المحاضرة الرابعة الفصل الخامس
النماذج النووية

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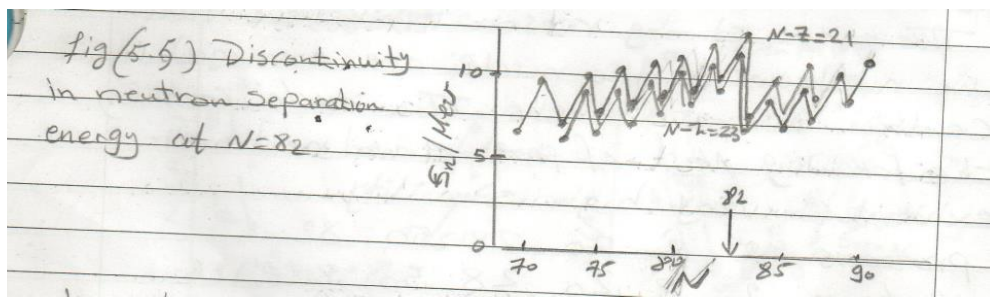
Fig. (b) shows the two mass parabola for the even A isobars with A=102. The upper one is for the odd Z odd N isobars, while the lower one is for the even Z even N isobars. The most stable isobar in this case falls on the lower parabola. Using Eq. (5.7.6), we get (Z_A=44.2) actually a stable nuclide ^{102}Ru at (Z=44) iis observed at this mass No. resides, another stable e-e nuclide ^{102}Pd (Z=46) also exists at this A. the two stable isobars differ in Z by two unit. The o-o isobar ^{102}Rh with (Z=45) between these two falls on the upper parabola and has an atomic mass greater than those of either of the above two. Hence ^{102}Rh is not stable. It shows both β^+ and β^- activities. β^+ emission transforms it to ^{102}Ru while by β^- emission it transforms to ^{102}Pd.

5.8 Nuclear shell structure

The liquid drop model can explain the observed variation of the nuclear binding energy with the mass No. and the fission of the heavy nuclei. However, this model predicts very closely spaced energy level in nuclei which is contrary to observation at law energies**.** The low lying excited states in nuclei are actually quite widely spaced, which cannot be explained by the liquid drop model. This and certain other properties of the nucleus would require us to consider the motion of the individual nucleons in a potential well which would give rise to the existence of a nuclear shell structure, similar to the electronic shells in the atoms. There are strong reasons to believe that the nucleons in the nuclei are arranged in certain discrete shells. The nuclei containing the following No. of protons and neutrons exhibited very high stability .

Protons 2 8 20 28 50 82, neutrons 2 8 20 28 50 82 126, these Nos. are known as magic Nos. and there is a semi-magic No. of protons and neutrons both. Example ^{4}He (Z=2, N=2), ^{16}O (Z=8, N=8), ^{40}Ca (Z=20, N=20), ^{48}Ca (Z=20, N=28), ^{208}Pb (Z=82, N=126). They are doubly magic and show exceptionally high stability. Following are the main evidences to show the existence of shell structure within the nuclei.

(a) measurement shows that the separation energy S_n of a neutron from a nucleus containing a magic No. of neutrons is large compared to that for a nucleus containing one more neutron. Similarly the separation energy S_p of a proton from a nucleus containing a magic No. of proton is large compared to that for a nucleus containing one more proton (separation energy is meant the minimum energy needed for separating one neutron or proton from a nucleus). The sudden discontinuity in the value of S_n at the magic neutron No. is shown in Fig (5.5).



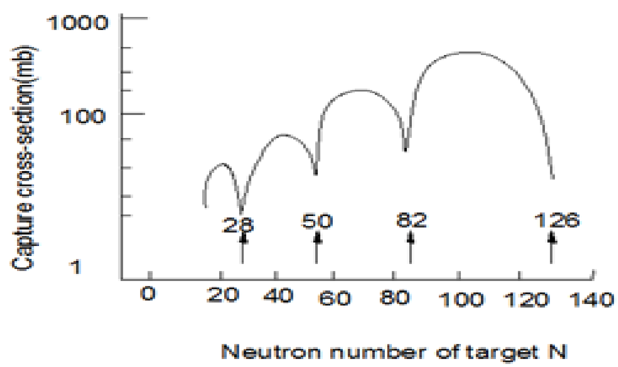
(b) The naturally occurring isotopes, whose nuclei contain magic Nos. of neutrons or protons have generally greater relative abundances (>60%). For example the isotopes ^{88}Sr (N=50), ^{138}Ba (N=82) and ^{140}Ce (N=82) have relative abundances of 82.56%, 71.66%, and 88.48% respectively.

(c) the No. of stable isotopes of an element containing a magic No. of proton is usually large compared to those for other elements. For example, calcium (Z=20) has 6 stable isotopes compared to 3 for Argon (Z=18) and 5 for titanium (Z=22).

(d) The No. of naturally occurring isotones with magic Nos. of neutrons is usually large compared to those in the neighborhood. As an example the No. of stable isotones at (N=82) is 7 compared to 3 at (N=80) and 2 at (N=84) respectively.

