

# المحاضرة السابعة-الفصل السادس

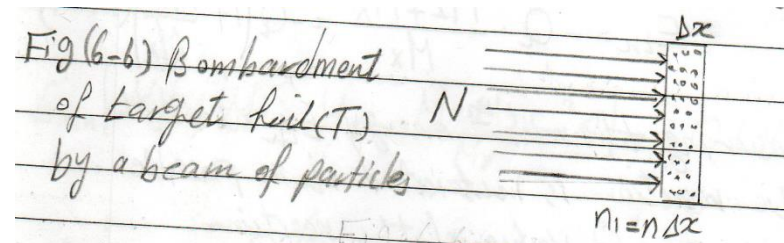
## التفاعلات النووية Nuclear Reaction

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## 6.6 Cross Section of Nuclear Reaction

The probability of the occurrence of a nuclear reaction is measured by the reaction cross section. It is designated by the symbol  $\Sigma$ . The cross section of the nuclear reaction  $X(x,y)Y$  can be written as  $\Sigma(x,y)$ . If a parallel beam of  $N$  projectiles is incident in a given interval of time upon a target foil  $T$  of thickness  $\Delta x$  and surface area  $S$ , then the No. of nuclei in  $T$  undergoing transformation due to the reaction of the type under consideration is proportional to the intensity of the incident beam of projectiles and to the total No. of target nuclei present in the foil see Fig. (6.6). The incident particle intensity is  $(N/S)$  and the No. of nuclei present in the foil is  $(nS \Delta x)$ . So the No. of nuclei transformed is  $\Delta N \propto (N/S) (nS \Delta x)$

$$\Delta N = \Sigma N n \Delta x = \Sigma N n_1 \quad \text{--- (66.1)}$$



Here  $n_1 = n \Delta x$  is the No. of target nuclei per unit area of the foil,  $n$  being the No. of nuclei per unit volume. Eq. (6.6.1) shows that since both  $\Delta N$  and  $N$  are pure No. and  $n_1 = n \Delta x$  has the dimension of the reciprocal of an area,  $\Sigma$  has the dimension of an area. Hence it is called the cross section and measures. The probability of the occurrence of the reaction when a single particle ( $N = 1$ ) falls on a single target nucleus present per unit area ( $n_1 = 1$ ). Since the nuclear radii are of the order of  $10^{-14}$  to  $10^{-15}$  m, the cross section of the nuclear reaction is of the order of  $10^{-28}$  m<sup>2</sup>. The unit of the nuclear reaction cross section is a barn (1 barn =  $10^{-28}$  m<sup>2</sup>). In the case of charged particles, the cross section is reduced because of the strong electrostatic repulsion of the target nucleus.

## 6.7 Partial Cross Sections

When a nuclear projectile  $x$  is absorbed by a target nucleus  ${}^A_ZX$ , a very short lived compound nucleus is formed which can break up by the emission of different types of nuclear particles ( $y$ ), leaving a different residual nucleus ( $Y$ ). We may have reactions of the type  $X(x,y)Y$ ,  $X(x,y')Y'$ ,  $X(x,y'')Y''$  etc. in addition, we may have elastic and inelastic scattering. Each of these different reactions induced by the same projectile  $x$  in the same target nucleus  $X$  has different cross section. e.g.  $\sigma(x,y)$ ,  $\sigma(x,y')$ ,  $\sigma(x,y'')$ , etc. in addition to  $\sigma(x,x)$  and  $\sigma(x,x')$ . The total cross sections for the interaction of  $x$  with  $X$  for a given energy  $E_x$  of  $x$  can

be written as  $\sigma_t = \sigma_x = \sigma(x,x) + \sigma(x,x') + \sigma(x,y) + \sigma(x,y') + \sigma(x,y'') + \dots$

$$\sigma_t = \sigma_{sc} + \sigma_r \quad \text{--- (6.7.1)}$$

The cross section for the individual type of reactions are known as partial cross sections.

