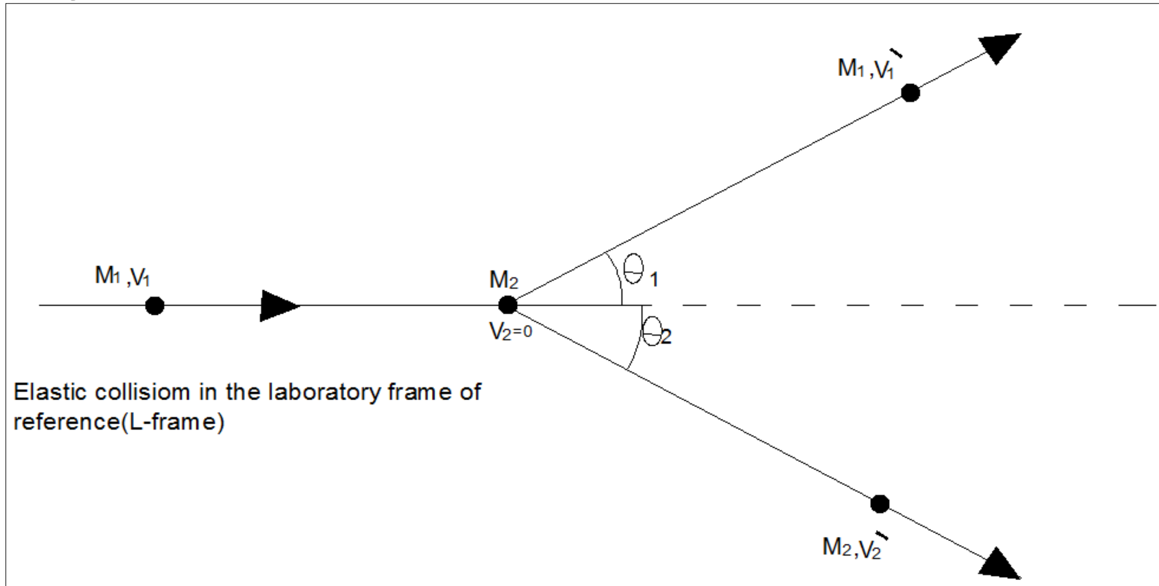
المحاضرةالسادسة –الفصل السادس
Nuclear Reactions

أ.د ألطاف عبد المجيد سلمان

Elastic Collision in L- System (NON-RELATIVISTIC)



Elastic collisiom in the laboratory frame of reference(L-frame)

Consider the elastic collision between a particle of mass M_1 and velocity V_1 (in the L-system) and a stationary target of mass M_2 (V_2= 0). After the collision, the two particles fly a part from the point of collision with the velocities V_1' and V_2' at the angles Θ_1 and Θ_2 respectively with respect to the incident direction. Referring to Fig. (6.1), we get from the laws of conservation of momentum along and perpendicular to the incident direction.

$P_1 = P_1'\cos\Theta_1 + P_2'\cos\Theta_2$ --- (6.4.1)

$0 = P_1'\sin\Theta_1 + P_2'\sin\Theta_2$ --- (6.4.2)

Squaring and adding the above two Eqs. We get $P_2'^2 = P_1^2 + P_1'^2 - 2P_1P_1'\cos\Theta_1$ --- (6.4.3)

We next apply the law of conservation of energy $E_1 = E_1' + E_2'$ --- (6.4.4)

In terms of the momenta, we get $\frac{P_1^2}{2M_1} = \frac{P_1'^2}{2M_1} + \frac{P_2'^2}{2M_2}$ --- (6.4.5)

Substituting for $P_2'^2$ from Eq. (6.4.3), we get $2M_2\left[\frac{P_1^2}{2M_1} = \frac{P_1'^2}{2M_1} + \frac{P_1^2}{2M_2} + \frac{P_1^2}{2M_2} - \frac{2P_1 P_1'}{2M_2}\cos\Theta_1\right]$

Or $P'^2_1\left(1 + \frac{M_2}{M_1}\right) - 2P_1 P_1' \cos\Theta_1 + P_1^2\left(1 - \frac{M_2}{M_1}\right) = 0$ --- (6.4.6)

