parallel-plate capacitor is charged with a battery to a charge Q_0 , as shown in Figure 4.6a. The battery is then removed, and a slab of material that has a dielectric constant (K)is inserted between the plates, as shown in Figure 4.6b. Find the energy stored in the capacitor before and after the dielectric is inserted.

Solution:-The energy stored in the absence of the dielectric is

$$U_0 = \frac{{Q_0}^2}{2C_0}$$

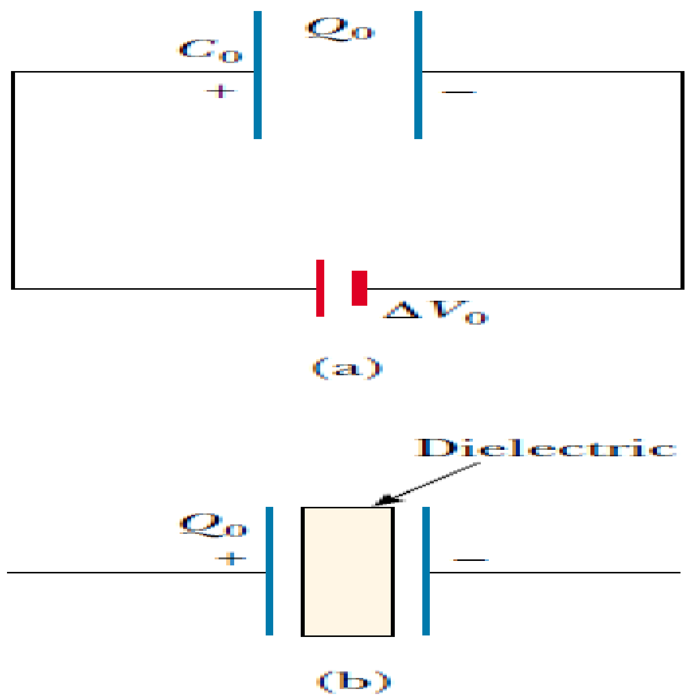
After the battery is removed and the dielectric inserted, the *charge on the capacitor remains the same.* Hence, the energy stored in the presence of the dielectric is

$$U = \frac{{Q_0}^2}{2C}$$

But the capacitance in the presence of the dielectric is$C$ =kC_0, so U becomes

$$U = \frac{{Q_0}^2}{2\kappa C_0} = \frac{U_0}{\kappa}$$

Because k>1, the final energy is less than the initial energy



(*Figure 4.6)*

SUMMARY

A capacitor consists of two conductors carrying charges of equal magnitude but opposite sign. The capacitance *C* of any capacitor is the ratio of the charge *Q* on either conductor to the potential difference Δ*V* between them:

$$C \equiv \frac{Q}{\Delta V}$$

This relationship can be used in situations in which any two of the three variables are known. It is important to remember that this ratio is constant for a given configurationof conductors because the capacitance depends only on the geometryof the conductors and not on an external source of charge or potential difference.

The SI unit of capacitance is coulombs per volt, or the farad (F), and
1 F =1 C/V.

If two or more capacitors are connected in parallel, then the potential differenceis the same across all of them. The equivalent capacitance of a parallel combinationof capacitors is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

If two or more capacitors are connected in series, the charge is the same on all of them, and the equivalent capacitance of the series combination is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

The work done in charging the capacitor to a charge *Q* equals the electric potential energy *U* stored in the capacitor, where

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\,\Delta V = \frac{1}{2}C(\Delta V)^2$$

TABLE 26.2 Capacitance and Geometry

Geometry	Capacitance	Equation
Isolated charged sphere of radius R (second charged conductor assumed at infinity)	$C = 4\pi\epsilon_0 R$	26.2
Parallel-plate capacitor of plate area A and plate separation d	$C = \epsilon_0 \frac{A}{d}$	26.3
Cylindrical capacitor of length ℓ and inner and outer radii a and b, respectively	$C = \frac{\ell}{2k_e \ln\left(\frac{b}{a}\right)}$	26.4
Spherical capacitor with inner and outer radii a and b, respectively	$C = \frac{ab}{k_e(b - a)}$	26.6

When a dielectric material is inserted between the plates of a capacitor, the capacitance increases by a dimensionless factor (k), called the dielectric constant:

$$C = \kappa C_0$$

WhereC_0 is the capacitance in the absence of the dielectric

c h a p t e r five (Current and Resistance)

ELECTRIC CURRENT

consider a system of electric charges in motion. Whenever there is a net flow of charge through some region, a current is said to exist. To define current more precisely, suppose that the charges are moving perpendicular to a surface of area A, as shown in Figure 3.1. (This area could be the cross-sectional area of a wire, for example.) The current is the rate at which charge flows through this surface. If $\Delta Q$is the amount of charge that passes through this area in a time interval Δt, the average current I_{av} is equal to the charge that passes through Aper unit time:

$$I_{av}=\Delta Q/\Delta t \qquad (3.1)$$

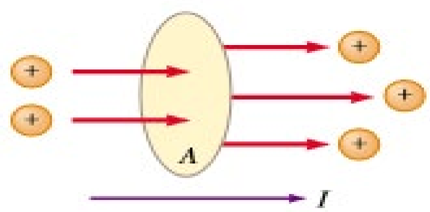


Figure 3.1 Charges in motion through an area A. The time rate at which charge flows through the area is defined as the current I. The direction of the current is the direction in which positive charges flow when free to do so.

If the rate at which charge flows varies in time, then the current varies in time; we define the **instantaneous current** I as the differential limit of average current:

$$I \equiv \frac{dQ}{dt} \qquad \textbf{(27.2)}$$

Electric current

The SI unit of current is the **ampere** (A):

$$1\text{ A} = \frac{1\text{ C}}{1\text{ s}} \qquad \textbf{(27.3)}$$

That is, 1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.

