(إحصاء 3)

متعدد المتغيرات

The Multivariate distribution:

1- The Trinomial dist.

The Binomial dist. can be generalized to the Trinomial dist., if n is a positive integer and P_1, P_2, P_3 are fixed constants, then:

$$f(x,y) = \begin{cases} \dfrac{n!}{x!\, y!\,(n-x-y)!} P_1^{x} P_2^{y} P_3^{n-x-y} & , x=y=0,1,2,\dots,n \\ 0 & , o/w \end{cases}$$

where x and y are nonnegative integers with $x+y \le n$, and P_1, P_2, P_3 are positive proper fractions with $P_1+P_2+P_3=1$.

Accordingly, $f(x,y)$ satisfies the conditions of being a joint p.d.f. of two r.vs. X and Y of the discrete type and said to have trinomial dist.

$$M_{x,y}(t_1,t_2) = \left(P_1e^{t_1} + P_2e^{t_2} + P_3\right)^n$$

for all real values of t_1 and t_2 the Marginal M.g.f. of X and Y are

$$M_X(t_1) = M_{x,y}(t_1,0) = \left[(1-P_1) + P_1e^{t_1}\right]^n$$

$$M_Y(t_2) = M_{x,y}(0,t_2) = \left[(1-P_2) + P_2e^{t_2}\right]^n$$

we see immediately, that X and Y are stochastically independent, addition, X is $b(n,P_1)$ and Y is $b(n,P_2)$

Accordingly, the Means and variance of X & Y are respectively:

$\mu_X = nP_1, \quad \mu_Y = nP_2$

$\sigma_X^2 = nP_1(1-P_1), \quad \sigma_Y^2 = nP_2(1-P_2)$

The correlation coefficient of X and Y are negative, we have:

$$\rho = -\sqrt{\frac{P_1 P_2}{(1-P_1)(1-P_2)}}$$

2) The bivariate normal distribution.

$$f(x,y) = \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1-\rho^2}} \operatorname{Exp}\left[\frac{-1}{2(1-\rho^2)}\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]$$

where $\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho$ are constants and parameters satisfies $-\infty < x < \infty$, $-\infty < y < \infty$, $-\infty < \mu_x < \infty$, $-\infty < \mu_y < \infty$

$\sigma_x > 0$, $\sigma_y > 0$, $-1 \le \rho \le 1$

$(x,y) \sim N(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$

The moment generating function of a bivariate normal dist. can determined as follows:-

$$\Gamma\tfrac{1}{2} = \sqrt{2} * \tfrac{1}{2}\sqrt{2\pi} = \sqrt{\pi}$$

~~Ex) Drive the M.g.f of~~

Ex) Find the value of C so that $f(x)$ is p.d.f. for following

H.W

1) $f(x) = \begin{cases} C x^2 (1-x)^4 & , 0 < x \leq 1 \\ 0 & , o/w \end{cases}$

2) $f(x) = \begin{cases} C e^{-\frac{(x-5)^2}{32}} & , -\infty < x < \infty \\ 0 & , o/w \end{cases}$

3) $f(x) = \begin{cases} C e^{-5x} x^4 & , x > 0 \\ 0 & , o/w \end{cases}$

4) $f(x) = \begin{cases} C x^5 e^{-\theta x} & , x > 0 \\ 0 & , o/w \end{cases}$

* Theorem: نظرية مهمة عن الهندسي

If X has the geometric dist. with Parameter p, then

$$P_r[X \geq i+j \mid X \geq i] = P_r[X \geq j] \quad , \quad i, j = 0, 1, 2, \ldots$$

Proof: ① الحل

$f(x,p) = \begin{cases} p q^x \\ 0 \end{cases}$

$$P_r[X \geq i+j \mid X \geq i] = \frac{P_r[X \geq i+j]}{P_r[X \geq i]}$$

$$= \frac{\sum_{x=i+j}^{\infty} p(1-p)^x}{\sum_{x=i}^{\infty} p(1-p)^x} = \frac{(1-p)^{i+j}}{(1-p)^i} = (1-p)^j$$

$$= P_r(X \geq j)$$

Ex) Let $X \sim b(2, P)$, $Y \sim b(4, P)$, if $P_r(X \geq 1) = \frac{5}{9}$

find $P_r(Y \geq 1)$.

Ex) The M.g.f. of X is $e^{4(e^t - 1)}$ find $P_r(X \leq \mu - 2\sigma)$

$$M_{x,y}(t_1, t_2) = Exp\left[\mu_x t_1 + \mu_y t_2 + \frac{\sigma_x^2 t_1^2 + 2\rho\sigma_x\sigma_y t_1 t_2 + \sigma_y^2 t_2^2}{2}\right]$$

It is interesting to note that if, in this M.g.f. $M_{x,y}(t_1, t_2)$, the Correlation Coefficient ρ is set equal to zero, then

$$M_{x,y}(t_1, t_2) = M_x(t_1, 0) \cdot M_y(0, t_2)$$

we note that:

$E(X) = \mu_x$, $Var(X) = \sigma_x^2$, $E(Y) = \mu_y$, $Var(Y) = \sigma_y^2$

$Cov(X, Y) = \rho\sigma_x\sigma_y$

Theorem: Suppose X and Y have a bivariate normal dist. with means μ_x, μ_y, positive variances σ_x^2, σ_y^2 and correlation coefficient ρ.

Then X and Y are stochastically independent iff $\rho = 0$

$f(x, y) = f(x) \cdot f(y)$

$P_r\left[-Y_n < \frac{Z_n}{n} - \theta\right]$

$$P_r\left[Y_n > \theta - \frac{Z_n}{n}\right] = \int_{\theta - \frac{Z_n}{n}}^{\theta} Y_n \, dy_n = \int_{\theta - \frac{Z_n}{n}}^{\theta} \frac{1}{\theta} \, dy_n$$

$$= \frac{1}{\theta}\left[Y_n\right]_{\theta - \frac{Z}{n}}^{\theta} = \frac{1}{\theta}\left[\theta - \right.$$

*Theorem) نظرية مهمة من التوزيع الهندسي

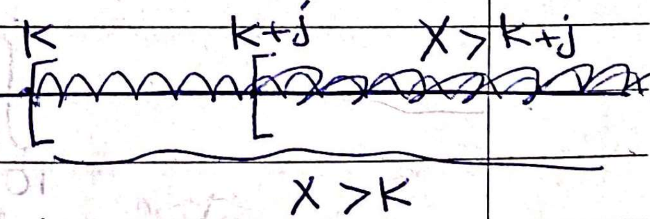
If X has a geometric dist. with Parameter p,
Then $P(X > k+j \mid X > k) = P(X > j)$ where $k, j \in I$

Proof:- (هام [illegible])

$$P(X > k+j) = \left[P(A^c)\right]^{k+j} = q^{k+j}$$

$$P(X > k) = \left[P(A^c)\right]^{k} = q^{k}$$

$$P(X > k+j \mid X > k) = \frac{P(AB)}{P(B)}$$



$AB: X > k+j$

$P(AB) = P(X > k+j) = q^{k+j}$

$B: X > k$

$P(B) = P(X > k) = q^{k}$

$$P(X > k+j \mid X > k) = \frac{q^{k+j}}{q^{k}} = q^{j} = P\left[(A^c)\right]^{j} = P(X > j)$$