**Remark: market equilibrium usually calculate for non-linear demand and supply functions, see the following example

A **polynomial function** has a rule of the type

$y = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \quad (n \in N)$

where $a_0, a_1 \ldots a_n$ are numbers called coefficients.

The degree of a polynomial is given by the value of n, the highest power of x with a non-zero coefficient.

Examples

i - $y = 2x + 3$ is a polynomial of degree 1.

ii - $y = 2x^2 + 3x - 2$ is a polynomial of degree 2.

iii - $y = -x^3 + 3x^2 + 9x - 7$ is a polynomial of degree 3.

****Quadratic function**

A quadratic equation has the general form

$ax^2 + bx + c = 0$

Ex:- suppose that the supply and demand functions for a particular market are given by the equtions:-

$P_s = Q^2 + 6Q + 9$ and $P_d = Q^2 - 10Q + 25$

(a) Graph each functions over the interval Q=0 to Q=5.
(b) Find the equilibrium price and quantity.

Sol:- (a)

Q	$P = Q^2 + 6Q + 9$	$P = Q^2 - 10Q + 25$
0	9	25
1	16	16
2	25	9
3	36	4
4	49	1
5	65	0

Note that the vertical intercepts of the demand function is p =25 (when Q = 0) and the vertical intercepts of the supply function is p = 9 (when Q = 0).

(b) market equilibrium is at the point of intersection of the two

Functions, market equilibrium exists when

$P_d = P_s$ and $Q_d = Q_s$

Now, $\mathbf{P_s = P_d}$

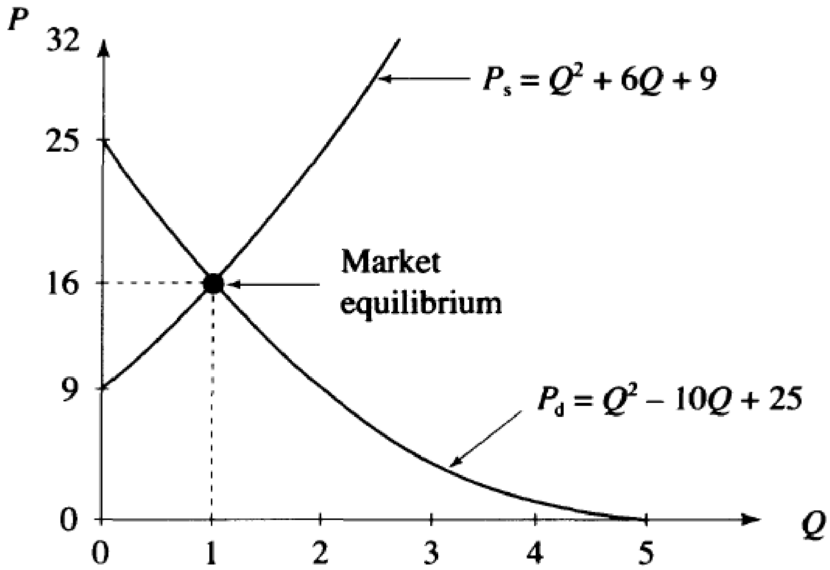
$Q^2 + 6Q + 9 = Q^2 – 10Q + 25$

$Q^2+ 6Q - Q^2+10Q = 25-9$

$16Q = 16$

$Q = 1$

when $Q = 1$, $P = 16$



EX:- The demand function for a monopolist is given by the equation P = 50 - 2Q.

(a) Write down the equation of the total revenue function.

(b) Graph the total revenue function for $0 \leq Q \leq 30$.