The charges passing through the surface in Figure 27.1 can be positive or negative, or both. **It is conventional to assign to the current the same direction as the flow of positive charge.** In electrical conductors, such as copper or alu-

The direction of the current

minum, the current is due to the motion of negatively charged electrons. Therefore, when we speak of current in an ordinary conductor, the direction of the current is opposite the direction of flow of electrons. However, if we are considering a beam of positively charged protons in an accelerator, the current is in the direction of motion of the protons. In some cases—such as those involving gases and electrolytes, for instance-the current is the result of the flow of both positive and negative charges. If the ends of a conducting wire are connected to form a loop, all points on the loop are at the same electric potential, and hence the electric field is zero within and at the surface of the conductor. Because the electric field is zero, there is no net transport of charge through the wire, and therefore there is no current. The current in the conductor is zero even if the conductor has an excess of charge on it. However, if the ends of the conducting wire are connected to a battery, all points on the loop are not at the same potential. The battery sets up a potential difference between the ends of the loop, creating an electric field within the wire. The electric field exerts forces on the conduction electrons in the wire, causing them to move around the loop and thus creating a current. It is common to refer to a moving charge (positive or negative) as a mobile charge carrier. For example, the mobile charge carriers in a metal are electrons.

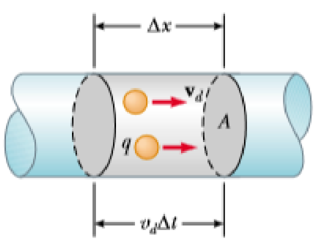


Figure 27.2 A section of a uniform conductor of cross-sectional area A. The mobile charge carriers move with a speed v_d, and the distance they travel in a time Δt is $\Delta x = v_d \Delta t$. The number of carriers in the section of length Δx is $nAv_d \Delta t$, where n is the number of carriers per unit volume.

Average current in a conductor

Microscopic Model of Current

We can relate current to the motion of the charge carriers by describing a microscopic model of conduction in a metal. Consider the current in a conductor of cross-sectional area A (Fig. 27.2). The volume of a section of the conductor of length Δx (the gray region shown in Fig. 27.2) is $A\,\Delta x$. If n represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the gray section is $nA\,\Delta x$. Therefore, the charge ΔQ in this section is

$$\Delta Q = \text{number of carriers in section} \times \text{charge per carrier} = (nA\,\Delta x)q$$

where q is the charge on each carrier. If the carriers move with a speed v_d, the distance they move in a time Δt is $\Delta x = v_d\,\Delta t$. Therefore, we can write ΔQ in the form

$$\Delta Q = (nAv_d\,\Delta t)q$$

If we divide both sides of this equation by Δt, we see that the average current in the conductor is

$$I_{av} = \frac{\Delta Q}{\Delta t} = nqv_dA \qquad \textbf{(27.4)}$$

The speed of the charge carriers v_d is an average speed called the **drift speed.** To understand the meaning of drift speed, consider a conductor in which the

The speed of the charge carriers vd is an average speed called the drift speed. To understand the meaning of drift speed, consider a conductor in which the charge carriers are free electrons. If the conductor is isolated-that is, the potential difference across it is zero-then these electrons undergo random motion that is analogous to the motion of gas molecules. As we discussed earlier, when a potential difference is applied across the conductor (for example, by means of a battery), an electric field is set up in the conductor; this field exerts an electric force on the electrons, producing a current. However, the electrons do not move in straight lines along the conductor. Instead, they collide repeatedly with the metal atoms, and their resultant motion is complicated and zigzag (Fig. 27.3). Despite the collisions, the electrons move slowly along the conductor (in adirection opposite that of E) at the drift velocity vd.

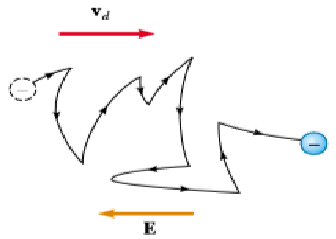


Figure 27.3 A schematic representation of the zigzag motion of an electron in a conductor. The changes in direction are the result of collisions between the electron and atoms in the conductor. Note that the net motion of the electron is opposite the direction of the electric field. Each section of the zigzag path is a parabolic segment.

We can think of the atom–electron collisions in a conductor as an effective internal friction (or drag force) similar to that experienced by the molecules of a liquid flowing through a pipe stuffed with steel wool. The energy transferred from the electrons to the metal atoms during collision causes an increase in the vibrational energy of the atoms and a corresponding increase in the temperature of the conductor.

Quick Quiz 27.1

Consider positive and negative charges moving horizontally through the four regions shown in Figure 27.4. Rank the current in these four regions, from lowest to highest.

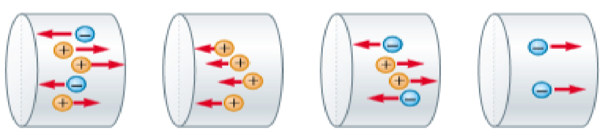


Figure 27.4

EXAMPLE 27.1 Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of 3.31×10^{-6} m^2. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is 8.95 g/cm^3.

Solution From the periodic table of the elements in Appendix C, we find that the molar mass of copper is 63.5 g/mol. Recall that 1 mol of any substance contains Avogadro's number of atoms (6.02×10^{23}). Knowing the density of copper, we can calculate the volume occupied by 63.5 g (= 1 mol) of copper:

$$V = \frac{m}{\rho} = \frac{63.5\ \text{g}}{8.95\ \text{g/cm}^3} = 7.09\ \text{cm}^3$$

Because each copper atom contributes one free electron to the current, we have

$$n = \frac{6.02 \times 10^{23}\ \text{electrons}}{7.09\ \text{cm}^3}(1.00 \times 10^6\ \text{cm}^3/\text{m}^3)$$
$$= 8.49 \times 10^{28}\ \text{electrons/m}^3$$

From Equation 27.4, we find that the drift speed is

$$v_d = \frac{I}{nqA}$$

where q is the absolute value of the charge on each electron. Thus,

$$v_d = \frac{I}{nqA}$$
$$= \frac{10.0\ \text{C/s}}{(8.49 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})(3.31 \times 10^{-6}\ \text{m}^2)}$$
$$= \boxed{2.22 \times 10^{-4}\ \text{m/s}}$$

Exercise If a copper wire carries a current of 80.0 mA, how many electrons flow past a given cross-section of the wire in 10.0 min?

