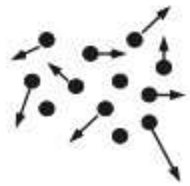
3.1. The Concept of Temperature:

A gas in thermal equilibrium has particles of all velocities.



If a sufficiently large number of collisions occurred between these particles the most probable distribution of these velocities is known as the **Maxwell Distribution**

For simplicity let's consider a gas in which the particles can move in only one direction (e.g. charged particles in a strong magnetic field). The one dimensional Maxwell Distribution is given by:

$$f(v) = A\, e^{-\frac{1}{2}mv^2/KT} \quad \ldots\ldots (\mathbf{1})$$

Where

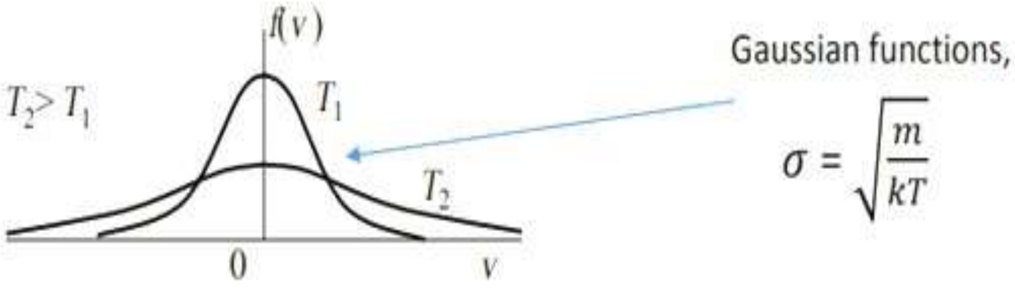
- fdv is the number of particles per m^3 with velocity between v and $v + dv$
- $\frac{1}{2}mu^2$ is the kinetic emergy.
- K is Boltzmann's constant.
- The density N or number of particles per m^3, given by

$$N = \int_{-\infty}^{\infty} f(v)dv \quad \ldots\ldots (\mathbf{2})$$

A is a normalization constant related to density

$$A = N\left(\frac{m}{2\pi KT}\right)^{1/2} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

The width of the distribution is characterized by a parameter **T** we call the Temperature.



T is related to the average kinetic energy **E_{av}**.

$$E_{av} = \frac{\int_{-\infty}^{\infty} \frac{1}{2} m v^2 f(v)\, dv}{\int_{-\infty}^{\infty} f(v)\, dv} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4)$$

We will define the thermal (most probable) velocity as:

$$E_{av} = \frac{1}{2} KT \dots\dots\dots (per\ degree\ of\ freedom) \dots\dots\dots (5),$$

And

$$E_{av} = \frac{3}{2} KT \dots\dots\dots (per 3 degree\ of\ freedom) \dots\dots\dots (6),$$

$$V_{Th} = \left(\frac{2kT}{m}\right)^{1/2} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

Substituting eq. (5) in eq. (1) we get.

$$f(v) = A \exp\left(-\frac{v^2}{V_{Th}}\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (8)$$

Defining

$$Y = \frac{v}{v_{Th}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$f(v) = A \exp(-Y^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Substituting in eq. (4) and multiplying and dividing v by $\mathbf{v_{Th}}$ to form Y:

$$E_{Av} = \frac{\frac{1}{2} m A v_{Th}^3}{N} \int_{-\infty}^{\infty} \left[\exp\left(-Y^2\right)\right] Y^2 dY \quad \ldots\ldots\ldots\ldots\ldots (11)$$

Integrating the numerator by parts:

$$\int_{-\infty}^{\infty} Y\left[\exp\left(-Y^2\right)\right] Y dY = \left[-\tfrac{1}{2}\exp(-Y^2)Y\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} -\tfrac{1}{2}\exp\left(-Y^2\right) dY = \quad \ldots\ldots\ldots\ldots (12)$$

$$= \tfrac{1}{2}\int_{-\infty}^{\infty} \exp(-Y^2)\, dY = \tfrac{1}{2}\sqrt{\pi}$$

$$E_{av} = \frac{\frac{1}{2} m A\, v_{Th}^3\, \frac{1}{2}\sqrt{\pi}}{N} = \frac{\frac{1}{2} m N \sqrt{\frac{m}{2\pi kT}} \left(\frac{2kT}{m}\right)^{3/2} \sqrt{\pi}}{N} = \tfrac{1}{2} kT \quad \ldots\ldots\ldots\ldots (13)$$

$$E_{av} = \tfrac{1}{2} kT$$

(Average kinetic energy in **one dimension**)

