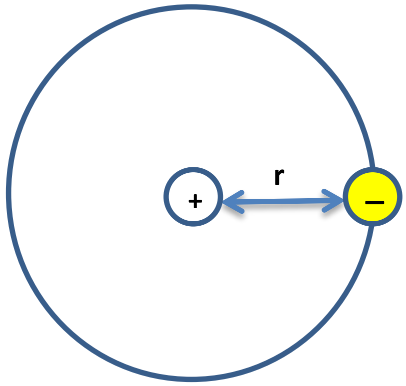
Bohr Model

Bohr used Rutherford model of atom and assumed that the H-atoms consists of a +ve nucleus and an electron moves in a circular orbit around the nucleus as shown in figure:



Hydrogen atom

E_k of the electron = ½ mv^2 ------- (1)

And

$$Ep = \frac{k\,(+Ze)(-e)}{r} = \frac{-kZe^2}{r} \quad \text{------- (2)}$$

الألكترون له قوة تجاذب مع النواة.

$$F_E = \frac{k\,(+Ze)(-e)}{r^2} = -\,m\frac{v^2}{r}$$

وحسب قانون نيوتن الثاني يجب أن يكون لدينا تعجيل مركزي.

الإشارة السالبة تعني أن التعجيل بإتجاه المركز.

Or $F_E = \frac{k\,Ze}{r} = mv^2$ ------- (3)

Eq. (3)/(2) ⇒ $\frac{1}{2}\,mv^2 = \frac{k\,Ze}{r} = E_k$ ------- (4)

Total energy $E = \underbrace{E_k}_{eq.(4)} + \underbrace{E_p}_{eq.(2)}$

$$E = \frac{k\,Ze^2}{2r} - \frac{k\,Ze^2}{r}$$

Or $E = -\frac{k\,Ze^2}{2r}$ ------- (5)

الإشارة السالبة: كلما زادت الـ r تزداد الـ E العلاقة طردية

The –ev sign in eq. (5) means that the total energy of the electron increases as r (distance between e and nucleus) increase.

To explain the line spectra of hydrogen , Bohr assume two postulates:

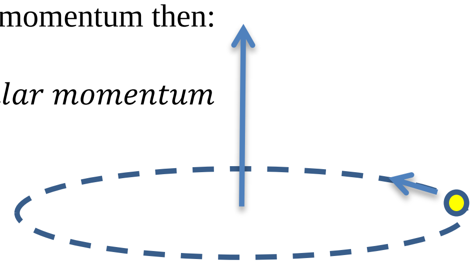
النظرية الأولى تحدد نصف قطر المدار وذلك لتحديد الزخم الزاوي.

First postulate:

To allow orbits of the electron are these for which the angular momentum then:

$\hbar = \frac{h}{2\pi}$ $\qquad P\theta = angular\ momentum$

$P\theta = (mr^2)\frac{v}{r} = mvr$



يكون عمودي على مدار الإلكترون

According to 1st postulate

$mvr = n\hbar$ ------- (6) $\qquad$ n= 1,2,3,4, …….. (Principle quantum no.)(K, L, M, N…..)

From eq. (3) we have:

$mv^2r = kZe^2$ ------- (7)

بتربيع معادلة (6) نحصل على:

$m^2v^2r^2 = n^2\hbar^2$ -------- (8)

To eliminate, divide eq. (8) by (7), we find:

$$\frac{m^2v^2r^2}{mv^2r} = \frac{n^2\hbar^2}{kZe^2}$$

$$\therefore mr = \frac{n^2\hbar^2}{kZe^2}$$

Divide this eq. by m

$\frac{mr}{m} = \frac{n^2\hbar^2}{kZe^2m}$ ------- (9) $\qquad$ When $\frac{\hbar^2}{ke^2m} = a_o$

Or $r = \left(\frac{n^2}{Z}\right) a_o$ ------- (10)

