Chapter -4-

Application for differential equations

Differential equations as mathematical models

Introduction

we introduce the notion of a differential equation as a mathematical model and discuss some specific models .

(1) Newton$n's$ law of cooling/warming

According to Newton$n's$ law empirical law of cooling /warming :

The rate at which the temperature of a body changes is proportional to the difference between the temperature of a body and the temperature of the surrounding medium the so-called ambient temperature.

Solution

T(t):the temperature of a body at time t

: the temperature of the surrounding mediumT_m

: The rate at which the temperature of the body changes$\frac{dT}{dt}$

The Newton$n's$ law of cooling /warming translates into the mathematical statement

$$\frac{dT}{dt} \propto T - T_m$$

$$\frac{dT}{dt} = k(T - T_m)$$

K is a constant of proportionality

$$\frac{dT}{k(T-T_m)} = dt$$

$$\frac{1}{k}\ln(T - T_m) = t + c$$

Is either case cooling or warming if T_m is a constant, it stands to reason that $k < 0$

(2) Population growth

The idea that the rate at which the population of a country growth at a certain time is proportional to the total population of the country at that time.

The more people there are at time t the more there are going to be in the future.

Solution

Let p(t): the total population at time t

Then this assumption can be expressed as:

$$\frac{dp}{dt} \propto p$$

$$\frac{dp}{dt} = kp$$

K is a constant of proportionality

$$\frac{dp}{p} = kdt$$

Lnp=kt+a

$$p = ce^{kt}$$

(3) Melting calf

If the average of melting calf is proportional to un equal calf. to find the follow over plus calf after t time where q_o is the calf in head start.

Solution

Let q :melting calf after time t

$$\frac{dq}{dt}: \text{the average of melting calf}$$

K>0 is a constant of proportionality

$$\frac{dq}{dt} \propto q$$

$$\frac{dq}{dt} = -kq$$

$$\frac{dq}{q} = -kdt$$

Lnq=-kt+c

Let q=q_o , $t = 0$

Ln$q_o = c$

Lnq=-kt+lnq_o

$$ln\frac{q}{q_o} = -kt$$

$$\frac{q}{q_o} = e^{-kt}$$

$$\therefore q = q_o e^{-\text{kt}}$$