المحاظرة الرابعة الفصل الخامس
النماذج النووية

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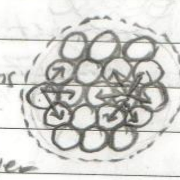
The total surface energy as $E_s = -a_2 A^{2/3}$, where a_2 is a constant = 13 MeV, $E_s = -13A^{2/3}$ ---- (5.6.2)



Fig(5-2) A nucleon at the surface of a nucleus interacts with fewer other nucleons than one in the interior of the nucleus and hence its binding energy is less. The larger the nucleus, the smaller the proportion of nucleon at the surface.

(iii) Coulomb Energy: The coulomb repulsion between the proton in the nucleus also tends to weaken the nuclear binding. This repulsive force being of the long range type, can act between all the proton pairs in the nucleus. The coulomb energy E_c of a nucleus is the work that must be done to bring together Z protons from in infinity into a spherical aggregate the size of the nucleus. The potential energy of a pair of protons r apart is equal to $V = e^2/(4\pi\varepsilon_0 r)$. Since there are Z(Z-1)/2 pairs of protons $E_c = Z(Z-1)V/2 = -[Z(Z-1)e^2/8\pi\varepsilon_0][1/r]_{av}$, where $(1/r)_{av}$ is the value of 1/r averaged over all proton pairs. If the protons are uniformly distributed throughout a nucleus of radius R, $(1/r)_{av}$ is proportional to 1/R and hence to $1/A^{1/3}$. So that coulomb energy $E_c = -a_3[Z(Z-1)/A^{1/3}]$, where a_3 is a constant = 0.72 MeV $E_c = -0.72[Z(Z-1)/A^{1/3}]$.

(iv) Asymmetry energy: In the light nuclei, there is a tendency for the neutron and proton No. to be equal (N=Z) to form the most stable configurations. As we go to the heavier nuclei, the increase in the number of protons tend to weaken the binding due to coulomb repulsive force between them. So some extra neutrons must be present to provide additional (n-n) bonds to compensate for this. However, this disturbs the condition of equality of Z and N to form the most stable configuration when coulomb effect is ignored.

Thus due to this asymmetry in the neutron-proton numbers, there will be a reduction in the binding energy by an amount E_a proportional to $(N-Z)^2$ which shows that the effect is the same, irrespective of whether the neutron No. is greater or less than the proton No. further the effect should decrease for the heavier nuclei. Since N-Z =A- 2Z, we can write the asymmetry energy as $E_a = - a_4 [(A - 2Z)^2/A]$.

(v) Pairing Energy: For a given A (even), the even Z even N (e-e) nuclei are more strongly bound than the odd Z odd N (o-o) nuclei. If the binding energies of these nuclei are compared with those of the neighboring odd A nuclei, both even Z odd N (e-o) and odd Z even N (o-e), it is found that the bindings of the odd A nuclei are intermediate between the (e-e) and (o-o) nuclei. The odd A nuclei are more strongly bound than the (o-o) nuclei while they are less strongly bound than (e-e) nuclei. These observations show that we must add a pairing energy term to the expression for E_B which arises due to the pairing of the nucleons of the same type with opposite spins. Such paring tends to increase the strength of binding which thus becomes maximum for (e-e) nuclei in which all nucleus of both types (P and n) are paired with oppositely aligned spin. In odd A nuclei, there is one unpaired nucleon which is a neutron for (e-o) nuclei and a proton for (o-e) nuclei. This weakens the binding in such nuclei by about 2 to 3 MeV. Finally in (o-o) nuclei, there are two unpaired nucleons, one proton and one neutron. As a result, the bindings of such nuclei are further weakened by about 2 to 3MeV. The pairing energy term (A,Z) depends on A only and is taken to be zero for odd A nuclei, positive for (e-e) nuclei and negative for (o-o) nuclei, its given by ş = $a_5 A^{-3/4}$ ---- (5.6.5), where a_5 is constant, $a_5=+33$ for (e-e) nuclei, $a_5=-33$ for (o-o) nuclei, $a_5=0$ for (o-e) or (e-o) nuclei, We can write the binding energy as $E_B(A,Z)= E_v - E_s - E_c - E_a +$ ş $= a_1A - a_2A^{2/3} - a_3Z^2/A^{1/3} - a_4(A-2Z)^2/A +$ ş --- (5.6.6)

This is the semi empirical formula for the nuclear binding energy and is known as the Bethe-Weizsacker formula. The atomic mass of an isotope can be written as $M(A,Z) = Z M_H + NM_n - E_B(A,Z)$ $M(A,Z) = ZM_H + (A-Z)M_n - a_1A + a_2A^{2/3} + a_3Z^2/A^{1/3} + a_4(A-2Z)^2/A$ - ş ---- (5.6.7).