Answer 3.0×10^{20} electrons.

Example 27.1 shows that typical drift speeds are very low. For instance, electrons traveling with a speed of 2.46 x 10^{-4} m/s would take about 68 min to travel 1 m! In view of this, you might wonder why a light turns on almost instantaneously when a switch is thrown. In a conductor, the electric field that drives the free electrons travels through the conductor with a speed close to that of light. Thus, when you flip on a light switch, the message for the electrons to start moving through the wire (the electric field) reaches them at a speed on the order of 10^8 m/s.

RESISTANCE AND OHM'S LAW

In Chapter 24 we found that no electric field can exist inside a conductor. However, this statement is true only if the conductor is in static equilibrium. The purpose of this section is to describe what happens when the charges in the conductor are allowed to move.

Charges moving in a conductor produce a current under the action of an electric field, which is maintained by the connection of a battery across the conductor. An electric field can exist in the conductor because the charges in this situation are in motion—that is, this is a *nonelectrostatic* situation.

Consider a conductor of cross-sectional area A carrying a current I. The **current density** J in the conductor is defined as the current per unit area. Because the current $I = nqv_dA$, the current density is

$$J \equiv \frac{I}{A} = nqv_d \qquad \textbf{(27.5)}$$

where J has SI units of A/m^2. This expression is valid only if the current density is uniform and only if the surface of cross-sectional area A is perpendicular to the direction of the current. In general, the current density is a vector quantity:

$$\mathbf{J} = nq\mathbf{v}_d \qquad \textbf{(27.6)}$$

From this equation, we see that current density, like current, is in the direction of charge motion for positive charge carriers and opposite the direction of motion for negative charge carriers.

A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor. If the potential difference is constant, then the current also is constant. In some materials, the current density is proportional to the electric field:

$$\mathbf{J} = \sigma\mathbf{E} \qquad \textbf{(27.7)}$$

where the constant of proportionality σ is called the **conductivity** of the conductor.[1] Materials that obey Equation 27.7 are said to follow **Ohm's law,** named after Georg Simon Ohm (1787–1854). More specifically, Ohm's law states that

> for many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.

We can obtain a form of Ohm's law useful in practical applications by considering a segment of straight wire of uniform cross-sectional area A and length ℓ, as shown in Figure 27.5. A potential difference $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current. If the field is assumed to be uniform, the potential difference is related to the field through the relationship[2]

$$\Delta V = E\ell$$

Therefore, we can express the magnitude of the current density in the wire as

$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

Because $J = I/A$, we can write the potential difference as

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I$$

The quantity $\ell/\sigma A$ is called the **resistance** R of the conductor. We can define the resistance as the ratio of the potential difference across a conductor to the current through the conductor:

$$R \equiv \frac{\ell}{\sigma A} \equiv \frac{\Delta V}{I} \tag{27.8}$$

Resistance of a conductor

From this result we see that resistance has SI units of volts per ampere. One volt per ampere is defined to be 1 **ohm** (Ω):

$$1\ \Omega \equiv \frac{1\ \text{V}}{1\ \text{A}} \tag{27.9}$$

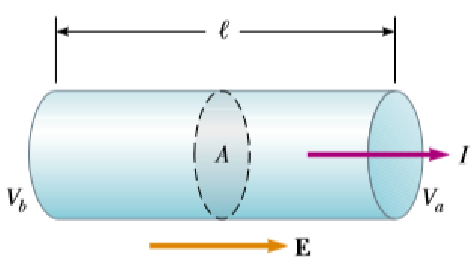


Figure 27.5 A uniform conductor of length ℓ and cross-sectional area A. A potential difference $\Delta V = V_b - V_a$ maintained across the conductor sets up an electric field $\mathbf{E}$, and this field produces a current I that is proportional to the potential difference.

[2] This result follows from the definition of potential difference:

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot d\mathbf{s} = E\int_0^\ell dx = E\ell$$

This expression shows that if a potential difference of 1 V across a conductor causes a current of 1 A, the resistance of the conductor is 1 Ω.

The inverse of conductivity is **resistivity**[3] ρ:

Resistivity

$$\rho \equiv \frac{1}{\sigma} \tag{27.10}$$

where ρ has the units ohm-meters ($\Omega \cdot m$). We can use this definition and Equation 27.8 to express the resistance of a uniform block of material as

Resistance of a uniform conductor

$$R = \rho \frac{\ell}{A} \tag{27.11}$$

Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature. Additionally, as you can see from Equation 27.11, the resistance of a sample depends on geometry as well as on resistivity. Table 27.1 gives the resistivities of a variety of materials at 20°C. Note the enormous range, from very low values for good conductors such as copper and silver, to very high values for good insulators such as glass and rubber. An ideal conductor would have zero resistivity, and an ideal insulator would have infinite resistivity.

Most electric circuits use devices called resistors to control the current level in the various parts of the circuit. Two common types of resistors are the composition resistor, which contains carbon, and the wire-wound resistor, which consists of a coil of wire. Resistors' values in ohms are normally indicated by color-coding, as shown in Figure 27.6 and Table 27.2. Ohmic materials have a linear current–potential difference relationship over a broad range of applied potential differences (Fig. 27.7a). The slope of the I-versus-ΔV curve in the linear region yields a value for 1/R. Nonohmic materials

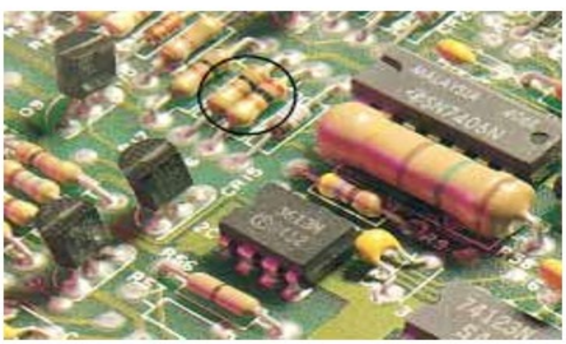


Figure 27.6 The colored bands on a resistor represent a code for determining resistance. The first two colors give the first two digits in the resistance value. The third color represents the power of ten for the multiplier of the resistance value. The last color is the tolerance of the resistance value. As an example, the four colors on the circled resistors are red (= 2), black (= 0), orange (= 10^3), and gold (= 5%), and so the resistance value is $20 \times 10^3\ \Omega$ = 20 kΩ with a tolerance value of 5% = 1 kΩ. (The values for the colors are from Table 27.2.)

