Some examples:

Ex1/ The infinite Taylor Series of

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots + \frac{x^{2n}}{n!} \longrightarrow (1)$$

when $\int_0^{1/2} e^{x^2}\,dx = 0.544987104184$

Determine the accuracy of the approximation obtained by replacing the integr and $f(x)=e^{x^2}$ with 1st, 2nd, ---, fifth terms from equation (1).

Sol:- By using one-term

$$\int_0^{1/2} 1\,dx = x\Big]_0^{1/2} = 0.5$$

By using two-terms

$$\int_0^{1/2} (1+x^2)\,dx = \left[x + \frac{x^3}{3}\right]_0^{1/2} = 1/2 + 1/24 = 0.541666666667$$

By using Three-terms

$$\int_0^{1/2} \left(1+x^2+\frac{x^4}{2!}\right)dx = \left[x + \frac{x^3}{3} + \frac{x^5}{10}\right]_0^{1/2} = \frac{1}{2} + \frac{1}{24} + \frac{1}{320} =$$

$$= 0.544791666667$$

By using four-terms

$$\int_0^{1/2} \left(1+x^2+\frac{x^4}{2!}+\frac{x^6}{3!}\right)dx = \left[x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{7\times 3!}\right]_0^{1/2} =$$

$$= 0.544977678571$$

By using five-terms

$$\int_0^{1/2}\left(1+x^2+\frac{x^4}{2!}+\frac{x^6}{3!}+\frac{x^8}{4!}\right)=\left[x+\frac{x^3}{3}+\frac{x^5}{10}+\frac{x^7}{7.3!}+\frac{x^9}{9\times 4!}\right]_0^{1/2}=$$

$$=0.5449867208\,17$$

نلاحظ من القيم اعلاه بزيادة حدود المتسلسلة فاننا نقترب من الحل المضبوط ويقل خطأ البتر .

(ill- Conditioned and Well- Conditioned Problems)

تسمى الطريقة غير مستقرة (unstable) أو (ill-conditioned) اذا كان التغير القليل في البيانات يسبب تغيراً كبيراً في الحل .

اما اذا كان التغير القليل في البيانات يسبب تغيراً صغيراً في الحل فنسمي الطريقة مستقرة (stable) أو (Well-conditioned)

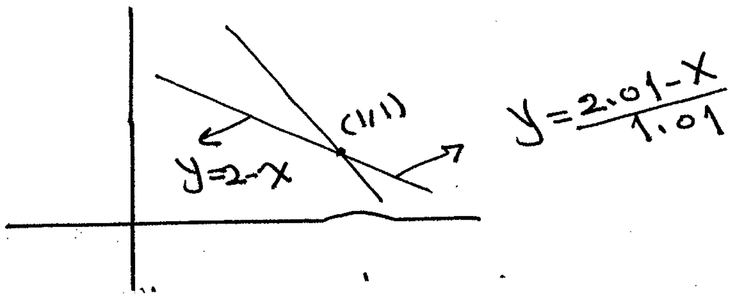
EX2/ suppose that $\boxed{y=\frac{x}{-x+1}}$ with $x=0.93$ and we get $y=13.28$, but when use $x=0.94$ we get $y=15.60$, So we would probably say that this expression is <u>ill- Conditioned</u> when evaluated for x <u>near</u> 0.93.

on the otherhand, if we use $x=-0.93$, we get $y=-0.4818$, and $x=-0.94$, we get $y=-0.4845$ we would say this it is <u>well-Conditioned</u> for x near <u>-0.93</u>.

<u>EX3:-</u> Consider the linear equations $x+y=2$
$x+1.01y=2.01$

which have solution $y=x=1$. if the number $\underline{2.01}$ is changed to $\underline{2.02}$ then the solution $x=0$ and $y=2$.

That means (small) change in the data produces a 100% change in the solution. (this is ill-Conditioned



<u>EX4:-</u> Consider the following system of two linear equations in two unknowns

$$\begin{bmatrix} 400 & -201 \\ -800 & 401 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ -200 \end{bmatrix}$$

نضرب المعادلة الاولى 2X

$800x_1 - 402x_2 = 400$

$-800x_1 + 401x_2 = -200$

$-x_2 = 200 \rightarrow x_2 = -2$

التعويض في المعادلة ①

$400x_1 + (201)(200) = 200$

$\rightarrow x_1 = -100$

The system can be solved and get $x_1 = -100$
$x_2 = -200$

Now, let us make a slight change in $\underline{a_{11}}$ from $\underline{400}$ to $\underline{40}$

$$\begin{bmatrix} 401 & -201 \\ -800 & 401 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ -200 \end{bmatrix}$$

(الحل في آخر الورقة)

In this case the solution is $x_1 = 40000$ and $x_2 = 79800$ $\underline{So}$ it is ill-Conditioned Problem.

اولاً نضرب المعادلة الاولى $2 \times$ تصبح

$$802x_1 - 402x_2 = 400$$

$$-\ 800x_1 + 401x_2 = -200$$

$$2x_1 - x_2 = 200$$

$$\boxed{x_2 = 2x_1 - 200}$$

نعوض في المعادلة الاولى الاصلية

$$401x_1 - 201x_2 = 200$$

$$401x_1 - (201)(2x_1 - 200) = 200$$

$$401x_1 - 402x_1 + 40200 = 200$$

$$-x_1 = -40000$$

$$\boxed{x_1 = 40000}$$

بالتعويض بالمعادلة الاولى

$$401x_1 - 201x_2 = 200$$

$$(401)(40000) - 201x_2 = 200$$

$$\boxed{x_2 = 79800}$$

EX5:- let $Q_n = 10^{-n}$ and let $P_n = 10^{-2^n}$ then $\{Q_n\}$ Converges to 0 linearly, and $\{P_n\}$ Converges to 0 quadratically.

Sol/ $\{Q_n\}$ Converges to 0 linearly, Since

$$|Q_{n+1} - 0| = 10^{-(n+1)} = 10^{-n} \cdot 10^{-1} = \frac{1}{10} \cdot 10^{-n} = \frac{1}{10} |Q_n - 0|$$

Thus, $P = \frac{1}{10}$ and $a = 1$.

Also, $\{P_n\}$ Converge to 0 quadratically, Since

$$|P_{n+1} - 0| = 10^{-2^{n+1}} = 10^{-2^n} \cdot 10^{2^1} = \left(10^{-2^n}\right)^2 = |P_n - 0|^2$$

Thus $P = 1$ and $a = 2$.

$u \longrightarrow P$

$$\lim_{k \to \infty} \frac{|x_{k+1} - r|}{|x_k - r|^a} = P$$