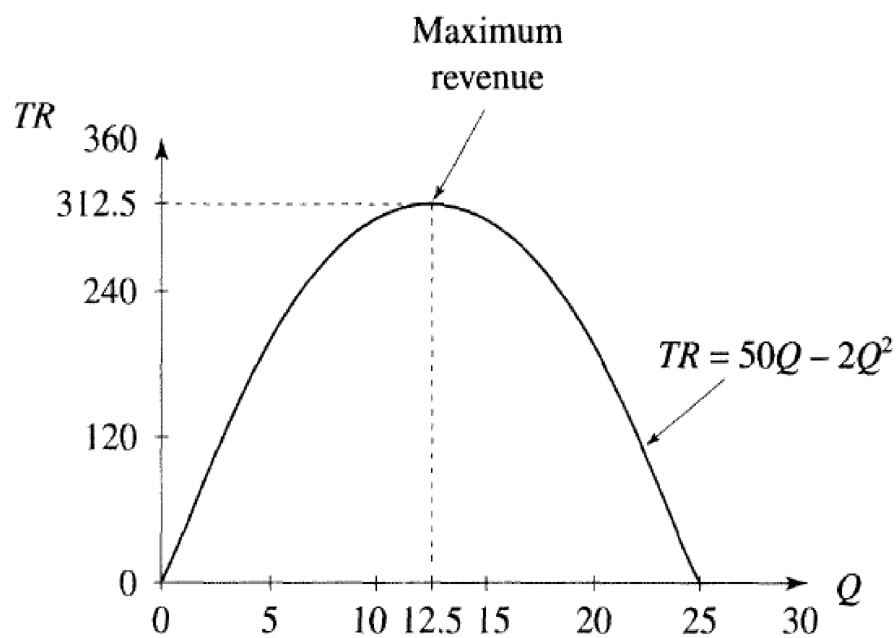
(c) Estimate the value of Q at which total revenue is a maximum and estimate the value of maximum total revenue.

Sol:- (a) Since P = 50 – 2Q, and TR = P * Q

then $TR = (50 - 2Q)Q = 50Q - 2Q^2$

(b) Calculate a table of values for $0 \leq Q \leq 30$such as those in next Table

Q	TR
0	0
5	200
10	300
15	300
20	200
25	0
30	−300



(c) A property of quadratic functions is that the turning point (in this case a maximum) lies halfway between the roots (solutions) of the quadratic function. The roots (solutions) of the Quadratic function, The roots of the TR fun. Is given as follows:

$TR = 50Q – 2Q^2$

$0 = 50Q – 2Q^2$

$0 = Q(50 – 2Q)$

$Q = 0$ and $Q = 25$

These roots are illustrated graphically as the points where the *TR* function intersects the *x*-axis. The turning point occurs halfway between these points, that is, at $Q = 12.5$. Substitute $Q = 12.5$ into the *TR* function and calculate maximum total revenue as:

$TR = 50Q – 2Q^2$

$= 50(12.5) – 2(12.5)^2 = 312.5$

****Cubic functions**

A cubic function is expressed by a cubic equation which has the general form

$ax^3 + bx^2 + cx + d = 0$ a, b, c and d are constants

EX:- A firm's total cost function is given by the equation,

$TC = Q^3$ The demand function for the good is $P = 90 - Q$.

(a) Write down the equations for total revenue and profit. Calculate the break-even points.

(b) Graph the total revenue and total cost functions on the same diagram for $0 \leq Q \leq 12$, showing the break-even points.

(c) Estimate the total revenue and total costs at break-even.

(d) From the graph estimate the values of Q within which the firm makes

(i) a profit, (ii) a loss.

Sol:-

(a) $TR = PQ = (90 – Q)Q = 90Q – Q^2$.