(e) Nuclei with magic Nos. of neutrons or protons have their first excited states at higher energies than in the cases of the neighborhood nuclei.

(f) The neutron capture cross-sections of the nuclei with magic Nos. of neutrons are usually low. Since the neutron shells are filled up in these nuclei, the probabilities of their capturing an additional neutron is small Fig. (5.6). similarly nuclei with magic proton Nos. have low proton capture.



(g) If the Alpha-disintegration energies of the heavy nuclei are plotted as functions of the mass No. A for a given Z, then usually a regular variation is observed till the magic neutron No. N=126 is reached, there is sudden discontinuity. This confirms the magic character of the neutron No. 126. Similar discontinuity are observed amongst the Beta-emitters at the magic neutron or proton Nos.

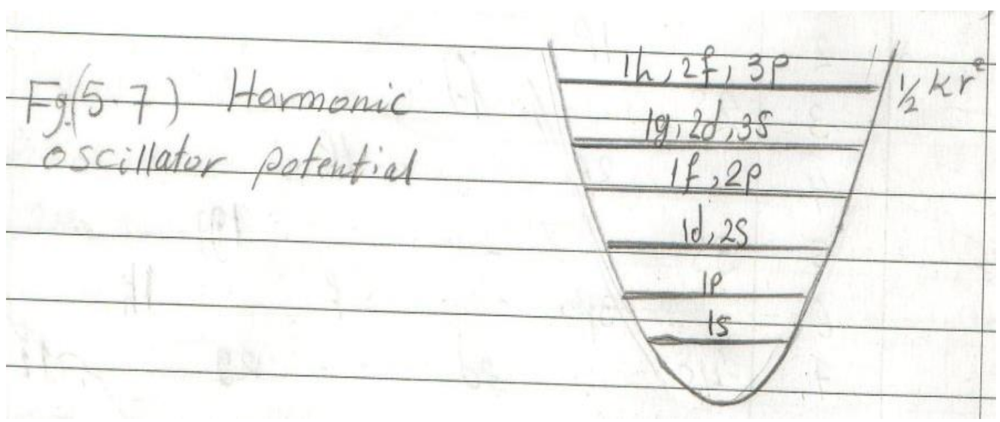
5.9 Single particle states in nuclei

The nuclear shell structure is based on the assumption of the existence of such a spherically symmetric central field of force governing the motion of the individual nucleons in the nuclei. The central field of force is believed to be an average field due to all the nucleons in the nucleus and it is assumed that no residual interaction exists between the nucleon. One factor influencing the observed characteristics of nuclear ground states is nuclear- spin – orbit effect for nucleons in nucleus and it is due to the nuclear force. When these effects are taken into account, the IPM [independent particle model] is able to account for the observed magic No.

If we assume that an average central potential V(r) gives rise to such a force, then it is possible to obtain a solution of the schrodinger wave Eq. governing the motion of an individual nucleon in this field. We shall assume an infinite three dimensional harmonic oscillator potential of the form

$V(r) = -V_0 + 1/2\ M\text{Ш}^2r^2$ --- (5.9.1)

Here M is the nucleon mass, V_0 is the well depth and Ш is the circular frequency of simple harmonic oscillations of the nucleon. The r-dependence of the potential. See Fig. (5.7)



Fg.(5-7) Harmonic oscillator potential

The three dimensional harmonic oscillator can have a discrete set of energy values E given by $E=(\Lambda+3/2)\,\hbar\,\omega$ ----(5.9.2)

where Λ is an oscillator No. $\Lambda=2n+l-2$ --- (5.9.3)

n and l are two integers n=1, 2, 3, --- and l = 0, 1, 2, --- Λ can assume the value 0, 1, 2, 3, --- from Eq. (5.9.3) an oscillator energy level with even value of Λ contains only states of even parity (even l). similarly all levels with odd Λ contain only states of odd parity (odd l) since (2n-2) is always even l is even or odd according as Λ is even or odd. The level of different azimuthal quantum No. l are designated by the usual symbols used in atomic spectroscopy, as given below.

Table (5-1)

L / n	0	1	2	3	4	5	6
1	1S						
2		1P					
3	2S		1d				
4		2P		1f			
5	3S		2d		1g		
6		3P		2f		1h	
7	4S		3d		2g		1i