If we put $r = M_2/M_1$, the above Eq. becomes $P'^2_1 (1 + r) - 2 P_1 P_1' \cos\Theta_1 + P_1^2 (1 - r) = 0$ --- (6.4.7)

In terms of energies, we get, $\frac{1}{E_1}[E_1' (1 + r) - 2\sqrt{E_1 E_1'} \cos\Theta_1 + E_1 (1 - r)$

Or $\frac{E_1'}{E_1}(1 + r) - 2\sqrt{E_1' / E_1}\cos\Theta_1 + (1 - r) = 0$ --- (6.4.8)

Eq. (6.4.8) is quadratic in $\sqrt{E_1}$ to solve this Eq. let $\frac{E_1'}{E_1} = X^2$ and $\sqrt{E_1' / E_1} = X$

Eq. (6.4.8) become, $X^2 (1 + r) - 2X\cos\Theta_1 + (1 + r) = 0$

$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = (1 + r)$, $b = -2\cos\Theta_1$ and $c = (1 - r)$

$$X = \frac{2\cos\Theta_1 \pm \sqrt{4\cos^2\Theta_1 - 4(1+r)(1-r)}}{2(1+r)} = \frac{2\cos\Theta_1 \pm \sqrt{4\cos^2\Theta_1 - 4(1-r^2)}}{2(1+r)}$$

$$X = \frac{2\cos\Theta_1 \pm 2\sqrt{\cos^2\Theta_1 - (1-r^2)}}{2(1+r)} = \frac{\cos\Theta_1 \pm \sqrt{\cos^2\Theta_1 - (1-r^2)}}{(1+r)}$$

$$\sqrt{E_1' / E_1} = \frac{\cos\Theta_1 \pm \sqrt{\cos^2\Theta_1 + (r^2 - 1)}}{(r+1)} \quad \text{--- (6.4.9)}$$

If the target particle is heavier, there $r > 1$. Since $\sqrt{E_1' / E_1} = \frac{P_1'}{P_1}$, we have to choose the sign

in Eq. (6.4.9) so that $\frac{P_1'}{P_1} > 0$. we then have to take the (+) sign before the square root symbol

Squaring we get, $\frac{E_1'}{E_1} = \frac{2\ cos^2\Theta_1 + (r^2-1) + 2\cos\Theta_1\sqrt{cos^2\Theta_1 + r^2 - 1)}}{(r+1)^2}$

So that ratio of the energy received by the struck particle (M_2) to the incident particle energy becomes

$$1 - \frac{E_1'}{E_1} = 1 - \frac{2\ cos^2\Theta_1 + (r^2-1) + 2\cos\Theta_1\sqrt{cos^2\Theta_1 + r^2 - 1)}}{(r+1)^2}$$

$$\frac{E_1 - E_1'}{E_1} = \frac{(r+1)^2 - 2\ cos^2\Theta_1 - (r^2-1) - 2\cos\Theta_1\sqrt{cos^2\Theta_1 + r^2 - 1)}}{(r+1)^2} =$$

$$\frac{r^2 + 2r + 1 - 2\ cos^2\Theta_1 - r^2 + 1 - 2\cos\Theta_1\sqrt{cos^2\Theta_1 + r^2 - 1)}}{(r+1)^2}$$

$$\frac{E_1 - E_1'}{E_1} = \frac{2(r+1 - cos^2\Theta_1) - 2\cos\Theta_1\sqrt{cos^2\Theta_1 + r^2 - 1)}}{(r+1)^2} \text{ --- (6.4.10)}$$

The maximum energy is received for $\Theta_1 = \pi$. We get in this case for $r >> 1$ (i.e. $M_2 >> M_1$)

$$\frac{E_1 - E_1'}{E_1} = \frac{2(r+1 - cos^2\pi) - 2\cos\pi\sqrt{cos^2\pi + r^2 - 1)}}{(r+1)^2} = \frac{2(r+1 - (-1)^2) - 2(-1)\sqrt{(-1)^2 + r^2 - 1)}}{(r+1)^2}$$

$$\frac{E_1 - E_1'}{E_1} = \frac{2(r+1-1) + 2(-1)\sqrt{1 + r^2 - 1)}}{(r+1)^2} = \frac{2_r + 2r}{(r+1)^2}$$

$$\frac{E_1 - E_1'}{E_1} = \frac{4_r}{(r+1)^2} \approx \frac{4}{r} = \frac{4M_1}{M_2} \text{ --- (6.4.11)}$$