TABLE 27.2 Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

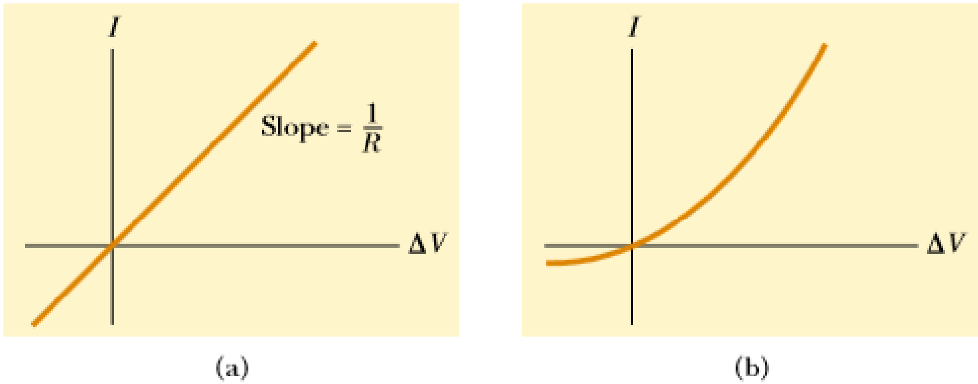


Figure 27.7 (a) The current–potential difference curve for an ohmic material. The curve is linear, and the slope is equal to the inverse of the resistance of the conductor. (b) A nonlinear current–potential difference curve for a semiconducting diode. This device does not obey Ohm's law.

EXAMPLE 27.2 The Resistance of a Conductor

Calculate the resistance of an aluminum cylinder that is 10.0 cm long and has a cross-sectional area of $2.00 \times 10^{-4}\ \text{m}^2$. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10}\ \Omega\cdot\text{m}$.

Solution From Equation 27.11 and Table 27.1, we can calculate the resistance of the aluminum cylinder as follows:

$$R = \rho\frac{\ell}{A} = (2.82 \times 10^{-8}\ \Omega\cdot\text{m})\left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2}\right)$$

$$= \boxed{1.41 \times 10^{-5}\ \Omega}$$

Similarly, for glass we find that

$$R = \rho\frac{\ell}{A} = (3.0 \times 10^{10}\ \Omega\cdot\text{m})\left(\frac{0.100\ \text{m}}{2.00 \times 10^{-4}\ \text{m}^2}\right)$$

$$= \boxed{1.5 \times 10^{13}\ \Omega}$$

As you might guess from the large difference in resistivities, the resistance of identically shaped cylinders of aluminum and glass differ widely. The resistance of the glass cylinder is 18 orders of magnitude greater than that of the aluminum cylinder.

Electrical insulators on telephone poles are often made of glass because of its low electrical conductivity.

EXAMPLE 27.3 The Resistance of Nichrome Wire

(a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

Solution The cross-sectional area of this wire is

$$A = \pi r^2 = \pi(0.321 \times 10^{-3}\ \text{m})^2 = 3.24 \times 10^{-7}\ \text{m}^2$$

The resistivity of Nichrome is $1.5 \times 10^{-6}\ \Omega\cdot\text{m}$ (see Table 27.1). Thus, we can use Equation 27.11 to find the resistance per unit length:

$$\frac{R}{\ell} = \frac{\rho}{A} = \frac{1.5 \times 10^{-6}\ \Omega\cdot\text{m}}{3.24 \times 10^{-7}\ \text{m}^2} = \boxed{4.6\ \Omega/\text{m}}$$

(b) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Solution Because a 1.0-m length of this wire has a resistance of 4.6 Ω, Equation 27.8 gives

$$I = \frac{\Delta V}{R} = \frac{10\ \text{V}}{4.6\ \Omega} = \boxed{2.2\ \text{A}}$$

Note from Table 27.1 that the resistivity of Nichrome wire is about 100 times that of copper. A copper wire of the same radius would have a resistance per unit length of only 0.052 Ω/m. A 1.0-m length of copper wire of the same radius would carry the same current (2.2 A) with an applied potential difference of only 0.11 V.

Because of its high resistivity and its resistance to oxidation, Nichrome is often used for heating elements in toasters, irons, and electric heaters.

Exercise What is the resistance of a 6.0-m length of 22-gauge Nichrome wire? How much current does the wire carry when connected to a 120-V source of potential difference?

Answer 28 Ω; 4.3 A.

Exercise Calculate the current density and electric field in the wire when it carries a current of 2.2 A.

Answer $6.8 \times 10^6\ \text{A/m}^2$; 10 N/C.

EXAMPLE 27.4 The Radial Resistance of a Coaxial Cable

Coaxial cables are used extensively for cable television and other electronic applications. A coaxial cable consists of two cylindrical conductors. The gap between the conductors is completely filled with silicon, as shown in Figure 27.8a, and current leakage through the silicon is unwanted. (The cable is designed to conduct current along its length.) The radius of the inner conductor is $a = 0.500$ cm, the radius of the outer one is $b = 1.75$ cm, and the length of the cable is $L = 15.0$ cm. Calculate the resistance of the silicon between the two conductors.

