

المحاضرة الثانية / رياضيات مالية

**Mathematical Modelling

Modeling is a set of processes and treatments for building models that are intended to facilitate complex phenomena

The idea of a model is used in many areas of life to represent aspects of reality. in the broadest sense, model may be classified as physical or abstract.

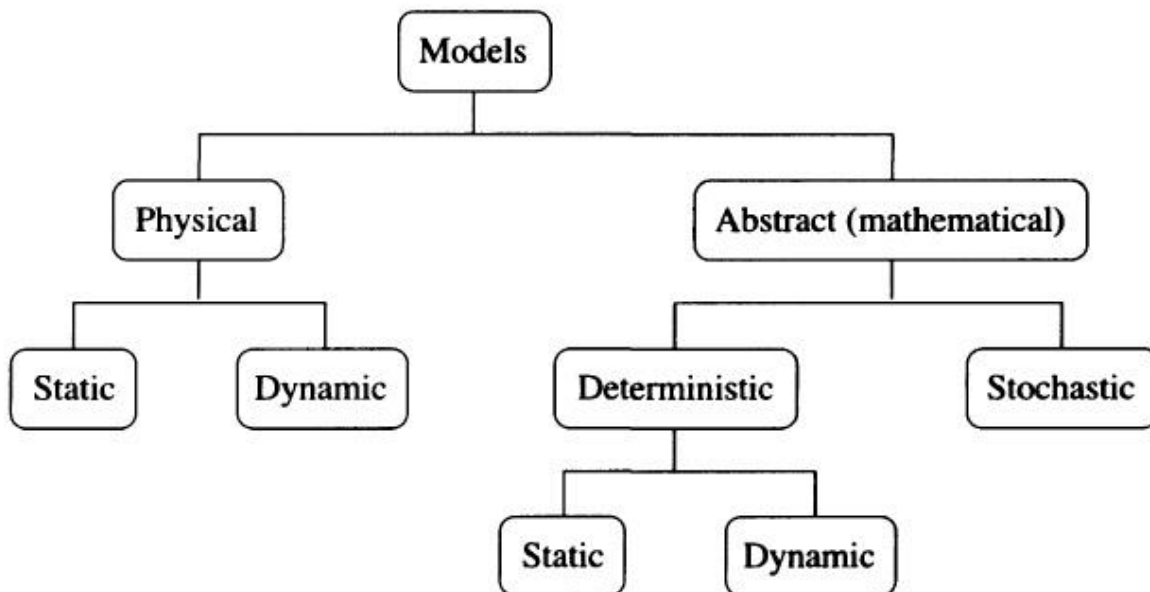
def:- a mathematical model is a mathematical construct often an equation or a relationship designed to study the behavior of a real phenomena.

**Examples for some real problems requiring mathematical model

- 1- find the mass of the earth
- 2- find the volume of blood inside the body
- 3- estimate the population of china in the year 2050

