Exercises

1- Show that:

a. $z = 0$ iff $Re(z) = 0$ and $Im(z) = 0$

b. $Re(iz) = -Im(z)$ and $Im(iz) = Re(z)$

c. The two numbers $z = 1 \pm i$ satisfies the equation $z^2 - 2z + 2 = 0$

d. $\overline{iz} = -i\bar{z}$

e. $\overline{(2-\imath)^2} = 3 - 4i$

4- Use -1=(-1,0) and z=(x,y) to show that (-1)z=-z

5- Reduce each the following to a real number:

a. $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$ $\quad b. (1-i)^4$

6- Find $|z|$ where:

a. z=3-4i

b. $z = -2 + \sqrt{12}\, i$

7- If z=x+iy then show that

a. $\frac{1}{z} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$

b. $\overline{iz} = -i\bar{z}$

8- Find the principal argument Arg z when $z = -i$.

9- Find $z_1 + z_2$ *and* $z_1 - z_2$ where:

a. $z_1 = \frac{2}{3} - i\,, z_2 = 2i$ $\quad b. z_1 = (-\sqrt{3}, 1), z_2 = (\sqrt{3}, 0)$

c. $z_1 = (-3,1), z_2 = (1,4)$

9- Find the following by using polar form

a. $(\sqrt{3} + \imath)^7$

b. $(-8i)^3$

c. $(2i)^2$

d. $(6i)^{1/4}$

Exercises

1- Find the domain for the following:

a. $f(z) = \frac{1}{z^2+1}$ b. $f(z) = \frac{z}{z+\bar{z}}$

2- If z=x+iy then write the function $f(z) = z^3 + z + 1$ in the form f(z)=u(x,y)+iv(x,y)

3- Find Limits for the following:

a. $\lim_{z\to 0} \frac{\frac{2}{z}+i}{\frac{1}{z}+1}$

b. $\lim_{z\to 1} \frac{z^2-1}{z-1}$

4- Find $f'(z)$ when:

a. $f(z) = \frac{1}{z}$ $b. f(z) = 3z^2 - 2z + 4$ $c. f(z) = (1 - 4z^2)^3$

5- Is $f'(z)$ exist by using Cauchy-Riemann equations:

a. $f(z) = |z|^2$

b. $f(z) = 2x + ixy^2$