Solution In this type of problem, we must divide the object whose resistance we are calculating into concentric elements of infinitesimal thickness dr (Fig. 27.8b). We start by using the differential form of Equation 27.11, replacing ℓ with r for the distance variable: $dR = \rho\, dr/A$, where dR is the resistance of an element of silicon of thickness dr and surface area A. In this example, we take as our representative concentric element a hollow silicon cylinder of radius r, thickness dr, and length L, as shown in Figure 27.8. Any current that passes from the inner conductor to the outer one must pass radially through this concentric element, and the area through which this current passes is $A = 2\pi rL$. (This is the curved surface area—circumference multiplied by length—of our hollow silicon cylinder of thickness dr.) Hence, we can write the resistance of our hollow cylinder of silicon as

$$dR = \frac{\rho}{2\pi rL}\, dr$$

Because we wish to know the total resistance across the entire thickness of the silicon, we must integrate this expression from $r = a$ to $r = b$:

$$R = \int_a^b dR = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r} = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$

Substituting in the values given, and using $\rho = 640\ \Omega\cdot\text{m}$ for silicon, we obtain

$$R = \frac{640\ \Omega\cdot\text{m}}{2\pi(0.150\ \text{m})}\ln\left(\frac{1.75\ \text{cm}}{0.500\ \text{cm}}\right) = 851\ \Omega$$

Exercise If a potential difference of 12.0 V is applied between the inner and outer conductors, what is the value of the total current that passes between them?

Answer 14.1 mA.

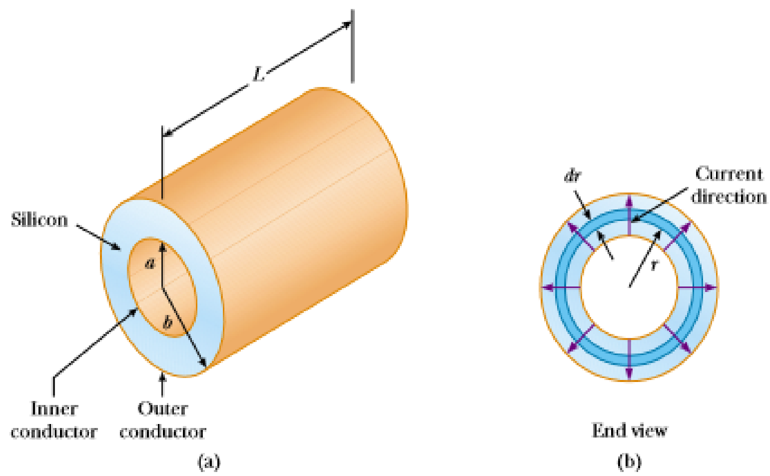


Figure 27.8 A coaxial cable. (a) Silicon fills the gap between the two conductors. (b) End view, showing current leakage.

ELECTROMOTIVE FORCE

In Section 27.6 we found that a constant current can be maintained in a closed circuit through the use of a source of emf, which is a device (such as a battery or generator) that produces an electric field and thus may cause charges to move around a circuit. One can think of a source of emf as a "charge pump." When an electric potential difference exists between two points, the source moves charges "uphill" from the lower potential to the higher. The emfε describes the work done per unit charge, and hence the SI unit of emf is the volt. Consider the circuit shown in Figure 28.1, consisting of a battery connected to a resistor. We assume that the connecting wires have no resistance. The positive terminal of the battery is at a higher potential than the negative terminal. If we neglect the internal resistance of the battery, the potential difference across it (called the terminal voltage) equals its emf. However, because a real battery always has some internal resistance r, the terminal voltage is not equal to the emf for a battery in a circuit in which there is a current. To understand why this is so, consider the circuit diagram in Figure 28.2a, where the battery of Figure 28.1 is represented by the dashed rectangle containing an emfε in series with an internal resistance r. Now imagine moving through the battery clockwise from a to b and measuring the electric potential at various locations. As we pass from the negative terminal to the positive terminal, the potential increases by an amount ε. However, as we move through the resistance r, the potential decreases by an amount Ir, where I is the current in the circuit. Thus, the terminal voltage of the battery is1

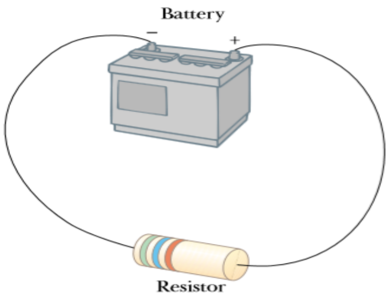
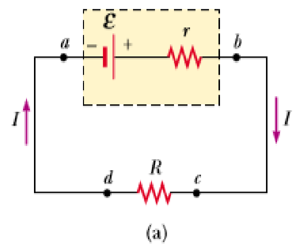


Figure 28.1 A circuit consisting of a resistor connected to the terminals of a battery.



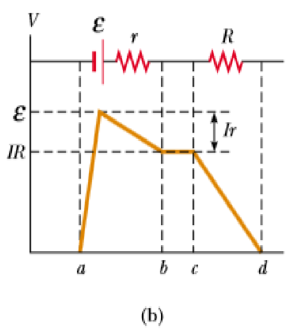


Figure 28.2 (a) Circuit diagram of a source of emf $\mathcal{E}$ (in this case, a battery), of internal resistance r, connected to an external resistor of resistance R. (b) Graphical representation showing how the electric potential changes as the circuit in part (a) is traversed clockwise.

$$\Delta V = \mathcal{E} - Ir \quad \textbf{(28.1)}$$

From this expression, note that $\mathcal{E}$ is equivalent to the **open-circuit voltage**—that is, the *terminal voltage when the current is zero.* The emf is the voltage labeled on a battery—for example, the emf of a D cell is 1.5 V. The actual potential difference between the terminals of the battery depends on the current through the battery, as described by Equation 28.1.

Figure 28.2b is a graphical representation of the changes in electric potential as the circuit is traversed in the clockwise direction. By inspecting Figure 28.2a, we see that the terminal voltage ΔV must equal the potential difference across the external resistance R, often called the **load resistance.** The load resistor might be a simple resistive circuit element, as in Figure 28.1, or it could be the resistance of some electrical device (such as a toaster, an electric heater, or a lightbulb) connected to the battery (or, in the case of household devices, to the wall outlet). The resistor represents a *load* on the battery because the battery must supply energy to operate the device. The potential difference across the load resistance is $\Delta V = IR$. Combining this expression with Equation 28.1, we see that

$$\mathcal{E} = IR + Ir \quad \textbf{(28.2)}$$

Solving for the current gives

$$I = \frac{\mathcal{E}}{R + r} \quad \textbf{(28.3)}$$

This equation shows that the current in this simple circuit depends on both the load resistance R external to the battery and the internal resistance r. If R is much greater than r, as it is in many real-world circuits, we can neglect r.