Where $a_o = \frac{\hbar^2}{kme^2} = 0.529 \times 10^{-10}\ m$, $\quad a_o = 0.53$ Å Bohr radius

From eq. (5) and (10) we find

$$E = E_n = -\frac{kZ^2e^2}{2ra_on^2} \quad \text{-------} \ (11)$$

Or $E_n = -\frac{13.6Z^2}{n^2}\,(eV)$ ------- (12)

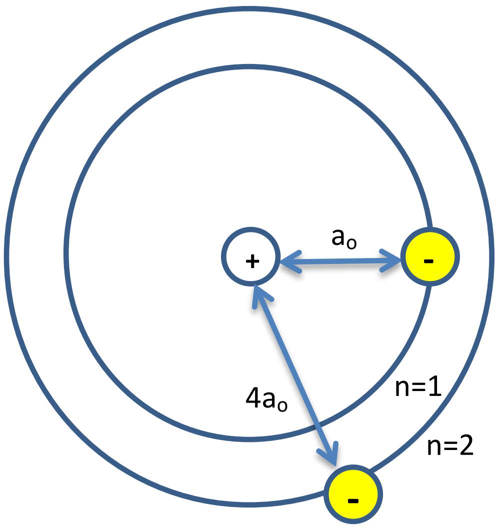
For Hydrogen Z=1

$$E_n = -\frac{13.6}{n^2}\,(eV)$$

E_1 When $n = 1$, E_2 When $n = 2$, ----------

$n = 1, 2, 3, 4, \dots \dots \dots \infty$

أعلى مستوى (مستوى التأين) عند $n = \infty \, , E_n = 0$



	n		$E_n = -\frac{13.6}{n^2}$
∞	∞	-------------------------	0
$16a_o$	4	-------------------------	$E_4 = -\frac{13.6}{16}$
$9a_o$	3	-------------------------	$E_3 = -\frac{13.6}{9}$
$4a_o$	2	-------------------------	$E_2 = -\frac{13.6}{4}$
a_o	1	-------------------------	$E_1 = -\frac{13.6}{1}$

Second postulate:

The electron doesn't radiate energy as long as it remains in the same energy level (orbit).

Radiation occurs when the electron goes from high energy level to a lower one.

The energy radiation (photon) emitted equal the energy difference between the two levels.

$$hf = E_{n_2} - E_{n_1}$$

Or $\frac{hc}{\lambda} = E_{n_2} - E_{n_1}$

Or $\frac{1}{\lambda} = \frac{1}{hc}\left(E_{n_2} - E_{n_1}\right)$ ------- (13)

From eq. (11) and (13) we get:

$\frac{1}{\lambda} = \frac{ke^2Z^2}{2a_ohc}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ ------- (14)   when $\frac{ke^2Z^2}{2a_ohc} = R$ (Rydberg constant)

Or $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ ------- (15)

Where $R = R_H = \frac{ke^2}{2a_ohc} = 1.0973731 \times 10^7\ (m^{-1})$

For H-atom

Z=1

$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ ------- (16)

Bohr eq. for H-atom.

Principal Quantum Number (n) ----------- 1,2,3, ...

Orbital Quantum Number (ℓ) ------------- 0, 1, 2, -------- , n-1   الحالات المتاحة n

Magnetic Quantum Number (m_ℓ) -------- $-\ell\ to\ \ell$ الحالات المتاحة $(2\ell + 1)$

Spin Magnetic Quantum Number (m_s) $(spin\ up\ \frac{1}{2}, spin\ down\ -\frac{1}{2})$

If $n = 1$

$\ell = 0\ and\ m_\ell = 0$

When $n = 2$

$\ell = 0,1$ When $\ell = 0,\ m_\ell = 0$

When $\ell = 1,\ m_\ell = -1,0,1$

Lyman series (from highest levels to the n=1)

Palmer series (from highest levels to the n=2)

Paschen series (from highest levels to the n=3)

Brackett series (from highest levels to the n=4)

Pfond series