Maxwell's velocity distribution in **three dimensions** can be written as

$$f(v_x, v_y, v_z) = A_3 \exp\left[-\tfrac{1}{2}\left(v_x^2 + v_y^2 + v_z^2\right)/kT\right] \quad \ldots\ldots\ldots\ldots\ldots (14)$$

$$A_3 = N\left[\left(\frac{m}{2\pi kT}\right)^{1/2}\right]^3 \quad \text{.............} \quad (15)$$

The average kinetic energy is:

$$E_{av} = \frac{A_3 \iiint_{-\infty}^{\infty} \frac{1}{2} m\left(v_x^2+v_y^2+v_z^2\right) \exp\left[-\frac{1}{2} m\left(v_x^2+v_y^2+v_z^2\right)/kT\right] dv_x dv_y dv_z}{A_3 \iiint_{-\infty}^{\infty} \exp\left[-\frac{1}{2} m\left(v_x^2+v_y^2+v_z^2\right)/kT\right] dv_x dv_y dv_z} \quad \text{..........} \quad (16)$$

The expression is symmetric in v_x , v_y, v_z since the Maxwell distribution is isotropic:

$$E_{av} = \frac{3A_3 \int \frac{1}{2} m v_x^2 \exp\left[-\frac{1}{2} m v_x^2/kT\right] dv_x \iint \exp\left[-\frac{1}{2} m\left(v_y^2+v_z^2\right)/kT\right] dv_y dv_z}{A_3 \int \exp\left(-\frac{1}{2} m v_x^2/kT\right) dv_x \iint \exp\left[-\frac{1}{2} m\left(v_y^2+v_z^2\right)/kT\right] dv_y dv_z} \quad \text{.............} \quad (17)$$

$$E_{av} = \frac{3}{2} kT \quad \text{.............} \quad (18)$$

(Average kinetic energy in **Three Dimensions**)

Since **T** is so closely relate to $\mathbf{E_{av}}$ it is common in plasma physics to give the temperature in units of energy. To avoid confusion in the number of dimensions involved it is not $\mathbf{E_{av}}$ but the energy corresponding to $\mathbf{kT_e}$ that is used to denote temperature.

For $kT = 1\,\text{eV} = 1.6\ 10^{-19}\,\text{J} \rightarrow T = \dfrac{1.6\ 10^{-19}\,\text{J}}{1.38\ 10^{-23}\,\text{J}/^{o}\text{K}} = 11{,}600\ ^{o}\text{K}$

$$1\,\text{eV} \Leftrightarrow 11{,}600\ ^{o}\text{K}$$

By **2 eV** usually we mean: **kT = 2 eV** → $\mathbf{E_{av}}$ **= 3 eV** in three dimensions**.**

It is interesting that a plasma can have several temperatures at the same time. It often happens the ions and the electrons have separate Maxwellian distributions with different temperatures T_i and T_e. This can come about because the collision rate among ions or among electrons themselves is larger than the rate of collision between an ion and an electron. Then each species can be in its own thermal equilibrium, but the plasma may not last long enough for the two temperatures to equalize. When there is a magnetic field *B*, even a single species, say ions, can have two temperatures this is because the forces acting on an ion along B are different from those acting perpendicular to B (due to the Lorentz force). The components of velocity perpendicular to B and parallel to B may the belong to different Maxwellian distributions with temperatures $T_{\perp}$ and $T_{\parallel}$

People are usually amazed to learn that the electron temperature inside a fluorescent light bulb is about $20000^{o}K$. "it doesn't feel that hot". Of course, the heat capacity must also be taken into account. The density of electrons inside a fluorescent tube is much less than that of a gas at atmospheric pressure, and the total amount of heat transferred to the wall by electrons striking it at their thermal velocities is not that great. Many laboratory plasmas have temperatures of the order of 1000000^o K $(100\ eV)$, but at densities of $10^{11}-10^{19}$ per m^3, the heating of the walls is not a serious consideration.

