** Elasticity of Demand

The concept of elasticity, as used in economics, is the ratio of the percentage change in the quantity demanded (or supplied) to a percentage change in an economic variable, such as price, income, etc. Elasticity may be used to predict the responsiveness of demand (and supply) to changes in such economic variables.

There are various symbols used to denote elasticity. In this text it is denoted by the symbol ε, pronounced 'epsilon'.

**Price elasticity of demand

Price elasticity of demand measures the responsiveness (sensitivity) of quantity demanded to changes in the good's own price.

$$\varepsilon_d = \frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}} = \frac{\%\Delta Q_d}{\%\Delta P} = \frac{\frac{\Delta Q}{Q}.100}{\frac{\Delta P}{P}.100}$$

$$\varepsilon_d = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

The numerical value or coefficient for price elasticity of demand is normally negative since $\Delta Q / \Delta P$ is negative. That is, the linear demand function, P = a - bQ has slope

$$= \frac{\Delta P}{\Delta Q} = -\frac{b}{1}$$

hence, inverting both sides gives

$$\frac{\Delta Q}{\Delta P} = -\frac{1}{b}$$

which is negative.

Point elasticity of demand

Given the linear demand function, P = a- bQ, the formula for point elasticity of demand at any point (P_0,Q_0) is:

$$\varepsilon_d = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

OR

$$\varepsilon_d = -\frac{1}{b} \cdot \frac{P_0}{Q_0} \text{ since slope } = \frac{\Delta Q}{\Delta P} = -\frac{1}{b}$$

EX:- Given the demand function for computers as

P = 2400 - 0.5Q.

(a) Determine the coefficient of point elasticity of demand when (i) P = 1800, (ii) P = 1200 and (iii) P = 600.

Give a verbal description of each result.

(b) If the price of computers increases by 12%, calculate the percentage change in the quantity demanded at P = 1800, P = 1200 and P = 600.

Sol:-

(a)

(i) At P = 1800 the quantity of computers demanded, Q, is calculated by substituting P = 1800 into the demand function:

P = 2400 - 0.5Q

1800 = 2400 - 0.5Q

0.5Q = 600

Q = 1200

The value of point elasticity of demand at (P = 1800, Q = 1200) is calculated by substituting these values along with b = 0.5 into formula:

$$\varepsilon_d = -\frac{1}{b} \cdot \frac{P_0}{Q_0}$$

$$= -\frac{1}{0.5} \cdot \frac{1800}{1200}$$

$$= -\frac{1800}{600} = -3$$

The coefficient of point elasticity of demand is ε_d = -3. which indicates that at the price P = 1800 a 1% increase (decrease) in price will cause a 3% decrease (increase) in the quantity of computers demanded. Demand is elastic, $|\varepsilon_d| > 1$.

P_0	1800	1200	600
Calculate Q_0	$P_0 = 2400 - 0.5Q_0$ $1800 = 2400 - 0.5Q_0$ $0.5Q_0 = 600$ $Q_0 = 1200$	$P_0 = 2400 - 0.5Q_0$ $1200 = 2400 - 0.5Q_0$ $0.5Q_0 = 1200$ $Q_0 = 2400$	$P_0 = 2400 - 0.5Q_0$ $600 = 2400 - 0.5Q_0$ $0.5Q_0 = 1800$ $Q_0 = 3600$
	$\varepsilon_d = -\frac{1}{b}\frac{P_0}{Q_0}$ $= -\frac{1}{0.5}\frac{1800}{1200} = -3$	$\varepsilon_d = -\frac{1}{b}\frac{P_0}{Q_0}$ $= -\frac{1}{0.5}\frac{1200}{2400}$ $= -\frac{1200}{1200}$ $= -1$ (see notes)	$\varepsilon_d = -\frac{1}{b}\frac{P_0}{Q_0}$ $= -\frac{1}{0.5}\frac{600}{3600}$ $= -\frac{1}{3}$ ≈ -0.33 (notes)
Demand	Demand is elastic $\lvert\varepsilon_d\rvert > 1$	Demand is unit elastic $\lvert\varepsilon_d\rvert = 1$	Demand is inelastic $\lvert\varepsilon_d\rvert < 1$