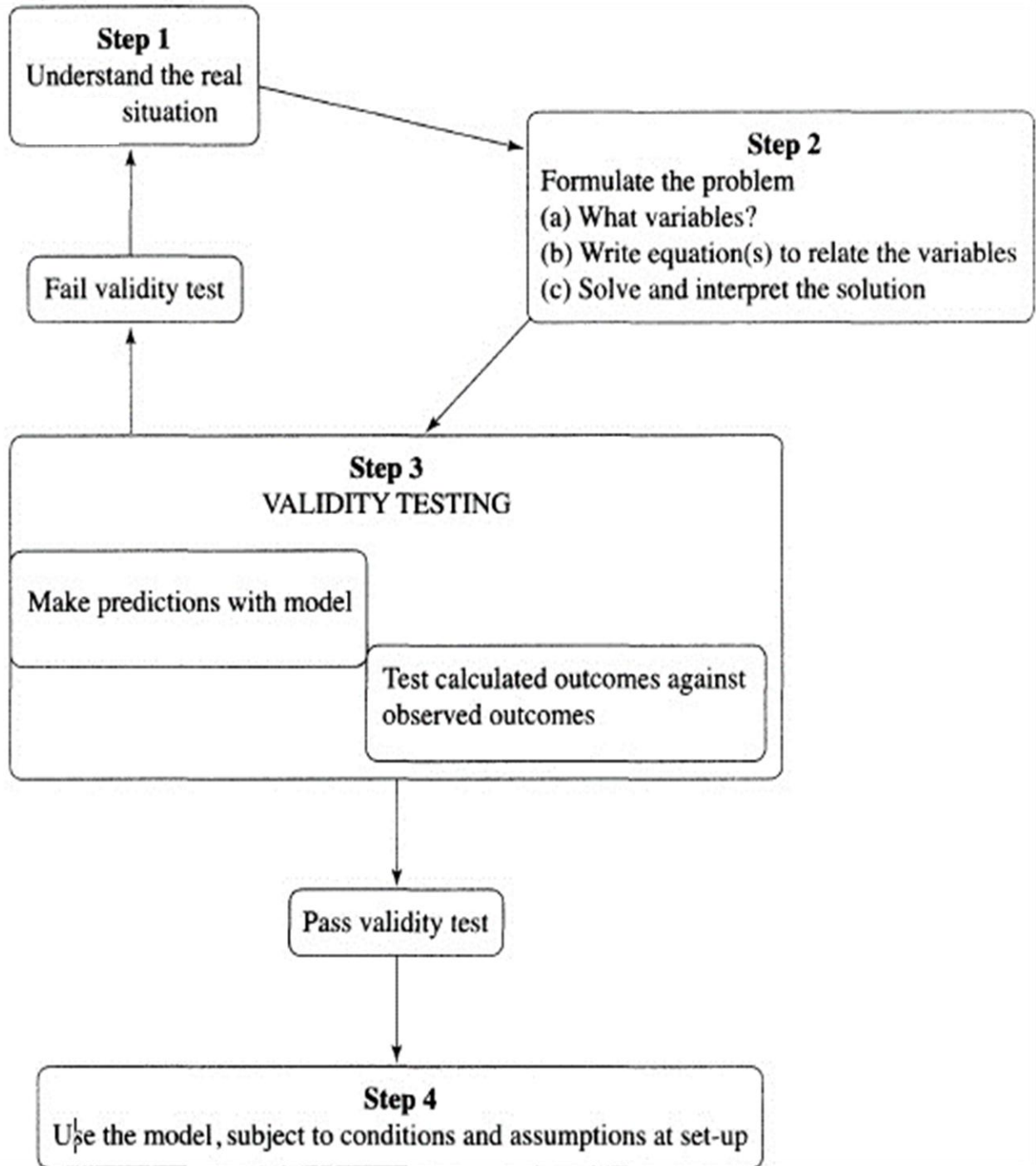
** now we can classify model as follows:

models may be classified as physical or abstract.



****Suggested steps in the construction of a mathematical model**

The suggested steps in mathematical modelling can be seen graphically as follows



*****Economic models**

Economics is a social science which studies how individuals within an economy make economic decisions on the allocation, distribution and utilisation of resources in order to satisfy their needs and wants.

Remark: we shall refer to mathematical models as economic models since they are used in the context of economics.

Economic models fall into two types:

- 1- Microeconomics: this studies the economic decisions of individual households and firms.
- 2- Macroeconomics: this studies the economy as a whole. It studies the aggregate of all economic decisions. such as total planned savings and total planned investment in the economy.

**** Items of goods**

- 1- A substitute good is one that can be used instead of another good, such as trains and buses.
- 2- A complementary good is one that is consumed in conjunction with another, such as petrol/cars.

**** The demand function**

Let X be a good, the following linear function $Q = 200 - 2p$ is called the demand function, where Q is the quantity demanded of good X , and p is the price of good X .

Remark :- The demand fun. $Q = f(p)$ is an example of economic model, we notice that there are several variables that influence the demand for a good X . this relation can be expressed as

$$Q = f(P, Y, P_s, P_c, T_a, A, \dots)$$

where

Q is the quantity demanded of good X

P is the price of good X

Y is the income of the consumer

PS is the price of substitute goods

Pc is the price of complementary goods

Ta is the taste or fashion of the consumer

A is the level of advertising

We see that Y, Ps, Pc, Ta, A, \dots are constant therefore Q depends on price only, Therefore the demand equation can be written as :

$Q = f(p)$ can be modelled by the simple linear equation

$Q = a - bp$ the demand fun. In general

Note: The relation between quantity demand and price is negative because when the price of a good increase, the quantity demand will decrease.