$$\sigma_r = \sigma(x,x') + \sigma(x,y) + \sigma(x,y') + \sigma(x,y'') \quad \text{--- (6.7.2)}$$

It is called the reaction cross section to distinguish from the elastic scattering cross section  $\sigma_{sc} = \sigma_{el}$ . Reaction cross sections are expressed in terms of the reaction channels, which are specified in terms of the energy, angular momentum and radius. A nuclear reaction can be written as  $X + x \rightarrow Y + y$

For a given  $E_x$ , the L.H.S. of this Eq.  $X + x$  is known as the entrance channel, ( $X$  at rest). The right hand side giving the final products constitutes the exit channel. For elastic scattering the exit channel is identical with the entrance channel. A channel is a possible pair of product nuclei, each in a definite quantum state. The total cross section  $\sigma_t = \sigma_r + \sigma_{el}$  determines the absorption coefficient for the beam of particles incident on the target foil.

Using Eq. (6.6.1), we can write for a target foil having  $n$  nuclei per unit volume of infinitesimal thickness  $dx$  on which a particle beam of intensity  $N_s = N/S$  is incident

perpendicularly as

$$dn_s = - \mu_t n_s dx \text{ --- (6.7.3)}$$

Where the minus sign is introduced on the R.H.S. to indicate the diminution in the beam intensity as it comes out of the foil. Integration gives for a foil of finite thickness  $x$ ,

$$n_s = n_{s0} \exp (- \mu S) \text{ --- (6.7.4)}$$

Where  $\mu = \mu_t n$ , is the total absorption coefficient.  $N_{s0}$  is the intensity of the beam incident on the foil and  $n_s$  is the emergent intensity.

## 6.8 Reaction Induced by $\alpha$ -Particle

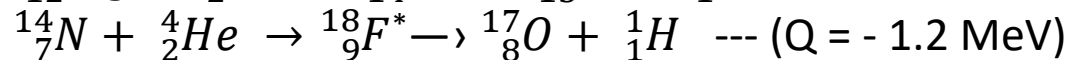
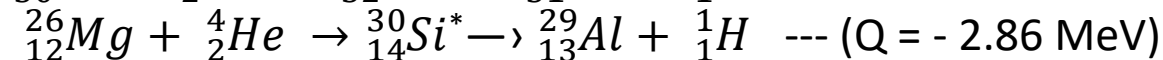
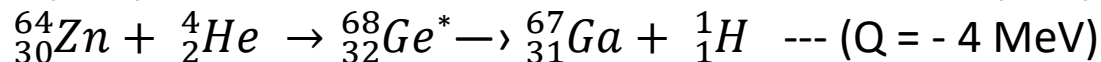
When nuclear reaction is induced by an  $\alpha$ -Particle the compound nucleus may break up by the emission of a proton, a neutron, a  $\gamma$ -ray photon etc.

(a)  $(\alpha, P)$  reaction:-  ${}^A_Z X + {}^4_2 \text{He} \rightarrow {}^{A+4}_{Z+2} C^* \rightarrow {}^{A+3}_{Z+1} Y + {}^1_1 H$ , using Eq. (6.5.3) the  $Q$  of this reaction can be as  $Q = B_Y - B_X - B_\alpha$ , where  $f_B = \frac{E_B}{A}$ ,  $E_B = f_B * A$

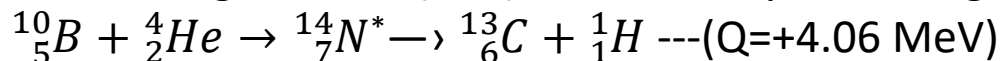
$$Q = (A + 3) f_{BY} - A f_{BX} - 4 f_{B\alpha} = A f_{BY} + 3 f_{BY} - A f_{BX} - 4 f_{B\alpha}$$