Actually all values Θ_1 are possible in this case. This type of scattering is observed when Beta particles are scattered by nuclei as they pass through matter. The energy received by the nucleus in such collision is negligibly small ($r = M_{nuc} / m_e >> 1$). When ($r < 1$) as in the collision of a nuclear particle with an electron, the solution given by Eq. (6.4.9) can be real only if $\cos^2\Theta_1 \geq 1 - r^2$ --- (6.4.12)

Both (+) and (-) signs are possible in Eq. (6.4.9). evidently $1 - \sin^2\Theta_1 \geq 1 - r^2$

$[- \sin^2\Theta_1 \geq - r^2] * (-1)$

$\sin^2\Theta_1 \leq r^2$ take the square root $\sin\Theta_1 \leq r$, or $\Theta_1 < \pi/2$ since $r < 1$.

For $r << 1$, $\Theta_1 \longrightarrow 0$. Thus a heavy projectile scattered by a very light target, such as an electron, goes on almost undeviated. The energy given to the target nucleus for $\Theta_1 = 0$ is (see Eq. (6.4.9))

$\frac{E_1 - E_1'}{E_1} = \frac{2_r \pm 2\sqrt{r^2}}{(r+1)^2} = \frac{4_r}{(r+1)^2}$ or 0 --- (6.4.13)

In this case the energy given to the target becomes $E_1 - E'_1 = E_1 \frac{4_r}{(r+1)^2}$ for $r << 1$

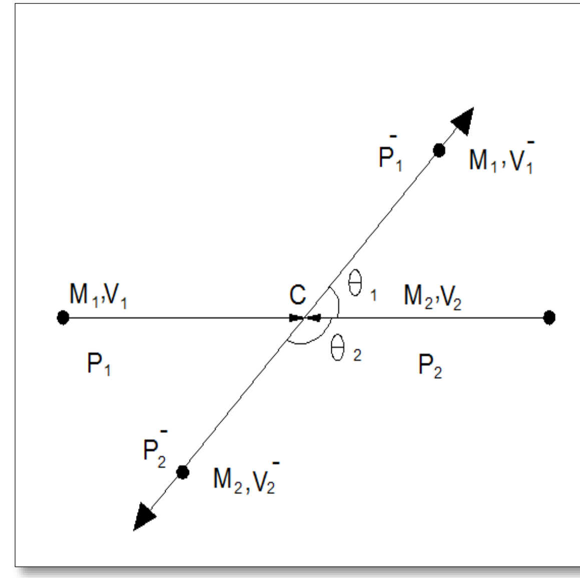
$(r + 1)^2 = (0 + 1)^2 = 1^2 = 1$

$E_1 - E'_1 = 4\, r\, E_1 = \frac{4M_2}{M_1} E_1$ --- (6.4.14)

The energy loss by the incident particles in a collision is small compared to the incident energy.

Elastic Collision in C-System (NON-RELATIVISTIC)

(Fig. 6.3) Momentum diagram for collision between two particles in the center of mass system



we now consider the collision between two particles from the point of view of an observer at rest relative to the center of mass C of the particles(Fig. 6.3). we shall denote the velocities and momenta in the C-System by the capital letters (V and P), while those in the L-System by the small letters (v and p) the energies and the angles of scattering will be denoted by E and ϴ in the C-System and by Ɛ and ϕ in the L-System. The particle M_2 is at rest in the L-System before collision ($v_2 = 0$). The velocity in the L-System of the center of mass is

$$v_c = \frac{M_1\, v_1 + M_2\, v_2}{M_1 + M_2} = \frac{M_1\, v_1}{M_1 + M_2} \quad \text{--- (6.4.15)}$$

the velocities of M_1 and M_2 in the C-System before collision are respectively

$$V_1 = v_1 - v_c = \frac{M_2\, v_1}{M_1 + M_2} \quad \text{--- (6.4.16)}$$

$$V_2 = v_2 - v_c = -\frac{M_1\, v_1}{M_1 + M_2} \quad \text{--- (6.4.17)}$$

The corresponding momenta are

$$P_1 = M_1 V_1 = \frac{M_1\, M_2\, v_1}{M_1 + M_2} = \mu\, v_1 \quad \text{---- (6.4.18)}$$

$$P_2 = M_2 V_2 = \frac{M_1\, M_2\, v_1}{M_1 + M_2} = -\mu\, v_1 \quad \text{---- (6.4.19)}$$