Remark :- the inverse demand function of $Q = 200 - 2p$ is given as :

$$Q = 200 - 2p$$

$$2p = 200 - Q$$

$$P = f^{-1}(Q) = 100 - 0.5 Q$$

In general the inverse of demand fun. $Q = a - bp$ is

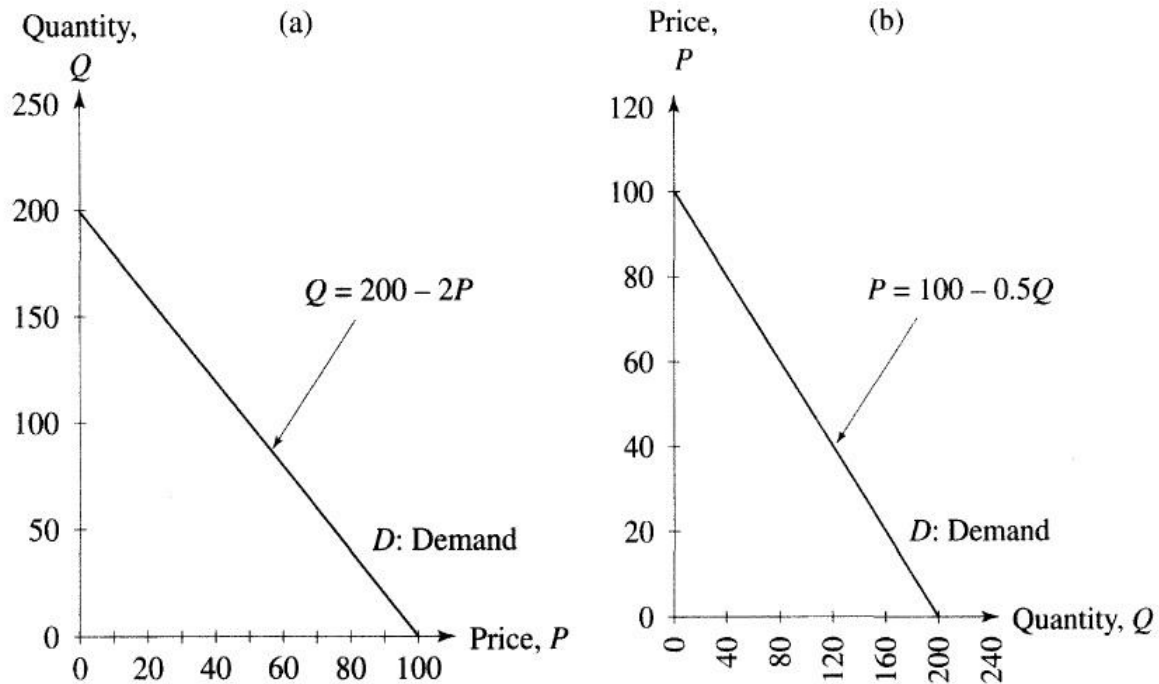
$$bP = a - Q$$

$$p = f^{-1}(Q) = \frac{a}{b} - \frac{1}{b} Q$$

** The graph of the demand function

The demand function $Q = 200 - 2P$ is graphed in the next Figure (a), Q plotted on the vertical axis and P plotted on the horizontal one.

The inverse demand function $P = f^{-1}(Q) = 100 - 0.5 Q$ is graphed in the next Figure (b), P on the vertical axis, and Q is plotted on the horizontal one.



Ex :- The demand function is given by the equation $P = 100 - 0.5Q$.

- State the slope and intercepts.
- What is the quantity demanded when $P = 5$?
- Plot the demand function $P = 100 - 0.5Q$ for $0 < Q < 200$.
- Find an expression for the demand function in the form $Q = f(P)$ and graph it.

Sol :

- The slope and vertical intercept may be deduced as follows

When $Q=0$, then $p=100$, therefore the vertical intercept is 100

The slope $\Delta P / \Delta Q = -0.5$ This indicates that the price drops by 0.5 units for each successive unit increase in quantity demanded When $p=0$, then $0 = 100 - 0.5Q$

$$0.5Q = 100$$

$Q = 200$, therefore the horizontal intercept is 200

(b) The quantity demanded when $P = 5$ is calculated by substituting $P = 5$ into the demand function