Where  $f_B$  denoted the binding fraction is almost constant for most medium heavy nuclei, being about 8 MeV per nucleon and  $f_{B\alpha}$  being about 7 MeV (so  $f_{BY} = f_{BX}$  )

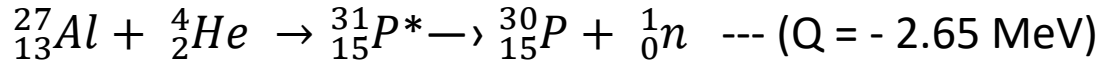
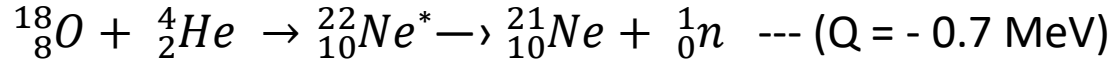
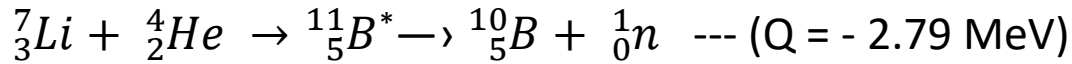
$Q(\alpha, P) \approx 3 \times 8 - 28 = - 4 \text{ MeV}$ , this shows that the  $(\alpha, P)$  reaction is endoergic



For some light nuclei  $(\alpha, P)$  reaction may be exoergic. e.g.

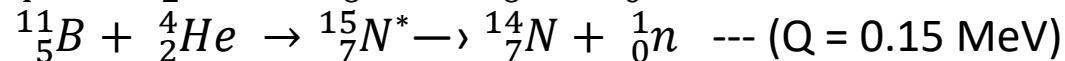
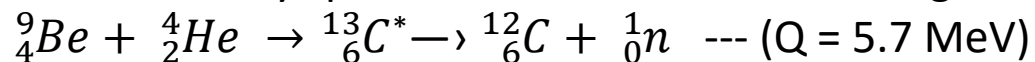


(b) ( $\alpha$ , n) reaction:-  ${}_Z^AX + {}_2^4He \rightarrow {}_{Z+2}^{A+4}C^* \rightarrow {}_{Z+2}^{A+3}Y + {}_0^1n$ , this reaction are endoergic for medium heavy nuclei



From last case the bombardment of aluminum by  $\alpha$ - particles led to the production of the isotopes  ${}_{15}^{30}P$  by  ${}_{13}^{27}Al$  ( $\alpha$ , n) reaction. The residual nucleus  ${}_{15}^{30}P$  produced in the reaction was radioactive and decayed by positron emission  ${}_{15}^{30}P \rightarrow \beta^+, {}_{14}^{30}Si$

In order to the substantiate this conclusion, it is verified the chemical nature of the new radioactive product by separating it from the target by standard radiochemical methods and showed that the positron emission took place from the separated phosphorus. The phenomenon is known as induced or artificial radioactivity most products of artificial transmutation of element are radioactive. They decay mainly by  $\beta^-$  or  $\beta^+$  emission or by orbital electron capture. In the case of some heavy elements they are found to decay by  $\alpha$  emission or by spontaneous fission. For some light nuclei, it may be exoergic as



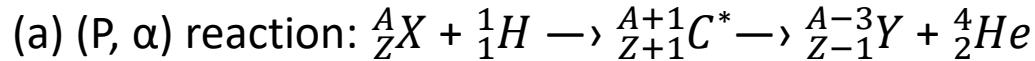
(c) ( $\alpha$ ,  $\gamma$ ) reaction, also known as the radioactive capture of an  $\alpha$ - particle has been observed in some case  ${}_3^7Li$  ( $\alpha$ ,  $\gamma$ )  ${}_5^{11}B$ . The general formula is  ${}_Z^AX + {}_2^4He \rightarrow {}_{Z+2}^{A+4}C^* \rightarrow {}_{Z+2}^{A+4}C + \gamma$ , these reaction are usually exoergic.