Where μ = $M_1\, M_2/(M_1 + M_2)$ is the reduced mass. The two particles have equal and opposite momenta before collision. So that their total momentum $P_1 + P_2 = 0$. Conservation of momemtum then requires that the total momentum of the two particles after collision is also zero $P_1' + P_2' = P_1 + P_2 = 0$

We have denoted the momentum after collision by putting primes (′) above the corresponding quantities before collision. Sums of the K. E. before and after collision are

$$E_1 + E_2 = \frac{P_1^2}{2\,M_1} + \frac{P_2^2}{2M_2} = \frac{P^2}{2\mu} \quad \text{---- (6.4.20)}$$

$$E_1' + E_2' = \frac{P_1'^2}{2M_1} + \frac{P_2'^2}{2M_2} = \frac{P'^2}{2\mu} \quad \text{---- (6.4.21)}$$

Where $|P_1| = |P_2| = |P|$ and $|P'_1| = |P'_2| = |P'|$

since energy conservation requires that $E_1' + E_2' = E_1 + E_2$

we get $P' = P$ so that the magnitudes of momenta of the particles before and after collision are all equal

$$P_1 = P_2 = P_1' = P_2' \quad \text{---- (6.4.22)}$$

The momentum diagram of the particles is shown in Fig. (6.3). the two particles fly apart from the point of collosion with equal and opposite momenta as shown so that $\Theta_1 + \Theta_2 = \pi$.

The K. E. of the centre of mass is $E_c = \frac{1}{2}(M_1 + M_2)\, v_c^2 = \frac{M_1}{M_1 + M_2}\,\varepsilon_1$ ---- (6.4.23)

Where $\varepsilon_1 = \frac{1}{2} M_1\, v_1^2$ is the K. E. of the L-System. The energy E_c given by Eq. (6.4.23) is not available for the production of any inelastic effect (e.g. reaction). The total amount of energy available for this purpose is $\varepsilon_1 - E_c = \varepsilon_1 - \frac{M_1}{M_1 + M_2}\,\varepsilon_1 = \frac{M_2}{M_1 + M_2}\,\varepsilon_1 = \frac{1}{2}\,\mu\, v_1^2$ ---- (6.4.24)

From the above discussions, it is clear that there is no change in the K. E. and momenta of the particles after collision in the C-System.

Nonelastic Collision: Nuclear reaction including inelastic scattering (e.g. PP$'$, nn$'$) bbelong to the this type of collision. The particles produced after collision are usually different from those before collision. If M_3 and M_4 are the masses of the two particles produced by thr reaction and their K. E. are E_3 and E_4. We can write the energy conservation equation as $M_1 + M_2 + E_1 + E_2 = M_3 + M_4 + E_3 + E_4$ --- (6.4.25)

Here the mass expressed in energy unite (i.e. Mc^2), $Q = M_1 + M_2 - M_3 - M_4$

Where $E_2 = 0$, then we get, $Q + E_1 = E_3 + E_4$ --- (6.4.26)

Eq. (6.4.26) along with the momentun conservation eq. have to be used to find the energy of the reaction products.

6.5 Energtics of Nuclear Reactions

During a nuclear reaction, energy is either evolved or absorbed. Reactions in which energy is evolved are known as exoergic reactions while those requiring absorption of energy are called endoergic. The total amount of energy evolved or absorbed during a nuclear reaction is called the Q-value or simply the Q of the reaction. So by definition $Q = E_Y + E_y - E_X - E_x = E_Y + E_y - E_x$ --- (6.5.1)

If the target nucleus X is at rest. If the atomic masses are expressed in energy units Eq. (6.3.3) can be rewritten as $M_X + M_x + E_x = M_Y + M_y + E_Y + E_y$

Then we get from Eq. (6.5.1) $Q = M_X + M_x - M_Y - M_y$ --- (6.5.2)

Written in terms of the binding energies of different nuclei, we can also write Q = B_Y + B_y – B_X – B_x --- (6.5.3) by definition (Q > 0) for an exorgic reaction, while (Q < 0) for an endorgic reaction. There is a net deficit of energy in the latter case. Some energy must be supplied for the reaction to occur. This usually comes from the K. E. E_x of the projectile. Eq. (6.5.2) shows that for an exorgic reaction M_X + M_x is greater than M_Y + M_y while for an endorgic reaction M_X + M_x is less than M_Y + M_y.