$$P = 100 - 0.5Q$$

$$5 = 100 - 0.5Q$$

$$0.5Q = 100 - 5$$

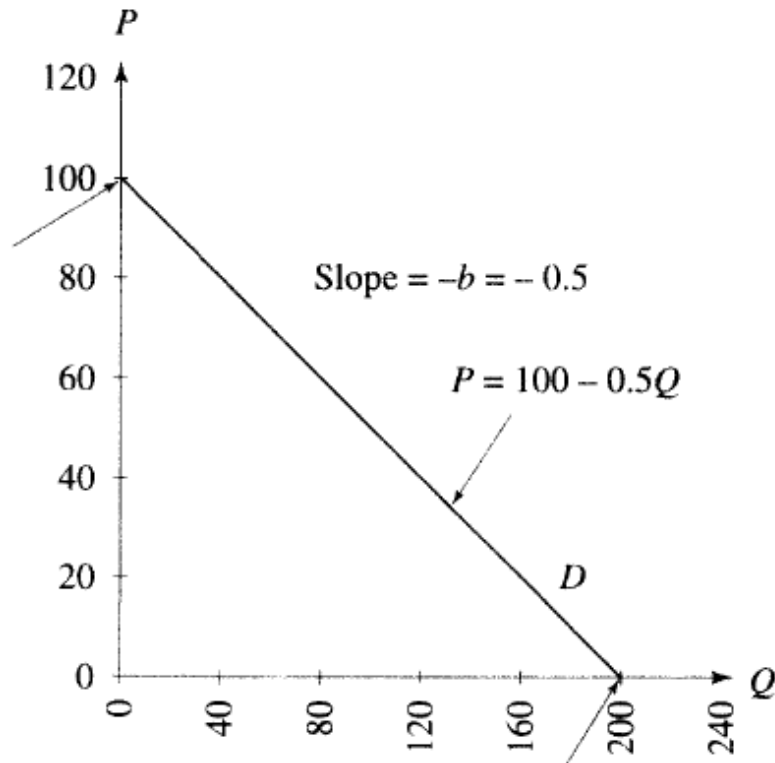
$$0.5Q = 95$$

$$Q = \frac{95}{0.5}$$

$$Q = 190$$

(c) To plot the demand function over the range, $0 < Q < 200$, choose various quantity values within this range.

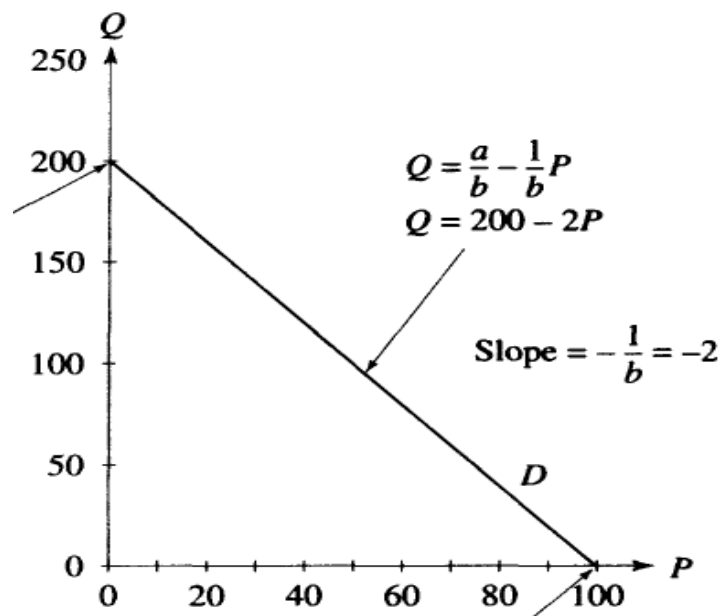
Quantity	Price
0	100
40	80
80	60
120	40
160	20
200	0



(d) $P = 100 - 0.5Q$

$$0.5Q = 100 - p$$

$$Q = 200 - 2p$$



**** The supply function**

Let X be a good, and Q the quantity supplied of a good X , and p is the price of the good itself. The supply function $P = h(Q)$ can be modelled by the following linear equation

$$P = c + dQ$$

where c and d are constants.

We note there is a positive relationship between quantity supplied and price, that is, when the price of a good increases, the quantity supplied will also increase.

EX:- The supply function is given by $P = 10 + 0.5Q$.

(a) State the slope and intercepts.

(b) Plot the supply function, $P = 10 + 0.5Q$ for $0 \leq Q \leq 100$.