(d) More than one particle emission: for high  $\alpha$ - energy, more than one particle may be emitted from the compound nucleus, producing such reactions as ( $\alpha$ , 2n), ( $\alpha$ , 2P), ( $\alpha$ , Pn), ( $\alpha$ , 3n) etc.



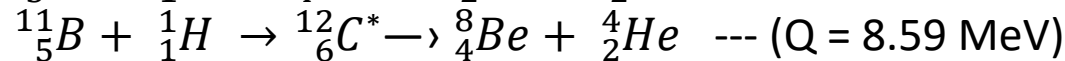
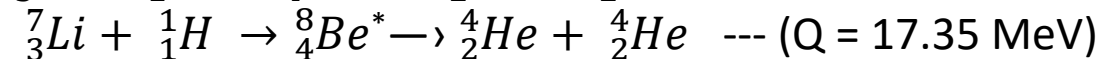
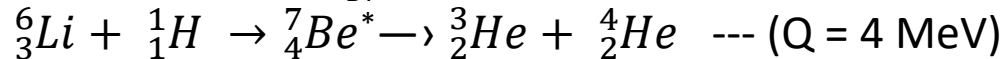
## 6.9 Proton Induced Reaction

When a high energy proton falls on a target nucleus the compound nucleus that is formed may disintegrate by the emission of different types of nuclear particles e.g. proton, neutron, deuteron,  $\alpha$ - particle,  $\gamma$ -ray etc. In the first case, we get elastic or inelastic scattering while in the other cases we get nuclear transmutation.



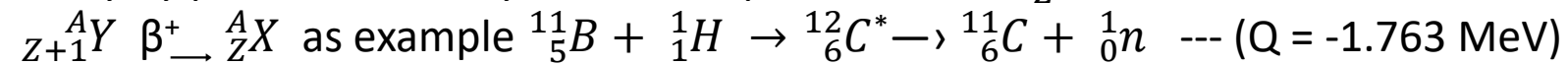
$$Q(P, \alpha) = (A-3) f_{BY} + 4 f_{B\alpha} - A f_{BX}$$

$$Q(P, \alpha) \approx 28 - 3 f_{BY} \approx 4 \text{ MeV.}$$



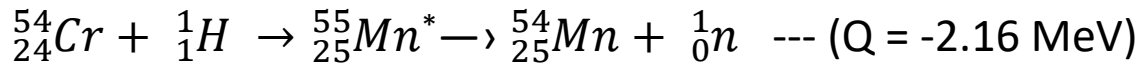
The residual nucleus  ${}^8_4Be$  formed in third reaction is highly unstable. It break up immediately after its production into two  $\alpha$ - particles ( ${}^8_4Be \rightarrow {}^4_2He + {}^4_2He$ ). The final products of this reaction are three  $\alpha$ - particles.

(b) (P, n) reaction:  ${}^A_ZX + {}^1_1H \rightarrow {}^{A+1}_{Z+1}C^* \rightarrow {}^{A+1}_{Z+1}Y + {}^1_0n$ . In this case the residual Y is isobaric (same A) with the target nucleus with the atomic number one unit higher. Since two isobars differing in Z by one unit cannot both be stable, the residual nucleus  ${}^{A+1}_{Z+1}Y$  must be  $\beta$ -active, the target nucleus being necessarily stable. Because of its higher Z, it will decay by  $\beta^+$  emission or by electron capture into the  ${}^A_ZX$ .