$Profit(\pi) = TR – TC = 90Q – Q^2 – Q^3$

The break-even points occur when TR = TC, therefore, solve

$90Q – Q^2 = Q^3$

$0 = Q3 + Q2 – 90Q$

$0 = Q(Q^2 + Q – 90)$

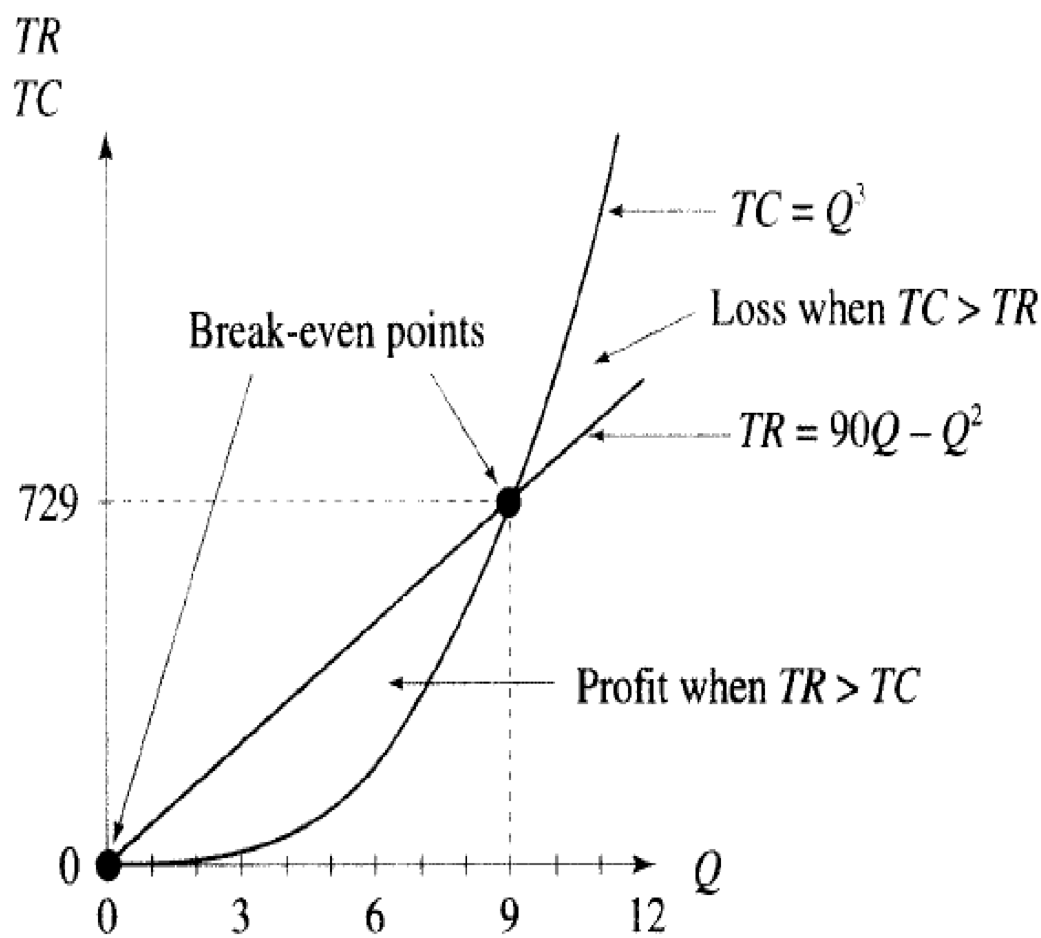
Therefore, Q = 0 and solving the quadratic Q2 + Q - 90 = 0 gives the solutions,

(Q+10) (Q-9)

Q = 9 and Q = -10.

(b)

Q	TR	TC
0	0	0
3	261	27
6	504	216
9	729	729
12	936	1728



(c) From the graph break-even is at Q = 0 and Q = 9.(Break-even at Q = –10 does not lie within the range of the graph. In fact, Q = –10 is not economically meaningful.)

At Q = 0, TR = TC = 0. At Q = 9, TR = TC = 729.