3.2. Plasma as State of Matter

Plasma is a state of matter that is often thought of as a subset of gases, but the two states behave very differently. Like gases, plasmas have no fixed shape or volume, and are less dense than solids or liquids. But unlike ordinary gases, plasmas are made up of atoms in which some or all of the electrons have been stripped away and positively charged nuclei, called ions, roam freely. "A gas is made of neutral molecules and atoms," . That is, the number of negatively charged electrons equals the number of positively charged protons. "Plasma is a charged gas, with strong Coulomb [or electrostatic] interactions,". Atoms or molecules can acquire a positive or negative electrical charge when they gain or lose electrons. This process is called ionization. Plasma makes up the sun and stars, and it is the most common state of matter in the universe as a whole. [**Jesse Emspak]** Charged particles in conducting gas result from detachment electrons from atoms or molecules. In order to understand the conditions for the existence of such a system, we compare it with an ordinary chemical system. Let us consider, for example, atmospheric air, consisting mainly of nitrogen and oxygen molecules. At high temperatures, along with the nitrogen and oxygen, nitrogen oxides can be formed. The following chemical equilibrium is maintained in air[**Boris M. Smirnov]**

$$N_2 + O_2 \leftrightarrow 2NO - 41.5\ \text{kcal/mol.} \qquad \text{………..……....(19)}$$

The sign $\leftrightarrow$ means that the process can proceed either in the forward or in the reverse direction. According to **Le Chatelier's** principle, an increase in the temperature of the air leads to an increase in the concentration of the *NO* molecules.

A similar situation takes place in the case of formation of charged particles

in a gas, but this process requires a high temperature. For example, the ionization equilibrium for nitrogen molecules has the form:

$$N_2 \leftrightarrow N_2^+ + e - 360\ \text{kcal/mol} \qquad \text{…………………... (20)}$$

Thus, the chemical and ionization equilibriums are analogues, but ionization of atoms and molecule proceeds at temperatures higher than that of chemical transformation. Table)1(shows contains examples of chemical and Ionization Equilibrium.

Table 1. Temperatures Corresponding to Dissociation of 0.1% of Molecules or Ionization of 0.1% of Atoms at a Pressure of 1 atm

Chemical Equilibrium	T, K	Ionization Equilibrium	T, K
$2CO_2 \leftrightarrow 2CO+O_2$	1550	$H_2 \leftrightarrow H^++e$	7500
$H_2 \leftrightarrow 2H$	1900	$H_e \leftrightarrow He^++e$	12000
$O_2 \leftrightarrow 2O$	2050	$C_S \leftrightarrow C_S{}^++e$	2500
$N_2 \leftrightarrow 2N$	4500		
$2H_2O \leftrightarrow 2H_2+O_2$	1800		

This table gives the temperatures at which 0.1% of molecules are dissociated in the case of chemical equilibrium or 0.1% of atoms are ionized for ionization equilibrium. The pressure of the gas is 1 atm. Thus, a weakly ionized gas, which we shall call a plasma, has an analogy with a chemically active gas. Therefore, though a plasma has characteristic properties which we shall describe, it is not really a new form or state of matter as is often asserted. In most actual cases plasma is a weakly ionized gas with a small degree of ionization. Table (2) gives some examples of real plasma and their

parameters the number of densities of electrons ($\boldsymbol{N}_{\mathrm{e}}$) and of atoms ($\boldsymbol{N}_{\mathrm{a}}$), the temperature (or the average of energy) of electrons ($\boldsymbol{T}_{\mathrm{e}}$), and the gas temperature ($\boldsymbol{T}_{\boldsymbol{g}}$).

Table 2: Parameters of Some Plasma

Types of Plasma	N_e (cm^{-3})	N_a(cm^{-3})	T_e(K)	T_g(K)
Sun's photosphere	10^{13}	10^{17}	6000	6000
E-layer of ionosphere	10^{5}	10^{13}	250	250
He-Ne laser	3×10^{11}	2×10^{16}	3×10^{14}	400
Argon laser	10^{13}	10^{14}	10^{5}	10^{3}

If the temperatures of electrons and neutral particles are identical, the plasma is called equilibrium plasma; in the opposite case we have **non-equilibrium plasma**.