Sol:-

- (a) The vertical intercept is $c = 10$. This means that the firm will supply no units at $P < 10$. The slope $\Delta P / \Delta Q = 0.5$. This means that price increases by 0.5 units for every successive unit increase in quantity supplied. The horizontal intercept is calculated by substituting $P = 0$ into the equation of the supply function, that is:

$$P = 10 + 0.5Q$$

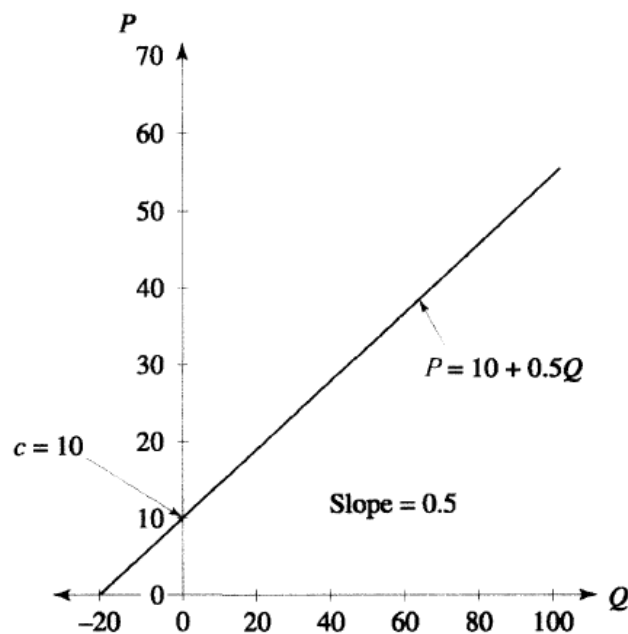
$$0 = 10 + 0.5Q$$

$$-10 = 0.5Q$$

$$Q = -20$$

(b) To plot the supply function

Quantity	Price
0	10
20	20
40	30
60	40
80	50
100	60

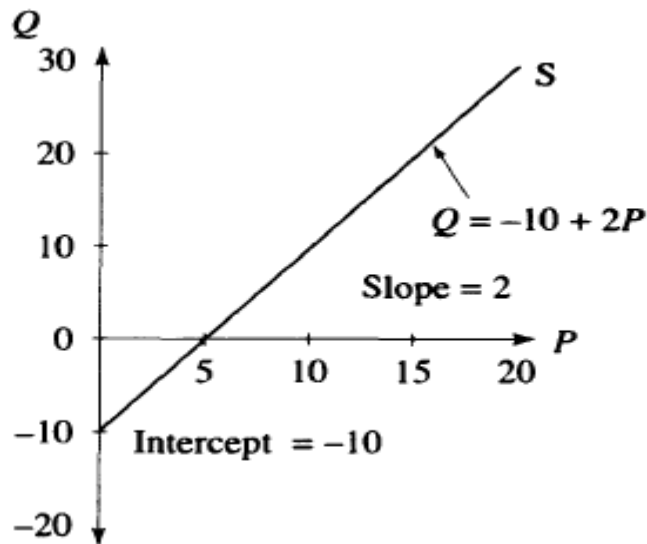


EX:- A supplier will only start to supply T-shirts when a price greater than £5 per unit is available. He will then increase output by 2 units (2 T-shirts) for every unit increase in price.

- Plot the supply function in the form $Q = f(P)$.
- Write down the equation of the supply function.
- Find the value of Q when $P = 15$ from the graph.
- Write the equation of the supply function in the form, $P = h(Q)$.

Sol:-

- (a) plot Q on the vertical axis and P on the horizontal one. In this case the graph crosses the horizontal axis at $P = 5$, that is, the supplier will supply no goods (T-shirts) at a price less than or equal to £5 per unit. Plot the point $P = 5, Q = 0$. From this point, draw a line with a slope of 2 (change in Q per unit change in P), this line cuts the vertical axis at $Q = -10$. where $m=2$ and point $(5,0)$ from horizontal axis and the other point on vertical axis is $(0,y)$ from the equation of slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ find the value of y



- (b) The general format for the equation of a line is $y = mx + c$. In this case, $y = Q$ and $x = P$. From (a) slope $m = 2$ and vertical intercept, $c = -10$. The required equation, therefore, for the supply function is, $Q = -10 + 2P$.
- (c) From the previous graph when, $p=15, Q=20$
 $Q = -10 + 2p$
 $Q = -10 + 2(15)$
 $Q = 20$

(d) The equation of the supply fun. Is the form

$P = h(Q)$ is derived as follows:

$$Q = -10 + 2p$$

$$-2p = -10 - Q$$

$$P = 5 + 0.5Q$$

