**probability theory**

is a branch of mathematics that allows us to reason about events that are inherently random. Or Probability is simply a way of measuring how likely some event is to happen.

For example, you might try to define probability as follows: Suppose I perform an action that can produce one of n different possible random OUT COMES, each of which is equally likely. (For example, I flip a fair coin to produce one of OUTCOMES two outcomes: heads or tails.) Then, the probability of each of those outcomes is 1n. (So, 1/2 for heads or tails.) The problem with this definition is that it says each random outcome is "equally likely".

**statistical experiment**

is an action or STATISTICAL occurrence that can have multiple different outcomes, all of which can be specified in advance, but where the particular outcome that will occur cannot be specified in advance because it depends on random chance. Flipping a coin is statistical experiments.

**The SAMPLE SPACE**

The SAMPLE SPACE of a statistical experiment is the set of all possible outcomes (also SAMPLE SPACE known as SAMPLE POINTS).

**An EVENT**

is a subset of the sample space.

**Example**.1.

Imagine I flip a coin, with two possible outcomes: heads (H) or tails (T). What is the sample space for this experiment? What about for three flips in a row?

**Solution:**

For the first experiment (flip a coin once), the sample space is just HT . For the second experiment (flip a coin three times), the sample space is HHHHHTHTHHTTTHHTHTTTHTTT . Order matters: HHT is a different outcome than HTH.

**Example 2.**

For the experiment where I flip a coin three times in a row, consider the event that I get exactly one T. Which outcomes are in this event?

**Solution:**

The subset of the sample space that contains all outcomes with exactly one T is HHTHTHTHH .

**The probability of an event**

let's consider the probability of an event. By definition,

1- an impossible event has probability zero.
2- a certain event has probability one.
3- The more interesting cases are events that are neither impossible nor certain. For the moment, let's assume that all outcomes in the sample space S are equally likely. If that is the case, then the probability of an event E, which we write as P(E), is simply the number of outcomes in E divided by number of outcomes in S: P(E) = E/S if outcomes S are equally likely (1) That is, the probability of an event is the proportion of outcomes in the sample space that are also outcomes in that event.

**H.W.1**

Imagine I flip a fair coin, with two possible outcomes: heads (H) or tails (T). What is the probability that I get exactly one T if I flip the coin once? What if I flip it three times?

**H.W. 2.**

Suppose I have two bowls, each containing 100 balls numbered 1 through 100. I pick a ball at random from each bowl and look at the numbers on them. What is the probability that the numbers add up to 200?

**MUTUALLY EXCLUSIVE (or DISJOINT) events**.

Two events MUTUALLY are mutually exclusive iff they contain no outcomes in common (i.e., both events cannot occur at the same time). For example, if I roll two dice, the events "get a total of 7" and "get a total of 8" are mutually exclusive. On the other hand, "get a total of 7" and "get a 6 on one die" are not mutually exclusive, since both could occur on the same roll.

**CONDITIONAL PROBABILITY**

is one of the most important concepts of probability theory. CONDITIONAL A conditional probability expresses the probability that some event A will occur, given that (conditioned on the fact that) event B occurred. The conditional probability of A given B, written P(A|B), where the is pronounced "given", is defined as P(A|B)= P(A B)/ P(B)

**Counting Techniques** PINs or passwords are needed for many things and the rules for creating them are often complicated. Many passwords are required to contain at least 8 characters and at least 1 each of

- lower-case letter
- upper-case letter
- number
- punctuation mark

How much difference does adding character choices and length make to the number of possible passwords?

Counting methods are used to identify the number of possible outcomes of an experiment in multiple stages. They are important in the discussion of probability since, when outcomes are equally likely, we can find the probability of an event by dividing the number of outcomes within that event by the total number of possible outcomes.

Probability Given Equally Likely Outcomes

$$P(\text{event A}) = \frac{\text{The number of outcomes in A}}{\text{The total number of outcomes}}$$

Methods for counting are based 'the multiplication principle' which states that, to find the number of possible outcomes for an experiment that takes place in multiple stages, we multiply the number of possibilities at each stage.

## Basic Probability Rules

1) Possible values for probabilities range from 0 to 1

    0 = impossible event
    1 = certain event

2) The sum of all the probabilities for all possible outcomes is equal to 1.

    Note the connection to the complement rule.

3) Addition Rule - the probability that one or both events occur

    mutually exclusive events:

    P(A or B) = P(A) + P(B)
    not mutually exclusive events: P(A or B) = P(A) + P(B) - P(A and B)

4) Multiplication Rule - the probability that **both** events occur together

    independent events:

    P(A and B) = P(A) * P(B)

    P(A and B) = P(A) * P(B|A)

5) Conditional Probability - the probability of an event happening **given** that another event has already happened

    P(A|B) = P(A and B) / P(B)
    *Note the line | means "given" while the slash / means divide

**Independent Events** : the probability of one event does not change based on the outcome of the other event

Consider a basketball player shooting 2 free throws. If the player's probability of making the second shot changes based on whether or not they make the first shot, then these events are dependent. If the probability does not change, then they would be independent.

## Intersections & Unions

In basic probability, we often work with multiple events, not just one. To start this discussion, let's learn about unions and intersections. A union is indicated by the word "or". For example, what is the probability that Event A or Event B happens? That means that outcomes in either group would be desired. In the Venn Diagram below, that means anything in the colored section would be considered a **desired outcome**. Anything outside of the circles, but still inside the sample space would not be a desired outcome.

An intersection is indicated by the word "and". For example, what is the probability that Event A and Event B happen? This refers to outcomes that are part of both groups, but not just one group. In the Venn Diagram below, that means only the outcomes contained in the overlap of the two circles would be considered **desired outcomes**. Anything in the white, teal, or light purple spaces would not be desired outcomes.