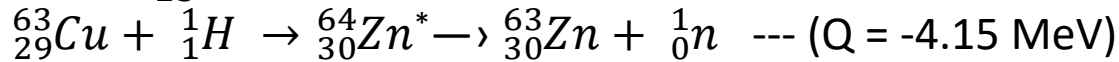


Where  ${}^{11}_6C \xrightarrow{\beta^+} {}^{11}_5B \quad \text{--- (}\tau = 2.5 \text{ min)}$





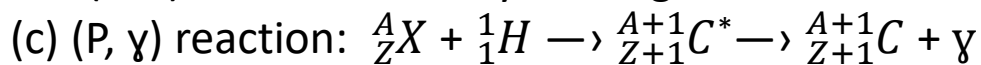
Where  ${}_{25}^{54}\text{Mn} \xrightarrow{\text{E.C.}} {}_{24}^{54}\text{Cr} \quad \text{--- } (\tau = 310 \text{ d})$



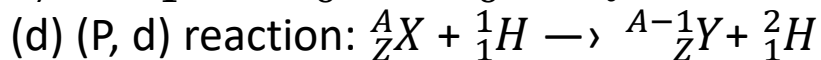
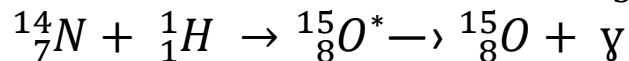
Where  ${}_{30}^{63}\text{Zn} \xrightarrow{\text{E.C.}} \beta^+ \rightarrow {}_{29}^{63}\text{Cu} \quad \text{--- } (\tau = 38.5 \text{ min})$

The product nucleus in each of the above reactions is radioactive. The Q of the (P, n) reaction can be written as  $Q(\text{P, n}) = M_X + M_H - M_Y - M_n$ , since Y is a  $\beta^+$  emitter, we have  $Q(\beta^+) = M_Y - M_X - 2m_e$

$Q(\text{P, n}) = -Q(\beta^+) - 2m_e - (M_n - M_H)$ , since  $M_n > M_H$ ,  $Q(\text{P, n}) < 0$  i.e. for a stable target nucleus, the (P, n) reaction is always endothermic



In some cases, the excited compound nucleus formed by the absorption of a proton by the target nucleus does not disintegrate by the emission of a nuclear particle, but goes down to the ground state by the emission of one or more  $\gamma$ -ray photons. This is the radiative capture of the proton or (P,  $\gamma$ ) reaction  ${}_3^7\text{Li} + {}_1^1\text{H} \rightarrow {}_4^8\text{Be}^* \rightarrow {}_4^8\text{Be} + \gamma \quad \text{--- } (E_\gamma = 17.2 \text{ MeV})$



Example  ${}_3^7\text{Li}(\text{P, d}){}_3^6\text{Li}$  and  ${}_4^9\text{Be}(\text{P, d}){}_4^8\text{Be}$ . This is an example of a pick up type direct reaction no compound nucleus is formed.

(e) More than one particle emission: if the incident proton beam has very high energy ( $E_p > 20 \text{ MeV}$ ), more than one particle may be emitted from the compound nucleus to produce reaction like (P, Pn), (P, 2n), (P, 2P), (P, 3n) etc.



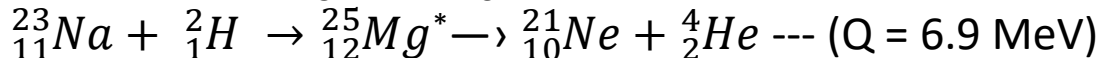
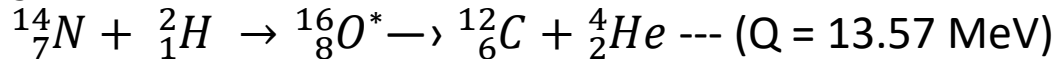
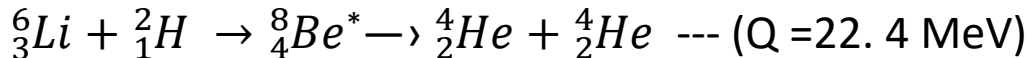
## 6.9 Deuteron Induced Reaction