ε_d = - 1 indicates that at P = 1200, a 1% increase (decrease) in price will cause a 1% decrease (increase) in the quantity of computers demanded. ε_d = -0.33 indicates that at P = 600, a 1 % increase (decrease) in price will cause a 0.33% decrease (increase) in the quantity of computers demanded

(b) By using the definition

$$\varepsilon_d = \frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}} = \frac{\%\Delta Q_d}{\%\Delta P}$$

may be rearranged as

$$\Delta Q(\%) = \Delta P\ (\%) * \varepsilon_d$$

At P = 1800 we have calculated that ε_d = -3. To calculate the percentage change in Q, substitute ε_d = -3 and ΔP = 12% into eqution

ΔQ = 12% * -3 = -36%

The quantity demanded decreases by 36%, a larger percentage decrease than the 12% price increase. Demand is strongly responsive to price change and is described as elastic demand.

P = 1200: ε_d = -1, ΔP = 12%. Substitute into equation:

ΔQ = 12% * -1 = -12%

The quantity demanded decreases by 12%; the percentage increase in price is 12%. This is described as unit elastic demand.

P = 600: ε_d = -0.33, ΔP = 12%. Substitute into equation:

ΔQ = 12% * -0.33 = -3.96%

The quantity demanded decreases by 3.96%, a smaller percentage decrease than the 12% price increase. So demand is weakly responsive to price, hence it is described as inelastic demand.

** Price elasticity of supply

The coefficient of price elasticity of supply is given by the formula

$$\varepsilon_s = \frac{\text{Percentage change in quantity supplied}}{\text{Percentage change in price}} = \frac{\%\Delta Q_S}{\%\Delta P} = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q}$$

Given the supply function, $P = c + dQ$, slope = $d = \Delta P/\Delta Q$. Then, $1/d = \Delta Q/\Delta P$; hence, the point elasticity of supply formula is

$$\varepsilon_s = \frac{1}{d} \cdot \frac{P}{Q}$$

Note: $\varepsilon_s > 1$: supply is elastic; $\varepsilon_s < 1$: supply is inelastic; $\varepsilon_s = 1$: supply is unit elastic.

****Taxes, subsidies and their distribution**

Taxes and subsidies are another example of government intervention in the market. A tax on a good is known as an indirect tax. Indirect taxes may be either:

• A fixed amount per unit of output, for example, the tax imposed on petrol

• A percentage of the price of the good; for example, value added tax.

**When a tax is imposed on a good, two issues of concern arise:

1- How does the imposition of the tax affect the equilibrium price and quantity of the good?
2- What is the distribution of the tax

In these calculations

• The consumer always pays the equilibrium price.

• The supplier receives the equilibrium price minus the tax.

EX:- The demand and supply functions for a good are given as

Demand function: $P_d = 100 - 0.5Q_d$

Supply function: $P_s = 10 + 0.5Q_s$

(a) Calculate the equilibrium price and quantity
(b) Assume that the government imposes a fixed tax of £6 per unit sold.

(i) Write down the equation of the supply function, adjusted for tax.

(ii) Find the new equilibrium price and quantity algebraically and graphically.

(i) Outline the distribution of the tax, that is, calculate the tax paid by the consumer and the producer.

Sol:- (a) $P_d = P_s$

$100 – 0.5Q = 10 + 0.5Q$

$100 – 10 = 0.5Q + 0.5Q$

$90 = Q$ equilibrium quantity

Now solve for the equilibrium price by substituting $Q = 90$ into either equation

$P = 100 - 0.5(90)$ substituting Q = 90 into equation

$P = 55$

Remember: the equilibrium price of £55 (with no taxes) means that the price the consumer pays is equal to the price that the producer receives.