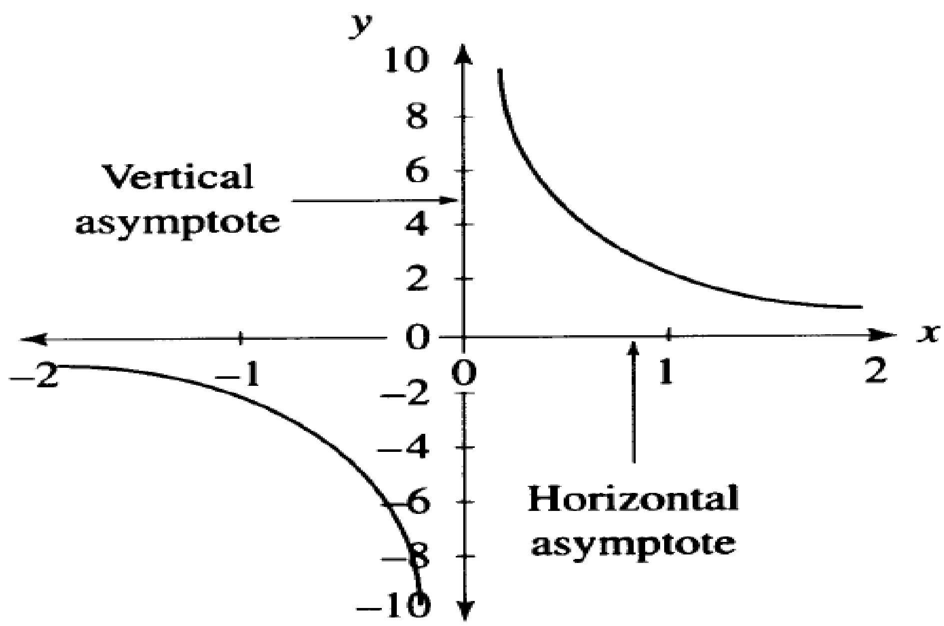
(d) The firm makes a profit when the TR curve is greater than the TC curve, that is between Q = 0 and Q = 9. When Q is greater than 9 the firm makes a loss.

Remark:- Functions of the form $a/(bx + c)$ model average cost, supply, demand and other functions which grow or decay at increasing rates.

the graph of the fun. Y= 1/x consists of two separate parts

To plot the graph, calculate a table of points such as next Table. Only one point presents a problem, 1 / 0. There is no number defined which gives a value for division by zero; no y-value can be defined for the point $x = 0$, so we cannot plot a point (x = 0, y = ?). Not only is there a point missing from the graph, but the shape of the graph increases or decreases dramatically on either side of the undefined point.

x	$y = \frac{1}{x}$
−2	−0.5
−1.5	−0.67
−1	−1
−0.5	−2
0	?
0.5	2
1	1
1.5	0.67
2	0.5



EX:- Sketch the functions $y = \frac{1}{x-0.23}$

Sol:-

Step 1: Calculate the value of x that gives rise to division by zero. This will determine the vertical asymptote.

X - 0.23 = 0

X = 0.23

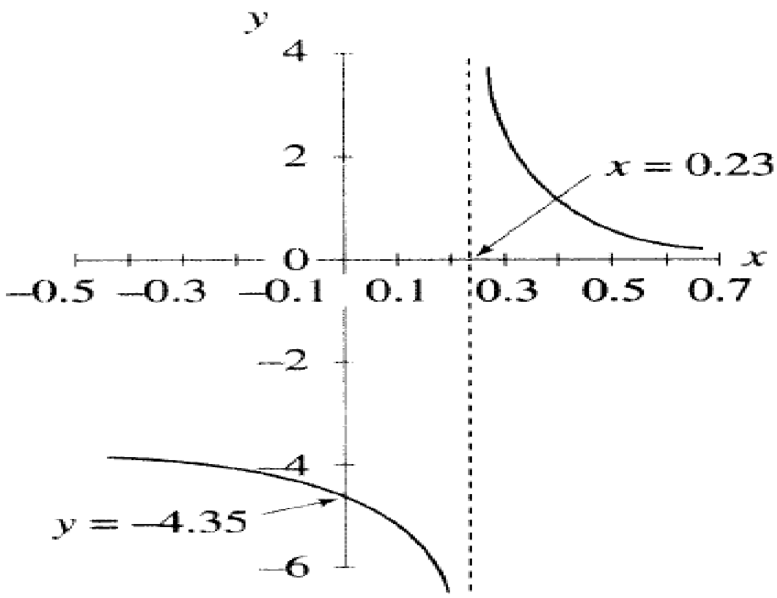
Step 2: Calculate the points of intersection with the y-axis, using the fact that x= 0 on the y-axis:

$$y = \frac{1}{x - 0.23}$$

$$y = \frac{1}{0 - 0.23} = -4.35$$

Step 3: To get some idea of the curvature as the graph approaches the vertical asymptote, calculate the coordinates of some points to its left and right:

x	0	0.1	0.2	0.3	0.4
y	−4.35	−0.13	−33.33	14.29	5.88



EX:- The demand function for a good is given by the equation $Q + 1 = \frac{200}{P}$ and the supply function is a linear function P = 5 + 0.5Q.