(a) (d,  $\alpha$ ) reaction:  ${}_Z^AX + {}_1^2H \rightarrow {}_{Z+1}^{A+2}C^* \rightarrow {}_{Z-1}^{A-2}Y + {}_2^4He$

$$Q(d, \alpha) = B_Y + B_\alpha - B_X - B_d = (A-2) f_{BY} + 28 - A f_{BX} - 2.2$$

$$Q(d, \alpha) = 25.8 - 2 f_{BY} \approx 25.8 - 16 = 9.8 \text{ MeV}$$

$Q(d, \alpha) > 0$ . Some example of (d,  $\alpha$ ) reactions are given

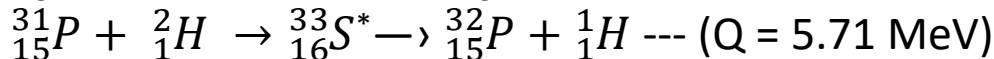
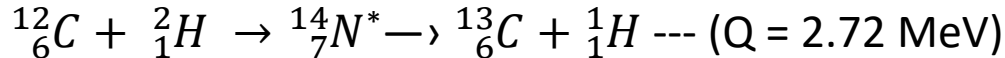
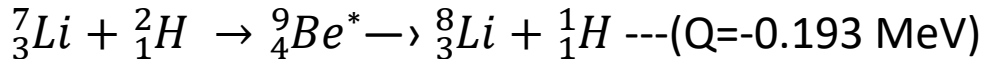


(b) (d, P) reaction:  ${}_Z^AX + {}_1^2H \rightarrow {}_{Z+1}^{A+2}C^* \rightarrow {}_Z^{A+1}Y + {}_1^1H$

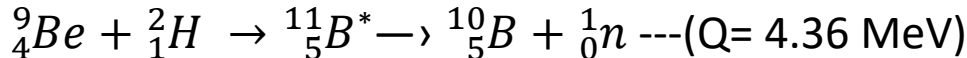
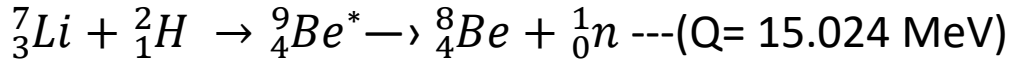
This reaction is usually exoergic  $Q(d, P) = B_Y - B_X - B_d = (A+1) f_{BY} - A f_{BX} - 2.2$

$$Q(d, P) \approx f_{BY} - 2.2 \approx 8 - 2.2 = 5.8 \text{ MeV, So } Q > 0$$

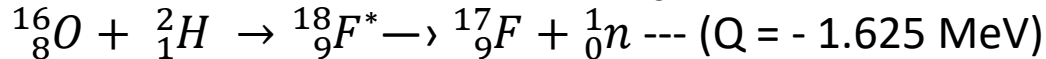
For some light nuclei Q may be negative (endoergic) as



(c) (d, n) reaction:  ${}_Z^AX + {}_1^2H \rightarrow {}_{Z+1}^{A+2}C^* \rightarrow {}_{Z+1}^{A+1}Y + {}_0^1n$  this reaction are usually exorgic



Some of them are endoergic as  ${}_6^{12}C + {}_1^2H \rightarrow {}_7^{14}N^* \rightarrow {}_7^{13}N + {}_0^1n \text{ --- } (Q = -0.283 \text{ MeV})$



(d) (d, t) reaction:  ${}_Z^AX + {}_1^2H \rightarrow {}_{Z+1}^{A+2}X^* \rightarrow {}_Z^{A-1}Y + {}_1^3H$

The product nucleus Y is an isotope of the target X with mass No. one unit lower. The cross section of this type of reaction is rather low, as  ${}_3^7Li + {}_1^2H \rightarrow {}_4^9Be^* \rightarrow {}_3^6Li + {}_1^3H$  --- (Q = - 0.996 MeV)

${}_4^9Be + {}_1^2H \rightarrow {}_5^{10}B^* \rightarrow {}_4^8Be + {}_1^3H$  --- (Q = 4.59 MeV)

(e) More than one particle emission: at higher energies of the deuteron ( $E_d > 20$  MeV), the reactions (d, 2n), (d, 2p), (d, 2p), etc.

