1

Ex The frequency of a damped Harmonic oscillator is one-half the frequency of the same oscillator with no damping. find the ratio of maxima of successive oscillations.

The damping frequency W_d

frequency without damping is W_0

$$W_d = \frac{1}{2} W_0 \Rightarrow (W_0^2 - \gamma^2)^{1/2} \Rightarrow \frac{1}{2} W_0 \Rightarrow W_0^2 - \gamma^2 = \frac{1}{4} W_0^2$$

$$W_0^2 - \frac{1}{4} W_0^2 = \gamma^2 \Rightarrow \frac{3}{4} W_0^2 = \gamma^2 \Rightarrow \boxed{W_0 \sqrt{\frac{3}{4}} = \gamma}$$

The damping period $T_d = \frac{2\pi}{W_d} = \frac{2\pi}{\frac{1}{2} W_0} = \frac{4\pi}{W_0}$

$x(t) = A e^{-\gamma T_d}$ معادلة الحركة في حالة damping $\Rightarrow \gamma T_d = \frac{4\pi}{W_0} \cdot \sqrt{\frac{3}{4}} W_0$

$\therefore \gamma T_d = 10.88$

$$x(t) = A e^{-10.88} \Rightarrow \frac{x(t)}{A} = e^{-10.88}$$

$\therefore \frac{x(t)}{A} = 0.00002$ maxima of successive oscillations

Forced Harmonic motion (Resonance) ②

we consider the motion of a damped Harmonic oscillator That is driven by external force F_{ext}. The Total applied force is then the sum of all three forces.

1 restoring force $-Kx$

2 the viscous damping force $-c\dot{x}$

3 driving force $F_{ext} \Rightarrow F_o e^{i\omega t}$

$$m\ddot{x} = -Kx - c\dot{x} + F_{ext} \Rightarrow m\ddot{x} + C\dot{x} + Kx = F_o e^{i\omega t} \quad \text{--- (1)}$$

$$x(t) = A e^{i(\omega t - \varphi)}$$ نفرض حل المعادلة

$$x(t) = A e^{i\omega t} \cdot e^{-i\varphi} \Rightarrow \dot{x}(t) = \frac{dx}{dt} = i\omega A e^{i\omega t} \cdot e^{-i\varphi}$$

$$\frac{d^2x}{dt^2} = \ddot{x}(t) = -\omega^2 A e^{i\omega t} \cdot e^{-i\varphi}$$

نعوض في معادلة (1)

A: is the amplitude

φ = phase difference.

$$m\frac{d^2x}{dt^2} = -\omega^2 A e^{i\omega t} \cdot e^{-i\varphi} = \ddot{x}$$

$$\dot{x} = i\omega A e^{i\omega t} \cdot e^{-i\varphi}$$

$$(-m\omega^2 A + i\omega cA + KA) e^{-i\varphi} \cdot e^{i\omega t} = f_o e^{i\omega t}$$

$$A[-m\omega^2 + i\omega C + K] = f_o e^{i\varphi} \qquad e^{i\varphi} = \cos\varphi + i\sin\varphi$$

$$A[-m\omega^2 + i\omega c + k] = f_o[\cos\varphi + i\sin\varphi]$$

نساوي الحقيقي مع الحقيقي والتخيلي مع التخيلي

$$A[-m\omega^2 + K] = f_o\cos\varphi \quad \text{---- (2)}$$

$$A\cancel{i}\omega c = f_o \cancel{i}\sin\varphi \Rightarrow A\omega C = f_o \sin\varphi \quad \text{--- (3)}$$

نقسم (3) على (2)

$$\frac{A\omega c}{A[-m\omega^2 + K]} = \frac{f_o\sin\varphi}{f_o\cos\varphi} \Rightarrow \tan\varphi = \frac{c\omega}{[K - m\omega^2]}$$

$$\boxed{\tan\varphi = \frac{c\omega}{K - m\omega^2}} \Rightarrow$$

If we squar both side of equations ③ and ② we have ③

$$A^2(K-m\omega^2)^2 = f_0^2 \cos^2\varphi$$

$$c^2\omega^2A^2 = f_0^2 \sin^2\varphi \quad +$$

$$A^2(K-m\omega^2)^2 + c^2\omega^2A^2 = f_0^2\left[\underbrace{\cos^2\varphi + \sin^2\varphi}_{1}\right]$$

$$A^2\left[(K-m\omega^2)^2 + c^2\omega^2\right] = f_0^2$$

$$A^2 = \frac{f_0^2}{(K-m\omega^2)^2 + c^2\omega^2} \Longrightarrow A = \frac{f_0}{((K-m\omega^2)^2 + c^2\omega^2)^{1/2}}$$

$\because \omega_0^2 = \frac{K}{m}$, $\gamma = \frac{c}{2m}$

$$\because \tan\varphi = \frac{c\omega}{K-m\omega^2} \quad \div m \Longrightarrow \frac{c\omega/m}{\frac{K}{m} - \frac{m\omega^2}{m}}$$

$$2\gamma = \frac{c}{m}, \; \frac{K}{m} = \omega_0^2 \Longrightarrow \tan\varphi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}$$

$$A(\omega) = \frac{F_0/m}{((\omega_0^2-\omega^2)^2 + 4\gamma^2\omega^2)^{1/2}}$$

ω_r = Resonance frequency $\Longrightarrow$ $\omega_r = \frac{dA(\omega)}{d\omega} = 0$

$$\omega_r = \frac{d}{d\omega}\left[F_0/m\left((\omega_0^2-\omega^2)^2 + 4\gamma^2\omega^2\right)\right]^{-3/4}$$

$$0 = \underbrace{-\frac{f_0}{m}}_{4}\left[\frac{1}{2}\left[(\omega_0^2-\omega^2)^2 + 4\gamma^2\omega^2\right] * \left[2(\omega_0^2-\omega^2)(-2\omega) + 8\gamma^2\omega\right]\right]$$

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$$0 = \frac{-f_0\,(-4\omega(\omega_0^2-\omega^2) + 8\gamma^2\omega]}{2m\left[(\omega_0^2-\omega^2)^2 + 4\gamma^2\omega^2\right]^{3/4}} = -4\omega(\omega_0^2-\omega^2) + 8\gamma^2\omega$$

$\therefore -4\omega(\omega_0^2 - \omega^2) + 8\gamma^2\omega = 0 \div -4\omega$

$\omega_0^2 - \omega^2 - 2\gamma^2 = 0$

$\omega_0^2 - 2\gamma^2 = \omega^2 \qquad \therefore \omega_r = \omega_0^2 - 2\gamma^2$

$\therefore \omega_r = \sqrt{\omega_0^2 - 2\gamma^2}$