APPROXIMATE SEQUENCE OF SINGLE-PARTICLE STATES

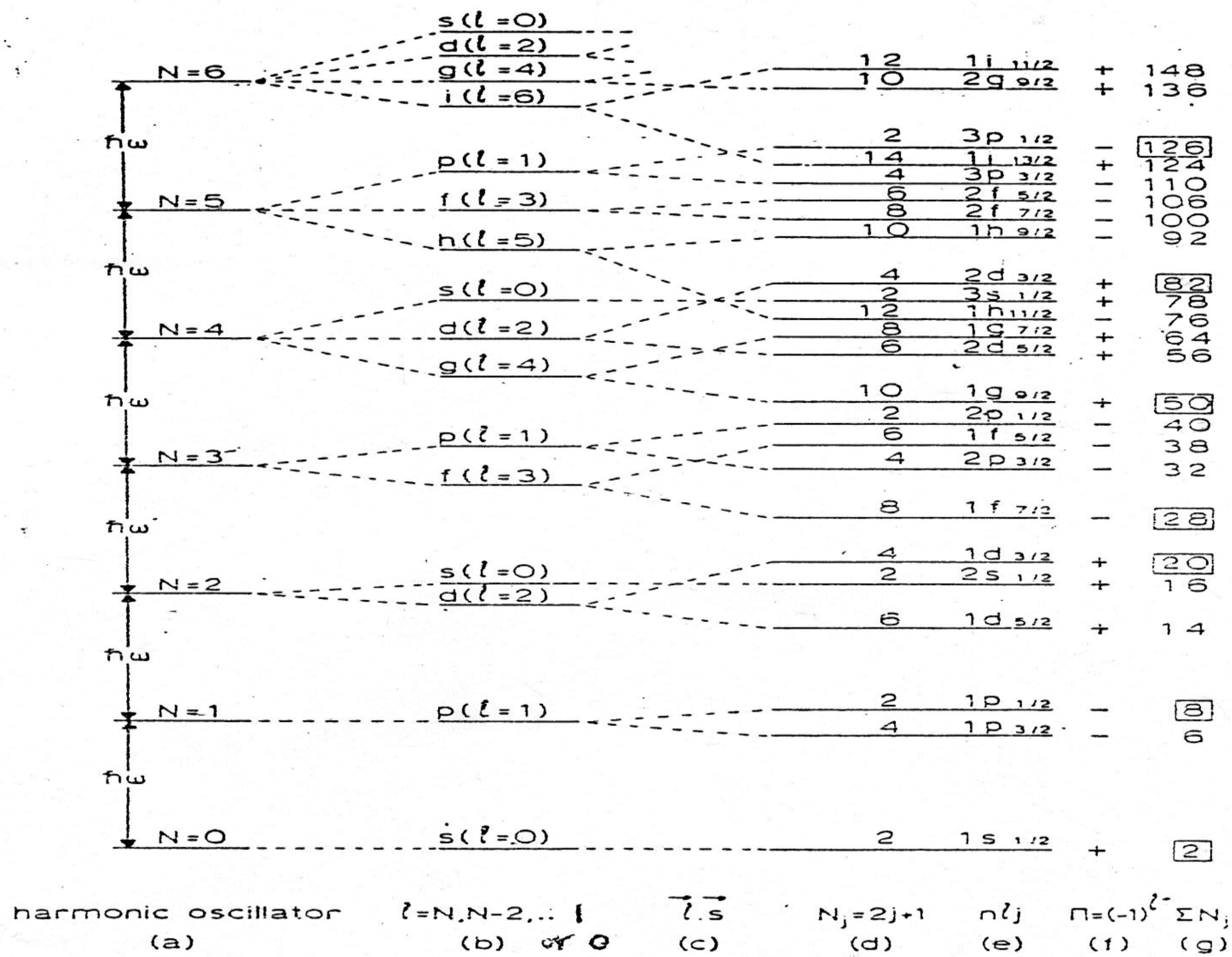


Fig. 2.2. (a) The single-particle energies of a harmonic-oscillator potential as a function of the oscillator quantum number N. (b) A schematic representation of the single-particle energies of a Saxon-Woods potential. (c) A schematic illustration of the level splitting due to the spin-orbit coupling term. (d) The number $N_j = 2j + 1$ of identical particles that can occupy each state. (e) The spectroscopic notation of the single-particle quantum numbers n, l and j. (f) The parity of each state, (g) The magic numbers are seen to appear at the energy gaps as the subtotals of the number of particles.

Table (5.2)					
Energy	ʎ	n	l	$\|nl\rangle$	Panty
3/2 ħɯ	0	1	0	1S	+
5/2 ħɯ	1	1	1	1P	-
7/2 ħɯ	2	1	2	1d	+
		2	0	2S	+
9/2 ħɯ	3	1	3	1f	-
		2	1	2P	-
11/2 ħɯ	4	1	4	1g	+
		2	2	2d	+
		3	0	3S	+

Table (5.2) quantum No. for the lower single- particle states of nucleon in the harmonic – oscillator potential.

The levels with the spin and orbital angular momentum parallel have a lower energy than those with the spin and orbital angular momentum anti parallel (i.e. higher j values have lower energies). The fact that the force of attraction between a nucleon and a nucleus is greater when spin and orbital angular momentum of the nucleon are parallel can also be shown from scattering experiments. The splitting is large for large l values. The splitting of the 1g level is so great that the large energy difference that occurs between the $1g_{9/2}$ (n=1, l=4, j=l + s = 9/2) and the $1g_{7/2}$ (n=1, l=4, j= l – s= 7/2) levels is enough to place those two sublevels in different major shells. Since there are (2j+1) values of N_j there can be 10 neutrons or protons in the $1g_{9/2}$ level. Making a total of 50 up through this level. This model also helps us to understand why nuclei tend to have more neutrons than protons. As in Fig (5.9a) the proton energy levels are higher than those for neutrons, due to the extra energy associated with coulomb repulsion.