If we multiply Equation 28.2 by the current I, we obtain

$$I\mathcal{E} = I^2R + I^2r \quad \textbf{(28.4)}$$

This equation indicates that, because power $\mathcal{P} = I\Delta V$, the total power output Iε of the battery is delivered to the external load resistance in the amount I^2R and to the internal resistance in the amount I^2r. Again, if then most of the power delivered by the battery is transferred to the loadresistance.

This equation indicates that, because power $\mathcal{P} = I\Delta V$ (see Eq. 27.22), the total power output $I\mathcal{E}$ of the battery is delivered to the external load resistance in the amount I^2R and to the internal resistance in the amount I^2r. Again, if $r \ll R$, then most of the power delivered by the battery is transferred to the load resistance.

EXAMPLE 28.1 **Terminal Voltage of a Battery**

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω. Its terminals are connected to a load resistance of 3.00 Ω. (a) Find the current in the circuit and the terminal voltage of the battery.

Solution Using first Equation 28.3 and then Equation 28.1, we obtain

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0\text{ V}}{3.05\ \Omega} = \boxed{3.93\text{ A}}$$

$$\Delta V = \mathcal{E} - Ir = 12.0\text{ V} - (3.93\text{ A})(0.05\ \Omega) = \boxed{11.8\text{ V}}$$

To check this result, we can calculate the voltage across the load resistance R:

$$\Delta V = IR = (3.93\text{ A})(3.00\ \Omega) = 11.8\text{ V}$$

(b) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

Solution The power delivered to the load resistor is

$$\mathcal{P}_R = I^2R = (3.93\text{ A})^2\,(3.00\ \Omega) = \boxed{46.3\text{ W}}$$

The power delivered to the internal resistance is

$$\mathcal{P}_r = I^2r = (3.93\text{ A})^2\,(0.05\ \Omega) = \boxed{0.772\text{ W}}$$

Hence, the power delivered by the battery is the sum of these quantities, or 47.1 W. You should check this result, using the expression $\mathcal{P} = I\mathcal{E}$.

RC CIRCUITS

So far we have been analyzing steady-state circuits, in which the current is constant. In circuits containing capacitors, the current may vary in time. A circuit containing a series combination of a resistor and a capacitor is called an RC circuit.

Charging a Capacitor

Let us assume that the capacitor in Figure 28.16 is initially uncharged. There is no current while switch S is open (Fig. 28.16b). If the switch is closed at $t = 0$, however, charge begins to flow, setting up a current in the circuit, and the capacitor begins to charge.[4] Note that during charging, charges do not jump across the capacitor plates because the gap between the plates represents an open circuit. Instead, charge is transferred between each plate and its connecting wire due to the electric field established in the wires by the battery, until the capacitor is fully charged. As the plates become charged, the potential difference across the capacitor increases. The value of the maximum charge depends on the voltage of the battery. Once the maximum charge is reached, the current in the circuit is zero because the potential difference across the capacitor matches that supplied by the battery.

To analyze this circuit quantitatively, let us apply Kirchhoff's loop rule to the circuit after the switch is closed. Traversing the loop clockwise gives

$$\mathcal{E} - \frac{q}{C} - IR = 0 \qquad \textbf{(28.11)}$$

where q/C is the potential difference across the capacitor and IR is the potential

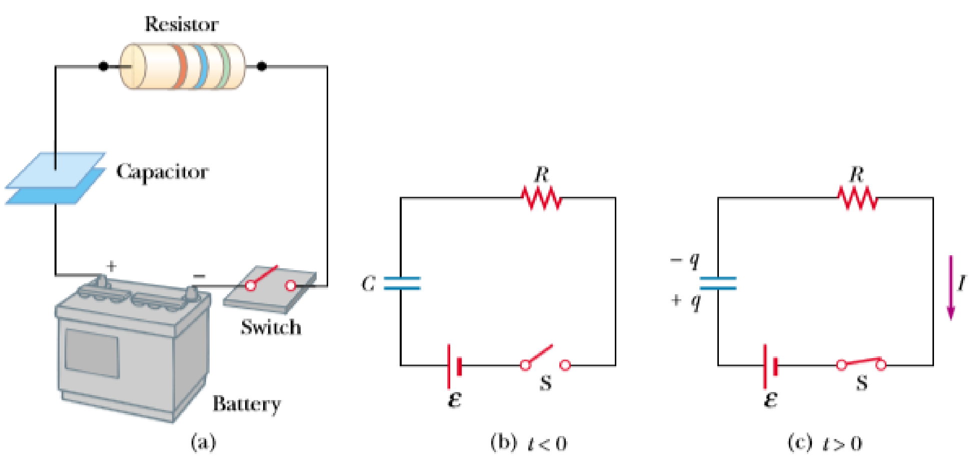


Figure 28.16 (a) A capacitor in series with a resistor, switch, and battery. (b) Circuit diagram representing this system at time $t < 0$, before the switch is closed. (c) Circuit diagram at time $t > 0$, after the switch has been closed.

difference across the resistor. We have used the sign conventions discussed earlier for the signs on $\mathcal{E}$ and IR. For the capacitor, notice that we are traveling in the direction from the positive plate to the negative plate; this represents a decrease in potential. Thus, we use a negative sign for this voltage in Equation 28.11. Note that q and I are *instantaneous* values that depend on time (as opposed to steady-state values) as the capacitor is being charged.

