Velocity dependent force ①

Fluid Resistance and terminal velocity

$F = F_o + F(v) = m\frac{dv}{dt}$

$F(v)$ = part of the force that depend on velocity

F_o = part of the force doesn't depend on the velocity

The force that act on the body is a function of the velocity of the body.

In the case of viscous resistance exerted on a body moving through a fluid. the force is function of velocity only.

$\therefore F(v) = -c_1 v - c_2 v|v|$

c_1 and c_2 are constant

$-c_1 v$ = we have Linear case.

$-c_2 v|v|$ = Qudratic case.

Ex/ Horizontal motion with Linear resistance

suppose a block is projected with intial velocity V_o on a smooth horizontal surface and there is air resistance such that the Linear term dominates. Find the displacement of Block.

$F_o = 0 \quad , \quad F(v) = -cv$

$$ma = m\frac{dv}{dt} = -cv = m\frac{dv}{dt}$$

②

$$-c_1 v\, dt = m\, dv \Rightarrow -c_1\, dt = m\int_{v_0}^{v} \frac{dv}{v}$$

$$= -ct = m\left[\ln v - \ln v_0\right]$$

$$-ct = m\ln\frac{v}{v_0} \Rightarrow \frac{-ct}{m} = \ln\frac{v}{v_0}$$

$$e^{-\frac{ct}{m}} = \frac{v}{v_0} \Rightarrow v = v_0 e^{-\frac{ct}{m}}$$

$$\frac{dx}{dt} = v_0 e^{-\frac{ct}{m}} \Rightarrow dx = v_0 e^{-\frac{ct}{m}} dt$$

$$\int_0^x dx = v_0\int_0^t e^{-\frac{ct}{m}} dt \Rightarrow x = v_0 \left.\frac{e^{-\frac{ct}{m}}}{-\frac{c}{m}}\right]_0^t$$

$$x = -\frac{v_0 m}{c}\left[e^{-\frac{ct}{m}} - 1\right]$$

$$= \frac{v_0 m}{c} - e^{-\frac{ct}{m}}$$

Vertical fall through a fluid
Terminal velocity.

③

ⓐ Linear Case: For an object falling vertically in a resisting fluid the force f_o is the weight of the object $(-mg)$. Find the terminal velocity

$$f_o = -mg$$

$$F_o + f(v) = f \Rightarrow ma = -mg - c_1 V$$

$$m\frac{dv}{dt} = -mg - c_1 V \quad \text{(Linear case for fluid)}$$

$$\int_0^t dt = \int_{v_0}^{v} \frac{m\,dv}{-mg - cV} \Rightarrow t = \int_{v_0}^{v} \frac{m\,dv}{-mg - c_1 V}$$

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$$u = -mg - c_1 V \qquad du = 0 - c_1 dV$$

تحتاج نضرب $\frac{-c_1}{-c_1}$

$$t = -m\frac{c_1}{-c_1}\int_{v_0}^{v} \frac{dv}{-(mg + c_1 V)} \Rightarrow -\frac{m c_1}{c_1}\int_{v_0}^{v} \frac{dv}{mg + c_1 V}$$

$$t = \frac{-m}{c_1}\int_{v_0}^{v} \frac{c_1\,dv}{mg + c_1 V} \Rightarrow t = -\frac{m}{c}\left[\ln\left[mg + c_1 V\right] - \ln\left[mg + c_1 V_0\right]\right]$$

$$t = \frac{-m}{c_1}\left[\ln(mg + c_1V) - \ln(mg + c_1V_0)\right]$$

$$t = \frac{-m}{c_1}\ln\frac{mg + c_1V}{mg + c_1V_0} \Rightarrow \frac{-t\,c_1}{m} = \ln\frac{mg + c_1V}{mg + c_1V_0}$$

$$\Rightarrow e^{-\frac{t c_1}{m}} = \frac{mg + c_1V}{mg + c_1V_0}$$

$$(mg + c_1V_0)\,e^{-\frac{t c_1}{m}} = mg + c_1V$$

$$(mg + c_1V_0)\,e^{-\frac{t c_1}{m}} - mg = c_1V \qquad \div c_1$$

$$\left(\frac{mg}{c_1} + V_0\right)e^{-\frac{t c_1}{m}} - \frac{mg}{c_1} = V$$

Ex

Find the velocity $\dot{x}$ and the position x as function of the time t for aparticle of mass m which starts from rest at $x=0$ and $t=0$. subjected to the following force functions:-

(a) $F_x = F_0 + ct$

(b) $F_x = F_0 \sin ct$

(c) $F_x = F_0 e^{ct}$

(c) $m\ddot{x} = f_0 e^{ct}$

$$\frac{dV}{dt} = \frac{f_0}{m} e^{ct} \Rightarrow dV = \frac{f_0}{m} e^{ct} dt$$

$$\int_0^v dV = \frac{f_0}{m} \int_0^t e^{ct} dt \Rightarrow V = \frac{f_0}{m} \left[\frac{e^{ct}}{c} \right]_0^t$$

$$v = \frac{f_0}{mc} \left[e^{ct} - 1 \right]$$

$$\frac{dx}{dt} = \frac{f_0}{mc} \left[e^{ct} - 1 \right] \Rightarrow \int_0^x dx = \int_0^t \frac{f_0}{mc} \left[e^{ct} - 1 \right] dt$$

$$X = \frac{f_0}{mc} \left[\left(\frac{e^{ct}}{c} \right)_0^t - t \right]$$

$$= \frac{f_0}{mc} \left[\frac{e^{ct}}{c} - \frac{1}{c} - t \right]$$