Thershold Energy of an Endoergic Reaction

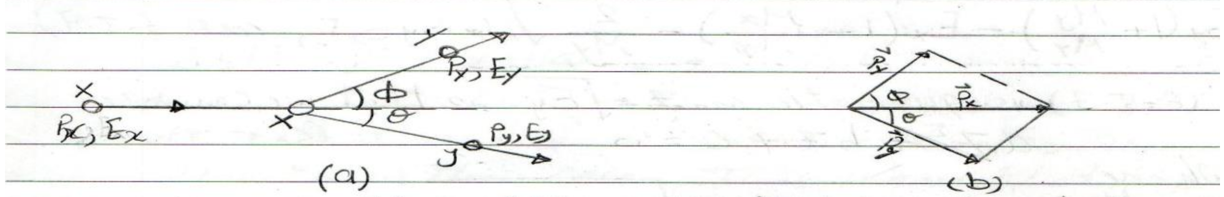


Fig 6-5 (a) Motion of the projectile (x) and product particles (y and Y) taking part in a nuclear reaction (b) Momentum diagram -

In view of the energy and momentum consevation laws, E_Y can be expressed in terms of E_x and E_y. in Fig.(6.5) we get from the law of consveration of momentum a long and perpendicular to the direction of motion of the projectile ($\mathrm{P} = \sqrt{2\mathrm{ME}}$)

$\sqrt{2\mathrm{M_x\,E_x}}$ = $\sqrt{2M_yE_y}$ cos Θ + $\sqrt{2M_YE_Y}$ cos ϕ --- (6.5.4)

0 = $\sqrt{2M_yE_y}$ sin Θ - $\sqrt{2M_YE_Y}$ sin ϕ --- (6.5.5)

Eq. (6.5.1) gives the law of conservation of energy Q = E_Y +E_y - E_x, squaring and adding Eq. (6.5.4) and (6.5.5) we get $\frac{1}{2M_Y}$ ($2\,M_Y\,E_Y = 2\,M_x\,E_x + 2\,M_y\,E_y - 4\sqrt{M_xM_yE_xE_y}$ cos Θ)

Or $E_Y = \frac{M_x}{M_Y}E_x + \frac{M_y}{M_Y}\,E_y$ - $\frac{2}{M_Y}\,\sqrt{M_xM_yE_xE_y}$ cos Θ ---(6.5.6)

Then from Eq. (6.5.1) and (6.5.6), we get Q= $E_y\left(1+\frac{M_y}{M_Y}\right)-E_x\left(1-\frac{M_x}{M_Y}\right)-$
$\frac{2}{M_Y}\sqrt{M_x M_y E_x E_y}$ cos Θ --- (6.5.7)

Eq. (6.5.7) is quadratic in $Z=\sqrt{E_y}$ so that we can write $aZ^2 + bZ + c = 0$ --- (6.5.8)

Where a = 1 + $\frac{M_y}{M_Y}$, b = - $\frac{2}{M_Y}\sqrt{M_x M_y E_x}$ cosΘ and c = $-E_x\left(1-\frac{M_x}{M_Y}\right)-\ Q$