### 6.10 Neutron Induced Reaction

(a) (n,α) reaction:  ${}_Z^AX + {}_0^1n \rightarrow {}_Z^{A+1}C^* \rightarrow {}_{Z-2}^{A-3}Y + {}_2^4He$  as,

${}_3^6Li + {}_0^1n \rightarrow {}_3^7Li^* \rightarrow {}_1^3H + {}_2^4He$  --- (Q = 4.785 MeV)

${}_5^{10}B + {}_0^1n \rightarrow {}_5^{11}B^* \rightarrow {}_3^7Li + {}_2^4He$  --- (Q = 2.79 MeV)

These reactions have large cross section, so they are utilized in the construction of neutron detector. (n,α) reactions are usually exothermic specially for medium heavy nuclei.

(b) (n, p) reaction:  ${}_Z^AX + {}_0^1n \rightarrow {}_Z^{A+1}C^* \rightarrow {}_{Z-1}^AY + {}_1^1H$ ,

the product nucleus Y is an isobar of this target nucleus X with atomic No. one unit lower. It is  $\beta^-$  active decaying to the target nucleus.  ${}_{Z-1}^AY \xrightarrow{\beta^-} {}_Z^AX$ , since  $Q(\beta^-) = M_Y - M_X$ , the Q of

the (n, p) reaction is  $Q(n, p) = B_Y - B_X = M_X + M_n - M_Y - M_H$

$Q(n, p) = (M_n - M_H) - Q(\beta^-) = 0.782 - Q(\beta^-)$  MeV

so if  $Q(\beta^-) < 0.782$  MeV, the reaction is exothermic, for  $Q(\beta^-) > 0.782$  MeV, the reaction is

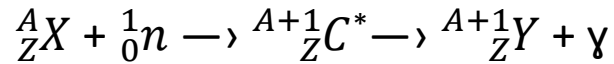
endothermic as,  ${}_2^3He + {}_0^1n \rightarrow {}_2^4He^* \rightarrow {}_1^3H + {}_1^1H$  --- (Q = 0.764 MeV)

${}_{13}^{27}Al + {}_0^1n \rightarrow {}_{13}^{28}Al^* \rightarrow {}_{12}^{27}Mg + {}_1^1H$  --- (Q = - 1.83 MeV)



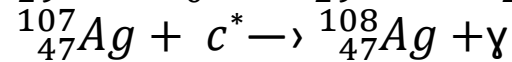
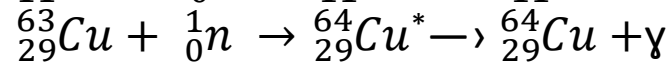
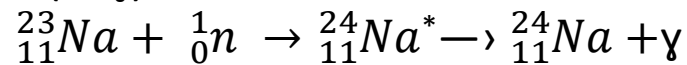
(c) (n, d) and (n, t) reactions: These reactions are known as pick up reactions which are belong to the category of direct reactions. The mechanism of these reactions is different from compound nuclear process as  ${}^{14}_7N + {}^1_0n \rightarrow {}^{12}_6C + {}^3_1H$

(d) (n,  $\gamma$ ) reaction: the most important neutron- induced reaction is the (n,  $\gamma$ ) reaction, known as the radiation capture of neutrons, the general formula is

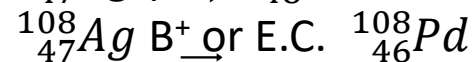
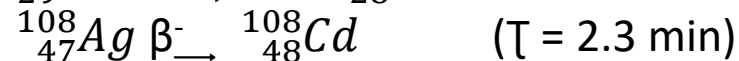
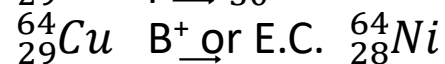