5.7 Applications of The Semi-Empirical Binding Energy Formula

Mass Parabola: Stability of nuclei against $_\beta$- decay equation (5.6.7)

without the ş term can be written as follows M(A,Z) = fA + pZ + qZ2 --- (5.7.1)

Where $f_A = A (M_n - a_1 + a_4) + a_2A^{2/3}$

$P = -4a_4 - (M_n - M_H)$

$q = \frac{1}{A} (a_3A^{2/3} + 4a_4)$

Eq. (5.7.1) is the Eq. is the Eq. to a parabola for a given A $(\frac{dM}{dZ})_A = p + 2qZ$ --- (5.7.2)

The lowest point of the parabola $Z = Z_A$ can be obtained by putting $(\frac{dM}{dZ})_A = 0$

This gives $p + 2qZ_A = 0$

or $Z_A = = -\frac{p}{2q} = (M_n - M_H + 4a_4) A / 2(a_3A^{2/3} + 4a_4)$ --- (5.7.3)

putting $Z = Z_A$ in Eq. (5.7.1) we get

$M(A,Z_A) = f_A + pZ_A + qZ_A^2$

$M(A,Z_A) = f_A + p (-\frac{p}{2q}) + q (p^2/4q^2)$

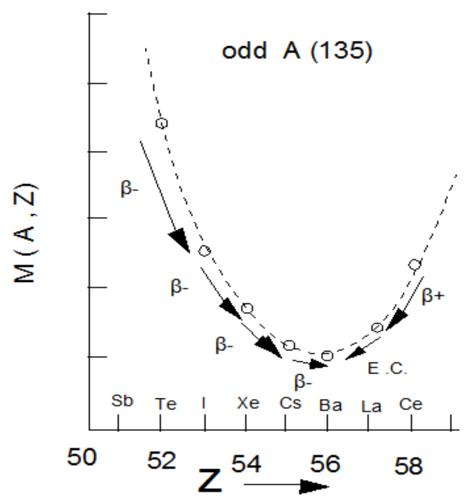
$M(A,Z_A) = f_A - (p^2/4q)$ --- (5.7.4)

Hence $M(A,Z_A) - M(A,Z_A) = p^2/4q + pZ + qZ^2$

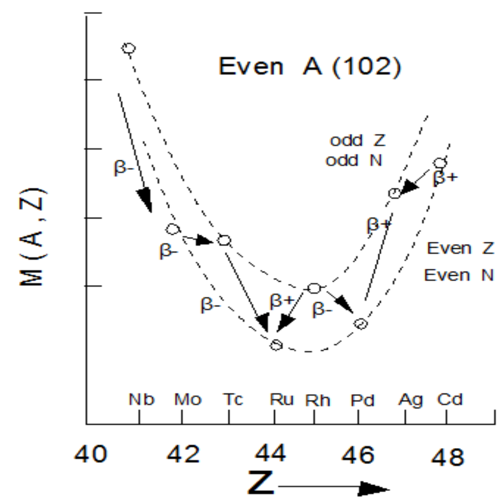
$M(A,Z_A) - M(A,Z_A) = q\,(Z - Z_A)^2$

This proves that the mass parabola for a given isobar has the lowest point at $Z = Z_A$ since the r.h.s. Eq. (5.7.5) is positive. Since M(A,Z) has the smallest value for a given A when $Z = Z_A$, this nucleus would have the largest binding energy amongst the isobars for the given A. (i.e.) Z_A would give the value of Z for the most stable isobar. Putting the numerical values of M_n, M_H , a_3 and $a_4$in Eq. (5.7.3), we get $Z_A = A/\,(1.98 + 0.015\,A^{2/3})$ --- (5.7.6)

In Fig.(a) is shown the plot of M(A,Z) against Z for odd A isobars with (A=135). This is a parabola for which the lowest point is at Z_A = 56.85. The stable isobar that is observed at this mass No. is ^{135}Ba for which (Z=56). The nuclides falling on either side of the stable isobar are all unstable. Those on the lower Z side (Z<56) are β^- active while those on the higher Z side (Z>56) are β^+ active or electron capturing. Each of these nuclei undergoes β-transformation into the product nucleus with Z one unit higher or lower respectively, as shown in the Fig.



(a)



(b)

Fig. (b) shows the two mass parabola for the even A isobars with A=102. The upper one is for the odd Z odd N isobars, while the lower one is for the even Z even N isobars. The most stable isobar in this case falls on the lower parabola. Using Eq. (5.7.6), we get (Z_A=44.2) actually a stable nuclide ^{102}Ru at (Z=44) iis observed at this mass No. resides, another stable e-e nuclide ^{102}Pd (Z=46) also exists at this A. the two stable isobars differ in Z by two unit. The o-o isobar ^{102}Rh with (Z=45) between these two falls on the upper parabola and has an atomic mass greater than those of either of the above two. Hence ^{102}Rh is not stable. It shows both β^+ and β^- activities. β^+ emission transforms it to ^{102}Ru while by β^- emission it transforms to ^{102}Pd.

5.8 Nuclear shell structure

The liquid drop model can explain the observed variation of the nuclear binding energy with the mass No. and the fission of the heavy nuclei. However, this model predicts very closely spaced energy level in nuclei which is contrary to observation at law energies**.** The low lying excited states in nuclei are actually quite widely spaced, which cannot be explained by the liquid drop model. This and certain other properties of the nucleus would require us to consider the motion of the individual nucleons in a potential well which would give rise to the existence of a nuclear shell structure, similar to the electronic shells in the atoms. There are strong reasons to believe that the nucleons in the nuclei are arranged in certain discrete shells. The nuclei containing the following No. of protons and neutrons exhibited very high stability .