Eq. (6.5.8) has the solution Z = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ --- (6.5.9)

$$\sqrt{E_y} = \frac{M_Y}{2(M_Y+M_y)}\,[\frac{2}{M_Y}\sqrt{M_x M_y E_x}\,\cos\Theta \pm (\sqrt{\frac{4}{M_Y^2}M_x M_y E_x \cos\Theta^2}$$

$$+\sqrt{4\left\{\frac{M_Y+M_y}{M_Y}\left(E_x\,\frac{M_{Y-}M_x}{M_y}+Q\right)\right\}}\,)]$$

$$\sqrt{E_y} = \frac{1}{(M_Y+M_y)}\,[(M_x M_y E_x)^{1/2} cos\theta$$

$$\pm\sqrt{M_x M_y\ E_x cos\theta^2 + M_Y(M_Y+M_y).\,(E_x\,\frac{M_Y-M_x+QM_Y}{M_Y})}\,]$$

$$\sqrt{E_y} = \frac{1}{(M_Y+M_y)}\,[(M_x M_y E_x)^{1/2} cos\theta \pm \{M_x M_y E_x cos^2\theta + \left(M_Y+M_y\right).\,(E_x(M_Y-M_x)$$

Since both a and Q′ are positive $Z = \sqrt{E_y}$ is imaginary in this case. This means that the reaction is not possible with E_x = 0. A minimum energy $E_x = E_{min}$ is needed to initiate endorgic reaction. In this case the term under the square root sign in Eq. (6.5.9) must be zero so that we get $b^2 - 4ac = 0$

Substitititing for a, b and c, we get $\frac{4}{M_Y^2}\left(M_x M_y E_{min}\right)cos^2\Theta$ = 4 (1 + $\frac{M_y}{M_Y}$)$\left\{-Q - E_{min}(1 - \frac{M_x}{M_Y}\right\}$

$$\frac{4}{M_Y^2}\left(M_x M_y E_{min}\right)cos^2\Theta + 4\left(\frac{M_Y+M_y}{M_Y}\right) E_{min}\left(\frac{M_Y-M_x}{M_Y}\right) = -4\,Q\left(\frac{M_Y+M_y}{M_Y}\right)$$

$$E_{min}\left[\frac{4}{M_Y^2}\left(M_x M_y\right)cos^2\Theta + \frac{4}{M_Y^2}(M_Y + M_y)(M_Y - M_x)\right] = -\frac{4}{M_Y}Q(M_Y + M_y)$$

$$E_{min}\left[\frac{M_x M_y cos^2\theta}{M_Y} + \frac{M_Y^2 - M_Y M_x + M_y M_Y - M_y M_x}{M_Y}\right] = -Q(M_Y + M_y)$$

$$E_{min}\left[\frac{M_x M_y (1-sin^2\theta)}{M_Y} + M_Y - M_x + M_y - \frac{M_y M_x}{M_Y}\right] = -Q(M_Y + M_y)$$

$$E_{min}\left[\frac{M_x M_y}{M_Y} - \frac{M_x M_y}{M_Y} sin^2\theta + M_y + M_Y - M_x - \frac{M_y M_x}{M_Y}\right] = -Q(M_Y + M_y)$$

$$E_{min}\left[M_y + M_Y - M_x - \frac{M_x M_y}{M_Y} sin^2\theta\right] = -Q(M_Y + M_y)$$

$$E_{min} = \frac{-Q(M_Y + M_y)}{M_y + M_Y - M_x - \frac{M_x M_y}{M_Y} sin^2\theta} \quad \text{--- (6.5.11)}$$

Since Q < 0, E_{min} > 0, using $Q = M_X + M_x - M_Y - M_y$

$$E_{min} = \frac{-Q(M_Y + M_y)}{M_x - Q - \frac{M_x M_y}{M_Y} sin^2\theta} \quad \text{--- (6.5.12)}$$

E_{min} depends on the angle at which the particle y is emitted. When Θ = 0 i.e. y is emitted in the forward direction, E_{min} has the lowest value and is known as the thershold energy for the endoergic reaction and is usually written as E_{th} from Eq. (6.5.12) we get E_{th} = - $\frac{(M_y + M_Y)Q}{M_x - Q}$ --- (6.5.13)

Since Q << M_X, we can neglect it in the denominator of Eq. (6.5.13), also we can replace M_y+M_Y in the numerator by M_x+M_X. so we get finally E_{th} = - Q $\frac{M_x + M_X}{M_X}$ = - Q (1 + $\frac{M_x}{M_X}$)--- (6.5.13)

So by measuring the minimum energy E_{th} at which an exoergic reaction is initiated it is possible to determine the Q value of the reaction.