Here the product nucleus is the same as the compound nucleus in the ground state ( $Y \equiv C$ ) the (n,  $\gamma$ ) reactions are always exoergic ( $Q > 0$ ) and can be induced by almost zero energy neutrons. The Q of the reaction is  $Q(n, \gamma) = M_X + M_n - M_Y$ , except for some light nuclei

$Q(n, \gamma) \approx 8 \text{ MeV}$  are emitted



The product nuclei in all the above reactions are radioactive capture of a neutron by the target nucleus increases the neutron-proton ratio and hence shifts the nucleus to the left above the stability line. The product nucleus usually becomes  $\beta^-$ -active, since it has an excess of neutrons compared to the No. of protons which would make it stable. However in the case of some odd-odd product nuclei (e.g.  ${}^{64}_{29}Cu$  and  ${}^{108}_{47}Ag$ ) both  $\beta^-$  and  $\beta^+$  or electron capture.



(e) More than one particle emission: at higher energies ( $E_n > 8 \text{ MeV}$ ), more than one particle may be emitted from the compound nucleus formed by the capture of a neutron. These reactions are  $(n, 2n)$ ,  $(n, 3n)$ ,  $(n, nP)$ ,  $(n, P2n)$  etc. for the proton emission along with one or two neutrons, the energy of excitation of the compound nucleus must be sufficiently high so that the proton is able to cross the potential barrier to come out.

### 6.11 Gamma Ray Induced Reaction

These reactions known as the photo-nuclear reactions occur if sufficiently high energy photons enter into a nucleus. The energy of the incident  $\gamma$ -ray must be greater than the energy of binding of a nuclear particle e.g. a neutron, a proton, an  $\alpha$ -particle etc. for the particle to be emitted from the nucleus producing reactions of the type  $(\gamma, n)$ ,  $(\gamma, P)$ ,  $(\gamma, \alpha)$  etc. these reactions are endoergic, as example  ${}^2_1\text{H} + \gamma \rightarrow {}^1_1\text{H} + {}^1_0\text{n}$  This is known as the photo-disintegration of the deuteron. The energy of the  $\gamma$ -ray which can induce this reaction must be greater than binding energy (2.226 MeV) of the deuteron.

These measurements  $\gamma$ -rays from the naturally radioactive isotope ( $E_\gamma = 2.62 \text{ MeV}$ ) and those from the artificially radioactive  ${}^{24}_{11}\text{Na}$  isotope ( $E_\gamma = 2.76 \text{ MeV}$ ) were used. Another example  ${}^9_4\text{Be} + \gamma \rightarrow {}^9_4\text{Be}^* \rightarrow {}^8_4\text{Be} + {}^1_0\text{n}$

The threshold of this reaction is (1.66 MeV),  $\gamma$ -ray from  ${}^{24}_{11}\text{Na}$  source is ( $E_\gamma = 2.04 \text{ MeV}$ ). It may be noted that the deuteron and  ${}^9_4\text{Be}$  are the only nuclei which undergo photo-disintegration by  $\gamma$ -ray from naturally radioactive substances.

For all other nuclei,  $\gamma$ -rays from nuclear reactions (mainly proton induced) of much higher energies ( $E_\gamma > 2.62 \text{ MeV}$ ) must be used to produce photo-nuclear reaction as

${}^7_3\text{Li} (P, \gamma) {}^8_4\text{Be}$  ( $E_\gamma = 17.2 \text{ MeV}$ ),  ${}^{11}_5\text{B} (P, \gamma) {}^{12}_6\text{C}$  ( $E_\gamma = 11.7 \text{ MeV}$ ),  ${}^{19}_9\text{F} (P, \gamma) {}^{16}_8\text{O}^*$   
 ${}^{16}_8\text{O}^* \rightarrow {}^{16}_8\text{O} + \gamma$  ( $E_\gamma = 6.13 \text{ MeV}$ ).