We can use Equation 28.11 to find the initial current in the circuit and the maximum charge on the capacitor. At the instant the switch is closed ($t = 0$), the charge on the capacitor is zero, and from Equation 28.11 we find that the initial current in the circuit I_0 is a maximum and is equal to

$$I_0 = \frac{\mathcal{E}}{R} \qquad \text{(current at } t = 0\text{)} \qquad \textbf{(28.12)}$$

Maximum current

At this time, the potential difference from the battery terminals appears entirely across the resistor. Later, when the capacitor is charged to its maximum value Q, charges cease to flow, the current in the circuit is zero, and the potential difference from the battery terminals appears entirely across the capacitor. Substituting $I = 0$ into Equation 28.11 gives the charge on the capacitor at this time:

$$Q = C\mathcal{E} \qquad \text{(maximum charge)} \qquad \textbf{(28.13)}$$

Maximum charge on the capacitor

To determine analytical expressions for the time dependence of the charge and current, we must solve Equation 28.11—a single equation containing two variables, q and I. The current in all parts of the series circuit must be the same. Thus, the current in the resistance R must be the same as the current flowing out of and into the capacitor plates. This current is equal to the time rate of change of the charge on the capacitor plates. Thus, we substitute $I = dq/dt$ into Equation 28.11 and rearrange the equation:

$$\frac{dq}{dt} = \frac{\mathcal{E}}{R} - \frac{q}{RC}$$

To find an expression for q, we first combine the terms on the right-hand side:

$$\frac{dq}{dt} = \frac{C\mathcal{E}}{RC} - \frac{q}{RC} = -\frac{q - C\mathcal{E}}{RC}$$

Now we multiply by dt and divide by $q - C\mathcal{E}$ to obtain

$$\frac{dq}{q - C\mathcal{E}} = -\frac{1}{RC}\,dt$$

Integrating this expression, using the fact that $q = 0$ at $t = 0$, we obtain

$$\int_0^q \frac{dq}{q - C\mathcal{E}} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC}$$

From the definition of the natural logarithm, we can write this expression as

Charge versus time for a capacitor being charged

$$q(t) = C\mathcal{E}\,(1 - e^{-t/RC}) = Q(1 - e^{-t/RC}) \qquad \textbf{(28.14)}$$

where e is the base of the natural logarithm and we have made the substitution $C\mathcal{E} = Q$ from Equation 28.13.

We can find an expression for the charging current by differentiating Equation 28.14 with respect to time. Using $I = dq/dt$, we find that

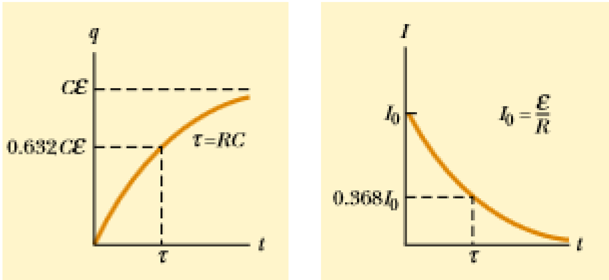
Current versus time for a charging capacitor

$$I(t) = \frac{\mathcal{E}}{R}\,e^{-t/RC} \qquad \textbf{(28.15)}$$

Plots of capacitor charge and circuit current versus time are shown in Figure 28.17. Note that the charge is zero at $t = 0$ and approaches the maximum value $C\mathcal{E}$ as $t \rightarrow \infty$. The current has its maximum value $I_0 = \mathcal{E}/R$ at $t = 0$ and decays exponentially to zero as $t \rightarrow \infty$. The quantity RC, which appears in the exponents of Equations 28.14 and 28.15, is called the **time constant** τ of the circuit. It represents the time it takes the current to decrease to $1/e$ of its initial value; that is, in a time τ, $I = e^{-1}I_0 = 0.368I_0$. In a time 2τ, $I = e^{-2}I_0 = 0.135I_0$, and so forth. Likewise, in a time τ, the charge increases from zero to $C\mathcal{E}\,(1 - e^{-1}) = 0.632C\mathcal{E}$.

The following dimensional analysis shows that τ has the units of time:

$$[\tau] = [RC] = \left[\frac{\Delta V}{I} \times \frac{Q}{\Delta V}\right] = \left[\frac{Q}{Q/\Delta t}\right] = [\Delta t] = \mathrm{T}$$



(a) (b)

Figure 28.17 (a) Plot of capacitor charge versus time for the circuit shown in Figure 28.16. After a time interval equal to one time constant τ has passed, the charge is 63.2% of the maximum value $C\mathcal{E}$. The charge approaches its maximum value as t approaches infinity. (b) Plot of current versus time for the circuit shown in Figure 28.16. The current has its maximum value $I_0 = \mathcal{E}/R$ at $t = 0$ and decays to zero exponentially as t approaches infinity. After a time interval equal to one time constant τ has passed, the current is 36.8% of its initial value.

Because $\tau = RC$ has units of time, the combination t/RC is dimensionless, as it must be in order to be an exponent of e in Equations 28.14 and 28.15.

The energy output of the battery as the capacitor is fully charged is $Q\mathcal{E} = C\mathcal{E}^2$. After the capacitor is fully charged, the energy stored in the capacitor is $\frac{1}{2}Q\mathcal{E} = \frac{1}{2}C\mathcal{E}^2$, which is just half the energy output of the battery. It is left as a problem (Problem 60) to show that the remaining half of the energy supplied by the battery appears as internal energy in the resistor.

Discharging a Capacitor

Now let us consider the circuit shown in Figure 28.18, which consists of a capacitor carrying an initial charge Q, a resistor, and a switch. The *initial* charge Q is not the same as the *maximum* charge Q in the previous discussion, unless the discharge occurs after the capacitor is fully charged (as described earlier). When the switch is open, a potential difference Q/C exists across the capacitor and there is zero potential difference across the resistor because $I = 0$. If the switch is closed at $t = 0$, the capacitor begins to discharge through the resistor. At some time t during the discharge, the current in the circuit is I and the charge on the capacitor is q (Fig. 28.18b). The circuit in Figure 28.18 is the same as the circuit in Figure 28.16 except for the absence of the battery. Thus, we eliminate the emf $\mathcal{E}$ from Equation 28.11 to obtain the appropriate loop equation for the circuit in Figure 28.18:

$$-\frac{q}{C} - IR = 0 \qquad \textbf{(28.16)}$$

When we substitute $I = dq/dt$ into this expression, it becomes

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{1}{RC}\,dt$$

Integrating this expression, using the fact that $q = Q$ at $t = 0$, gives

$$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

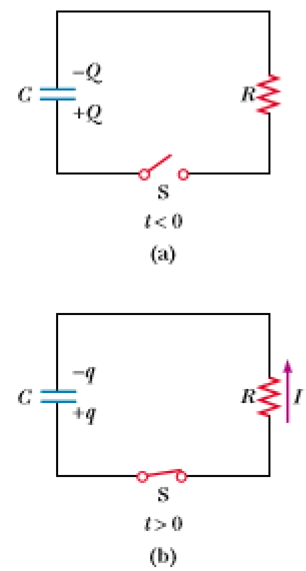


Figure 28.18 (a) A charged capacitor connected to a resistor and a switch, which is open at $t < 0$. (b) After the switch is closed, a current that decreases in magnitude with time is set up in the direction shown, and the charge on the capacitor decreases exponentially with time.

$$q(t) = Qe^{-t/RC} \qquad \textbf{(28.17)}$$

Charge versus time for a discharging capacitor

Differentiating this expression with respect to time gives the instantaneous current as a function of time:

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}(Qe^{-t/RC}) = -\frac{Q}{RC}e^{-t/RC} \qquad \textbf{(28.18)}$$

Current versus time for a discharging capacitor

where $Q/RC = I_0$ is the initial current. The negative sign indicates that the current direction now that the capacitor is discharging is opposite the current direction when the capacitor was being charged. (Compare the current directions in Figs. 28.16c and 28.18b.) We see that both the charge on the capacitor and the current decay exponentially at a rate characterized by the time constant $\tau = RC$.

Example 28.11 Charging a Capacitor in an *RC* Circuit

An uncharged capacitor and a resistor are connected in series to a battery, as shown in Figure 28.19. If $\mathcal{E} = 12.0$ V, $C = 5.00\ \mu$F, and $R = 8.00 \times 10^5\ \Omega$, find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and current as functions of time.

Solution The time constant of the circuit is $\tau = RC = (8.00 \times 10^5\ \Omega)(5.00 \times 10^{-6}\ \text{F}) = 4.00$ s. The maximum charge on the capacitor is $Q = C\mathcal{E} = (5.00\ \mu\text{F})(12.0\ \text{V}) = 60.0\ \mu$C. The maximum current in the circuit is $I_0 = \mathcal{E}/R = (12.0\ \text{V})/(8.00 \times 10^5\ \Omega) = 15.0\ \mu$A. Using these values and Equations 28.14 and 28.15, we find that

$$q(t) = (60.0\ \mu\text{C})(1 - e^{-t/4.00\text{ s}})$$

$$I(t) = (15.0\ \mu\text{A})\ e^{-t/4.00\text{ s}}$$

Graphs of these functions are provided in Figure 28.20.

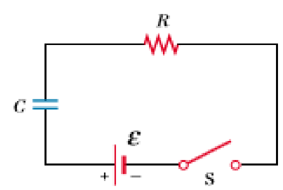


Figure 28.19 The switch of this series *RC* circuit, open for times $t < 0$, is closed at $t = 0$.

Exercise Calculate the charge on the capacitor and the current in the circuit after one time constant has elapsed.

Answer 37.9 μC, 5.52 μA.

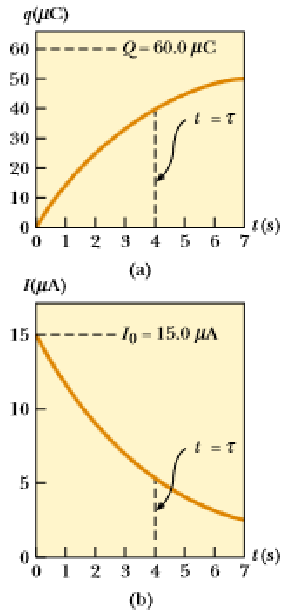


Figure 28.20 Plots of (a) charge versus time and (b) current versus time for the *RC* circuit shown in Figure 28.19, with $\mathcal{E} = 12.0$ V, $R = 8.00 \times 10^5\ \Omega$, and $C = 5.00\ \mu$F.

Example 28.12 Discharging a Capacitor in an *RC* Circuit

Consider a capacitor of capacitance C that is being discharged through a resistor of resistance R, as shown in Figure 28.18. (a) After how many time constants is the charge on the capacitor one-fourth its initial value?

Solution The charge on the capacitor varies with time according to Equation 28.17, $q(t) = Qe^{-t/RC}$. To find the time it takes q to drop to one-fourth its initial value, we substitute $q(t) = Q/4$ into this expression and solve for t:

$$\frac{Q}{4} = Qe^{-t/RC}$$

$$\tfrac{1}{4} = e^{-t/RC}$$

Taking logarithms of both sides, we find

$$-\ln 4 = -\frac{t}{RC}$$

$$t = RC(\ln 4) = 1.39RC = 1.39\tau$$

(b) The energy stored in the capacitor decreases with time as the capacitor discharges. After how many time constants is this stored energy one-fourth its initial value?

Solution Using Equations 26.11 ($U = Q^2/2C$) and 28.17, we can express the energy stored in the capacitor at any time t as

$$U = \frac{q^2}{2C} = \frac{(Qe^{-t/RC})^2}{2C} = \frac{Q^2}{2C}e^{-2t/RC} = U_0e^{-2t/RC}$$

where $U_0 = Q^2/2C$ is the initial energy stored in the capacitor. As in part (a), we now set $U = U_0/4$ and solve for t:

$$\frac{U_0}{4} = U_0e^{-2t/RC}$$

$$\tfrac{1}{4} = e^{-2t/RC}$$

Again, taking logarithms of both sides and solving for t gives

$$t = \tfrac{1}{2}RC(\ln 4) = 0.693RC = 0.693\tau$$

Exercise After how many time constants is the current in the circuit one-half its initial value?

Answer $0.693RC = 0.693\tau$.