(b)

(i) The tax of £6 per unit sold means that the effective price received by the producer is (Ps - 6). The equation of the supply function adjusted for tax is

$P_s – 6 = 10 + 0.50Q$

$P_s = 16 + 0.50Q$

(ii) The new equilibrium price and quantity are calculated by equating the original demand function, and the supply function adjusted for tax

$P_d = P_s$

$100 - 0.50Q = 16 + 0.50Q$

$Q = 84$

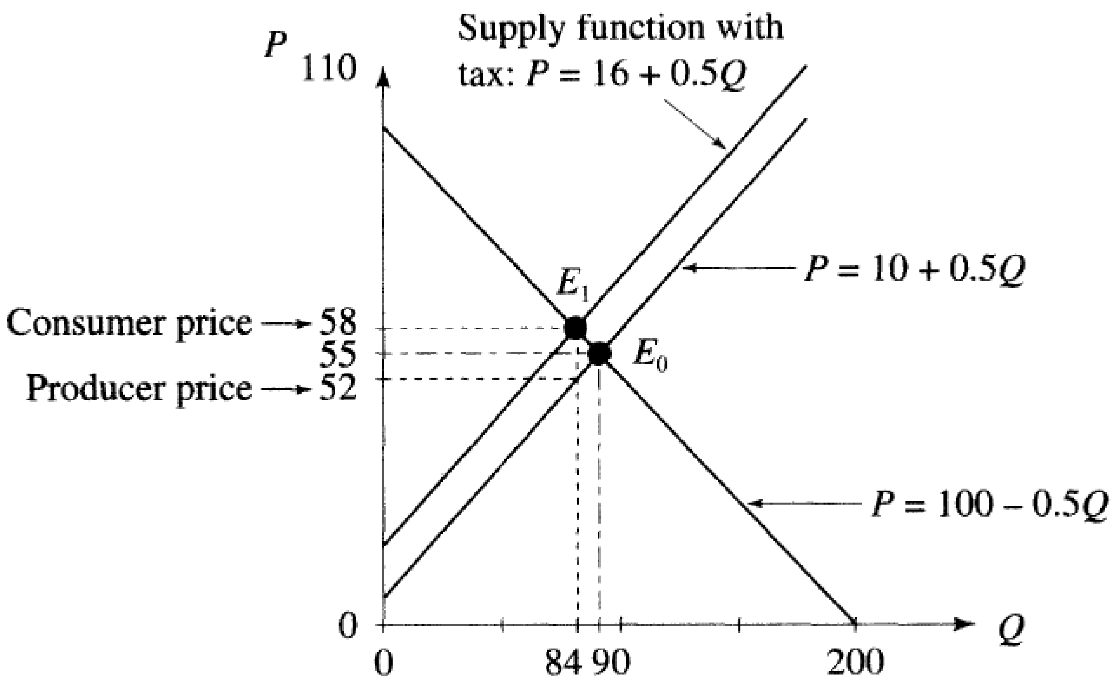
So

$P = 100 - 0.5(84)$

$P = 58$

(iii) The consumer always pays the equilibrium price, therefore the consumer pays £58, an increase of £3 on the original equilibrium price with no tax, which was £55. This means that the consumer pays 50% of the tax. The producer receives the new equilibrium price, minus the tax, so the producer receives £58 - £6 = £52, a reduction of £3 on the original equilibrium price of £55. This also means that the producer pays 50% of the tax. In this example, the tax is evenly distributed between the consumer and producer. The reason for the 50 : 50 distribution is due to the fact that the slope of the demand function is equal to the slope of the supply function.

Remark :- This suggests that changes in the slope of either the demand or supply functions will alter this distribution.



**** Subsidies and their distribution**

In the analysis of subsidies, a number of important points need to be highlighted:

• A subsidy per unit sold will trans the supply function vertically downwards, that is, the price received by the producer is (P + subsidy).

• The equilibrium price will decrease (the consumer pays the new lower equilibrium price).

The price that the producer receives is the new equilibrium price plus the subsidy.

• The equilibrium quantity increases.

EX:- The demand and supply functions for a good (£ *P* per ton of potatoes) are given as

Demand function: $Pd = 450 - 2Q_d$

Supply function: $Ps = 100 + 5Q_s$

(a) Calculate the equilibrium price and quantity.

