Probability Rules:

There are four basic probability rules. These rules are helpful in solving probability problems, in understanding the nature of probability, and in deciding if your answers to the problems are correct.

Probability rule 1:

The probability of any event E is a number (either a fraction or decimal) between and including 0 and 1. This is denoted by $0 \leq P(E) \leq 1$.

Rule 1 states that probability **cannot be negative or greater than 1**.

Probability rule 2:

If an event E cannot occur (i.e. the event contains no members in the sample space), its probability is 0.

Example 7:

When a single die is rolled, find the probability of getting a 9.

Solution:

Since the sample space is $\Omega = \{1,2,3,4,5,6\}$ it is impossible to get a 9. Hence, the probability is

$$P(9) = \frac{0}{6} = 0.$$

Probability rule 3:

If an event E is **certain**, then the probability of E is 1.

Example 8:

When a single die is rolled, what is the probability of getting a number less than 7?

Solution:

Since all outcomes in $\Omega = \{1, 2, 3, 4, 5, 6\}$ are less than 7, then the probability is

$$p(number\ less\ than\ 7) = \frac{6}{6} = 1.$$

The event of getting a number less than 7 is **certain**.

Probability rule 4:

The sum of the probability of all the outcomes in the sample space is 1.

Example 9:

In the roll of a fair die, each outcome in the sample space has a probability of $\frac{1}{6}$. Hence, the sum of probabilities of the outcomes is as shown:

Outcomes	**1**	**2**	**3**	**4**	**5**	**6**
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Sum of all probabilities $= \frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{6}{6}=1$

Complementary of an Event E:

Is the set of outcomes in the sample space that are not included in the outcomes of event E. The complement of E is denoted by $\boldsymbol{E^C}$.

Example 10:

Find complement (E^C) of each event

a. Rolling a die and getting a 4.
b. Selecting a letter of the alphabet and getting a vowel.
c. Selecting a month and getting a month that begins with a j.
d. Selecting a day of the week and getting a weekday.

Solution:

a. Getting a 1, 2, 3, 5, *or* 6. (all the outcomes except 4)
b. Getting a consonant (assume y is a consonant). (all the letters in the alphabet except the vowels)
c. Getting February, March, April, May, August, September, October, November, or December. (all months except (January, June and July))
d. Getting Saturday or Sunday. (only the weekends or holydays)

Rule for complementary events:

$$p(E)+p(E^C)=1.$$

Example 11:

If the probability that a person lives in an industrialized country of the world is $\frac{1}{5}$, find the probability that a person does not live in an industrialized country.

Solution:

Let E = the event of the person lives in an industrialized country

E^C = the event of the person not lives in an industrialized country

Hence, from the probability rule of complementary event

$$p(E) + p(E^C) = 1 \rightarrow p(E^C) = 1 - p(E) = 1 - \frac{1}{5} = \frac{4}{5}.$$

Mutually exclusive events:

Two events are ***mutually exclusive*** event if they cannot occur at the same time (i.e. they have no outcomes in common).

Example 12:

Determine which events are mutually exclusive and which are not, when a **single die is rolled**.

a. Getting an odd number and getting an even number.
b. Getting a 3 and getting an odd number.
c. Getting an odd number and getting a number less than 4.
d. Getting a number greater than 4 and getting a number less than 4.

Solution:

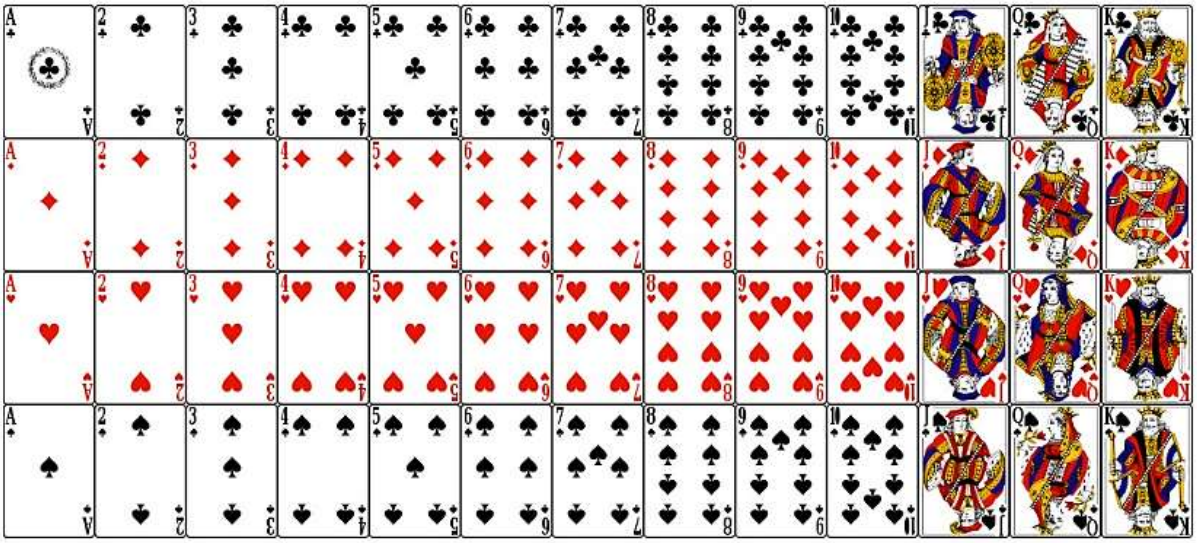
a. The events are **mutually exclusive**, since the first event can be 1, 3, or 5 and the second event can be 2, 4, or 6. (they cannot occur at the same time)
b. The events are **not mutually exclusive**, since the first event is a 3 and the second can be 1, 3, or 5. Hence, 3 is contained in both events. (they can occur at the same time)
c. The events are **not mutually exclusive**, since the first event can be 1, 3, or 5 and the second can be 1, 2, or 3. Hence, 1 and 3 are contained in both events. (they can occur at the same time)
d. The events are **mutually exclusive**, since the first event can be 5 or 6 and the second event can be 1, 2, or 3. (they cannot occur at the same time)

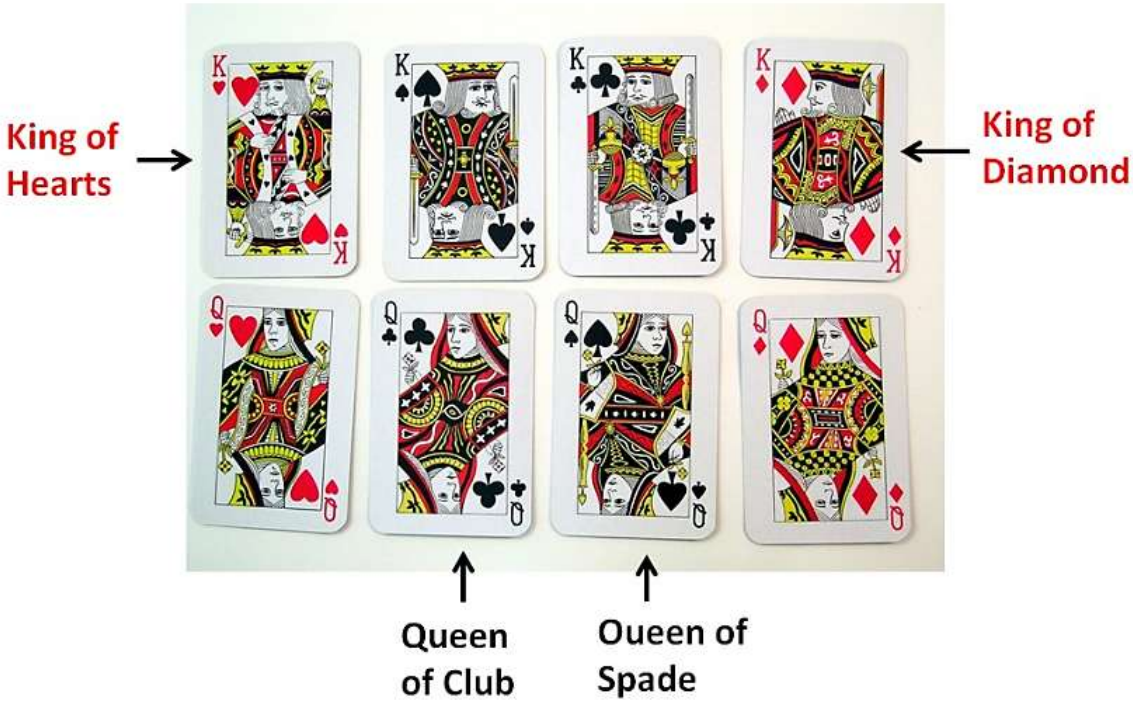
Example 13:

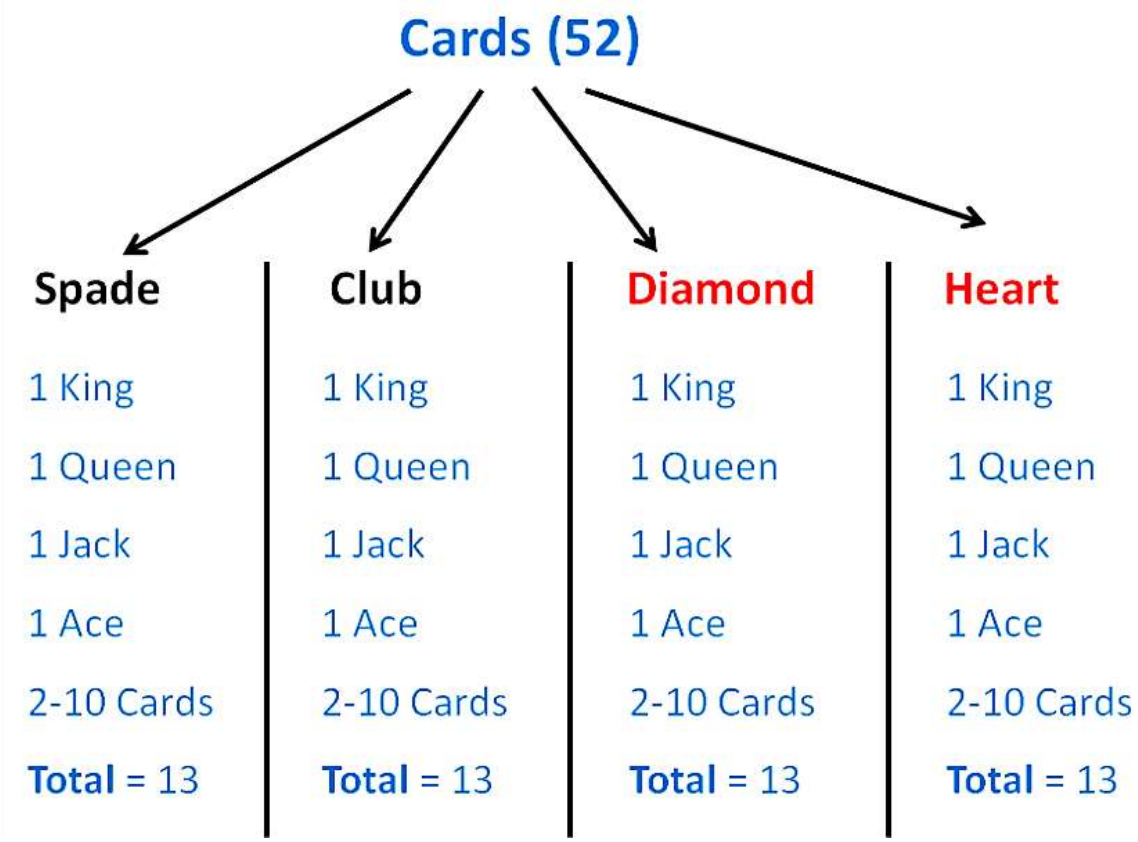
Determine which events are mutually exclusive and which are not, when **a single card is drawn from a deck**.

For each type of these there exist Ace, 2, 3, 4,…, 10, $\underbrace{\text{J(Jack), Q(Queen), K(king)}}_{pictures}$

Deck of playing Cards







a. Getting a 7 and getting a jack.
b. Getting a club and getting a king.
c. Getting a face card and getting an ace.
d. Getting a face card and getting a spade.

Solution:

a. The events are **mutually exclusive**, since the first event can be a 7 and the second event can be a jack. (they cannot be together)
b. The events are **not mutually exclusive**, since the first event can be a club and the second event can be a king. (they can be together)
c. The events are **mutually exclusive**, since the first event can be a face card (it can be a jack, queen or king) and the second event can be an ace. (they cannot be together)
d. The events are **not mutually exclusive**, since the first event can be a face card and the second event can be a spade. (they can be together)