Motion of partical

$$\vec{F} = \sum_i \vec{F_i} = m \frac{d^2\vec{r}}{dt^2} = m\vec{a}$$

F_i is one of the exerted on the aparticle with mass(m), $\vec{a}$ is the acceleration of particle

RectiLinear (motion in one dimension)

$F_x = m\ddot{x}$ قوانين الحركه الخطيه بتعجيل ثابت

constant acceleration

$$a = \ddot{x} = \frac{dv}{dt} = \int_{t_0}^{t} a\,dt = \int_{v_0}^{v} dv$$

$$a(t - t_0) = v - v_0 \Rightarrow v = at + v_0 \text{ —— } (1)$$

$$\frac{dx}{dt} = at + v_0 \Rightarrow dx = at\,dt + v_0 dt$$

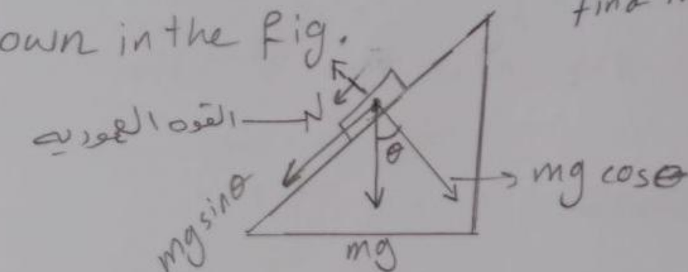
$$\therefore \int_{x_0}^{x} dx = \int_0^t at\,dt + \int_0^t v_0\,dt \Rightarrow x - x_0 = \frac{at^2}{2} + v_0 t \quad \cdots (2)$$

$$a = \ddot{x} = \frac{dv}{dt} \Rightarrow \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} \cdot v$$

$$a = \frac{dv}{dx} \cdot v \quad \Rightarrow a\,dx = dv \cdot v \Rightarrow \int_{x_0}^{x} a\,dx = \int_{v_0}^{v} v\,dv$$

$$a(x - x_0) = \left.\frac{v^2}{2}\right]_{v_0}^{v} \Rightarrow a(x - x_0) = \frac{v^2}{2} - \frac{v_0^2}{2}$$

Ex/ consider aparticle is sliding down asmooth plane inclined at an angle θ to the Horizontal as shown in the fig. find the acceleration



نستخدم قانون نيوتن الثاني

$F = ma$

$F = m\ddot{x} \Rightarrow \ddot{x} = \frac{F}{m} \Rightarrow$

القوة التي تسبب النزول هي

على المسار محصلة القوى على X هي

$F = mg\sin\theta$

$mg\sin\theta$

$\therefore \ddot{x} = \frac{\cancel{m}g\sin\theta}{\cancel{m}}$

$\ddot{x} = g\sin\theta$

قوة الاحتكاك هي

$f = \mu N$

μ = coefficient of Kinetic friction

$N = mg\cos\theta$ القوة العمودية

$f = \mu mg\cos\theta$ قوة الاحتكاك

$\sum F_x = mg\sin\theta - \mu mg\cos\theta$

$\therefore \ddot{x} = \frac{\sum F_x}{m} = g(\sin\theta - \mu\cos\theta)$

Force that depend on the Position
(Concepts of Kinetic and potential energy)

$$F(x) = m\ddot{x} \Rightarrow m\frac{dx}{dt}\cdot\frac{dv}{dx} \Rightarrow mv\frac{dv}{dx}$$

$$F(x) = m\frac{1}{2}\frac{dv^2}{dx} \qquad \boxed{\frac{dv^2}{dx} = 2v\frac{dv}{dx}}$$

$T = \frac{1}{2}mv^2$ Kinetic energy

$$F(x) = \frac{dT}{dx} \Rightarrow \int_{x_0}^{x} F(x)\,dx = \int_{T_0}^{T} dT \quad --- (1)$$

The work down by Impressed force on particle is equal to the change in the Kinetic energy of the Particle.

$$\Rightarrow F(x) = \frac{-dU(x)}{dx} \qquad U(x) \Rightarrow \text{Potential energy}$$

$$\int_{x_0}^{x} F(x)\,dx = \int_{U_0}^{U} -dU(x) \quad --- (2)$$

نعوض في معادلة (1)

$$-\int_{U_0}^{U} dU(x) = \int_{T_0}^{T} dT \Rightarrow -(U - U_0) = T - T_0$$

$$-U + U_0 = T - T_0 \Rightarrow \quad T + U = U_0 + T_0 \quad --- (3)$$

قانون حفظ الطاقة

equation (3) called the energy equation.
This mean's that for one dimensional motion If the Impressed force is a function of position only. the sum of Kinetic and potential energy remains constant

and the force is said to be conservative. ④

$$F = -\frac{du(x)}{dx}$$ conservative force.

$u = mgx$ =>

Ex: Free fall [the motion of freely falling Body]

A body projected upward with intial speed V_o From the origion $x=0$. Find the max hight

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mV_o^2 + mgh_o \to 0$$

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mV_o^2 \qquad \times 2 \quad \div m$$

$v^2 + 2gh = V_o^2$ => at the turning point $V=0$

$2gh = V_o^2$ => $2g X_{max} = V_o^2$

$$X_{max} = \frac{V_o^2}{2g}$$

The force as a function of time

The Concept of Impulse.

$F(t) = m\frac{dv}{dt}$ The force acting on a particle is Function of time.

① constant force

$\frac{F(t)}{m} = \frac{dv}{dt} \Rightarrow \int_{v_0}^{v} dv = \frac{F(t)}{m}\int_0^t dt$

$V - V_0 = \frac{F}{m}t \Rightarrow V = V_0 + \frac{F}{m}t$

$\frac{dx}{dt} = V_0 + \frac{F}{m}t \Rightarrow dx = V_0\,dt + \frac{F}{m}t$

$\int_{x_0}^{x} dx = \int_0^t V_0\,dt + \frac{F}{m}\int_0^t dt \Rightarrow x - x_0 = V_0 t + \frac{F}{2m}t^2$

step force: suppose a body of mass m is subject to a constant force F_1 that acts for a certian Length of tim t_1 and then the force suddenly changes to different constant value f_2 $\Rightarrow$ $\frac{F(t)}{m} = \frac{dV}{dt}$ نفس الخطوات السابقة

$V = V_0 + \frac{F_1 t_1}{m}$ $X = x_0 + V_0 t_1 + \frac{1}{2m}f_1 t_1^2$

$X = X_0 + V_0 t_1 + \frac{f_1 t_1^2}{2m} + V_0 t + \frac{F_2 t_2^2}{2m}$

(3) uniformaly Increasing force

$$m\frac{dv}{dt} = F(t)$$

$F(t)$ = aconstant Increasing force $F(t) = ct$

$$m\frac{dv}{dt} = ct \qquad dv = \frac{ct}{m}dt$$

$$V = \int_0^t \frac{ct}{m}dt = \frac{ct^2}{2m}$$

$$\frac{dx}{dt} = \frac{ct^2}{2m} \Rightarrow \int_{x_0}^{x} dx = \int_0^t \frac{ct^2}{2m}dt$$

$$x - x_0 = \frac{ct^3}{6m} \Rightarrow x = x_0 + \frac{ct^3}{6m}$$