(b) The government provides a subsidy of £70 per unit (ton) sold:

(i) Write down the equation of the supply function, adjusted for the subsidy

(ii) Find the new equilibrium price and quantity algebraically and graphically,

(iii) Outline the distribution of the subsidy, that is, calculate how much of the subsidy is received by the consumer and the supplier.

Sol :-

(a) $450 - 2Q_d = 100 + 5Q_s$

$450 – 100 = 7Q$

$Q = 50$

$P = 100 + 250 = 350$

(b)

(i) With a subsidy of £70 per unit sold, the producer receives (P_s + 70). The equation of the supply function adjusted for subsidy is :

$P_s + 70 = 100 + 5Q$

$P_s = 30 + 5Q$

The supply function is trans vertically downwards by 70 units.

(ii) The new equilibrium price and quantity are calculated by equating the original demand function and the supply function adjusted for the subsidy

$P_d = (P_s + \text{subsidy})$

450 - 2Q = 30 + 5Q

Q = 60

Now , P = 450 - 2Q

P = 450 - 2(60)

P =330

(iii) The consumer always pays the equilibrium price, therefore, the consumer pays £330, a decrease of £20 on the equilibrium price with no subsidy (£350). This means that the consumer receives 20/70 of the subsidy.

The producer receives the equilibrium price, plus the subsidy, so the producer receives £330 + £70 = £400, an increase of £50 on the original price of £350. The producer receives 50/70 of the subsidy.

Remark :- The producer receives a greater fraction of the subsidy than the consumer. The reason? The slope of the supply function is greater than the slope of the demand function.

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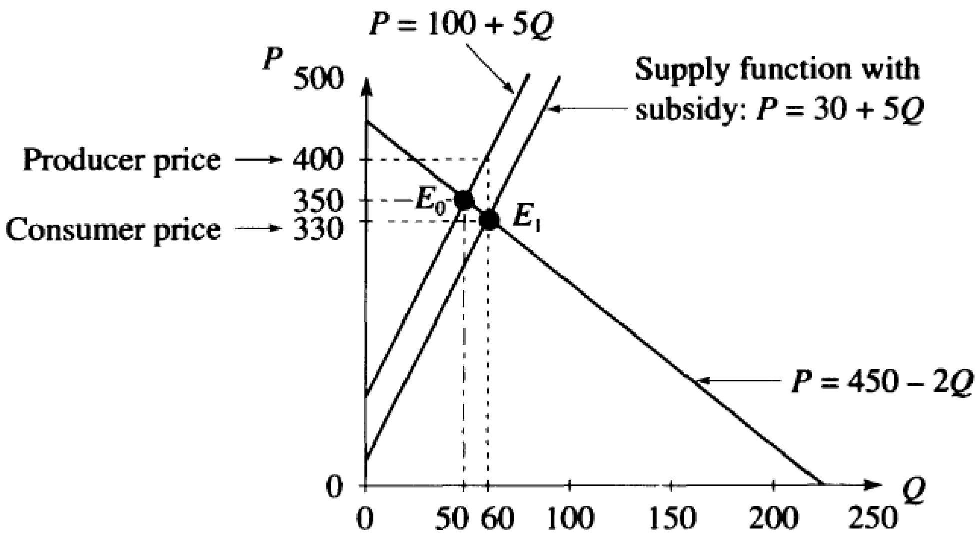

****Breakeven analysis**

The breakeven point for a good occurs when total revenue is equal to total cost.

Breakeven point: total revenue = total cost

EX:- The total revenue and total cost functions are given as follows:

TR = 3Q

TC = 10 + 2Q

(a) Calculate the equilibrium quantity algebraically and graphically at the breakeven point.

(b) Calculate the value of total revenue and total cost at the break-even point.

Sol:- (a) The break-even point occurs when :

3Q= 10 + 2Q

Q= 10 The equilibrium quantity at the break-even point

(b) TO calculate the value of total revenue and total cost at the break-even point is calculated we substitute Q=10 into the respective revenue and cost functions:

TR = 3Q = 3(10) = 30

TC= 10 + 2Q= 10+ 2(10) = 30

At Q = 10, TR = TC = 30.