(a) Write the demand function in the form P = f(Q).

(b) Sketch the supply and demand functions on the same diagram for $-1 \leq Q \leq 20$. From the graph estimate the equilibrium point. Confirm your answer algebraically.

Sol:-

(a) $Q + 1 = \frac{200}{P}$

$P(Q + 1) = 200$

$P = \frac{200}{1+Q}$

(b) To sketch the graph, follow these steps:

- For $P = \frac{200}{1+Q}$, this function has a vertical a symptote at Q = -1
- The graph cuts the p-axis at Q = 0, therefore

$P = \frac{200}{1+0}$, P = 200

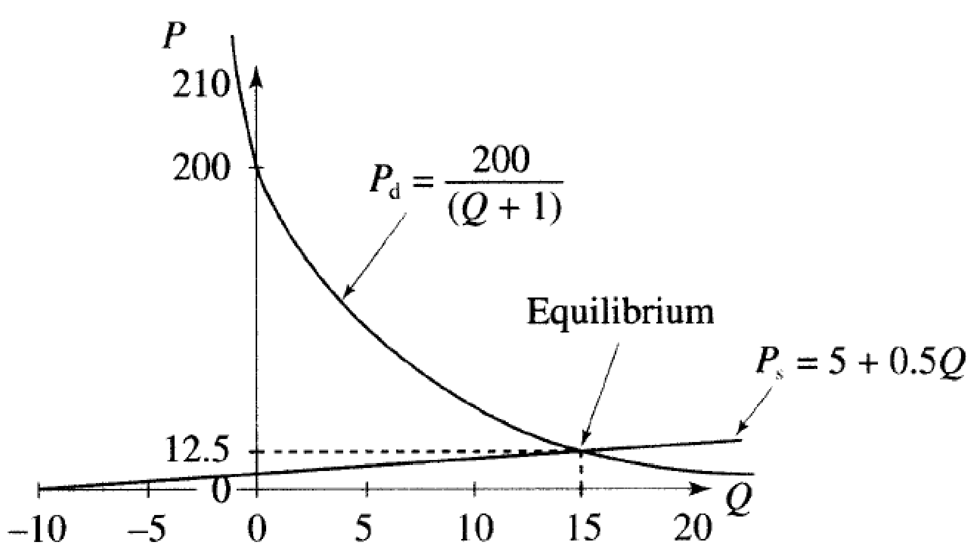
The horizontal axis is an asymptote. To show that the graph never cuts the horizontal axis as follows. Since $P = 0$ at every point on the horizontal axis (Q-axis), substitute $P = 0$ into the equation of the demand curve and find the value of Q at which the graph crosses the Q-axis, therefore

$$0 = \frac{200}{1+Q}$$

$$0(Q + 1) = 200$$

This equation has no solution hence, there is no point where the graph crosses the horizontal axis.

Now The supply function (P = 5 + 0.5Q) is a linear function with intercept 5 and slope 0.5. Since two points are sufficient to plot a straight line, we need one other point in addition to the intercept. Choose any value of Q then substitute this value into the equation of the supply function to calculate the corresponding P coordinate; for example, when Q = 4, P = 7



Algebraically, equilibrium exists when $P_d = P_s$ and $Q_d = Q_s$.

$= \frac{200}{1+Q} = 5 + 0.5Q$

$200 = (5+0.5Q)(Q+1)$

$200 = 0.5Q^2 + 5.5Q + 5$

$0 = 0.5Q^2 + 5.5Q - 195$

$(Q + 26)(Q - 15) = 0$

$Q = -26$ and $Q = 15$

The negative quantity is not economically meaningful, substitute Q=15 into either original equation and solve for the corresponding equilibrium price. Therefore market equilibrium is reached when Q=15 and P = 12.5.