

$\therefore$  the center is  $(h, k) = (6, -4)$

$$a^2 = 36 \Rightarrow a = 6, (6, 0), (-6, 0)$$

$$b^2 = 16 \Rightarrow b = 4, (0, 4), (0, -4)$$

$$c = \sqrt{a^2 - b^2} = 2\sqrt{5}, \text{ foci is } (6 + 2\sqrt{5}, -4), (6 - 2\sqrt{5}, -4)$$

the length of semimajor is  $2a = 12$

the length of semiminor is  $2b = 8$

Ex: Find the equation of Ellipse, Vertices, foci, Semimajor semiminor, center, length of major, length of minor

$$1. \frac{x^2}{64} + \frac{y^2}{100} = 1$$

$$2. \frac{x^2}{81} + \frac{y^2}{36} = 1$$

$$3. 4x^2 + 25y^2 = 100$$

#### 4. Hy Perabola.

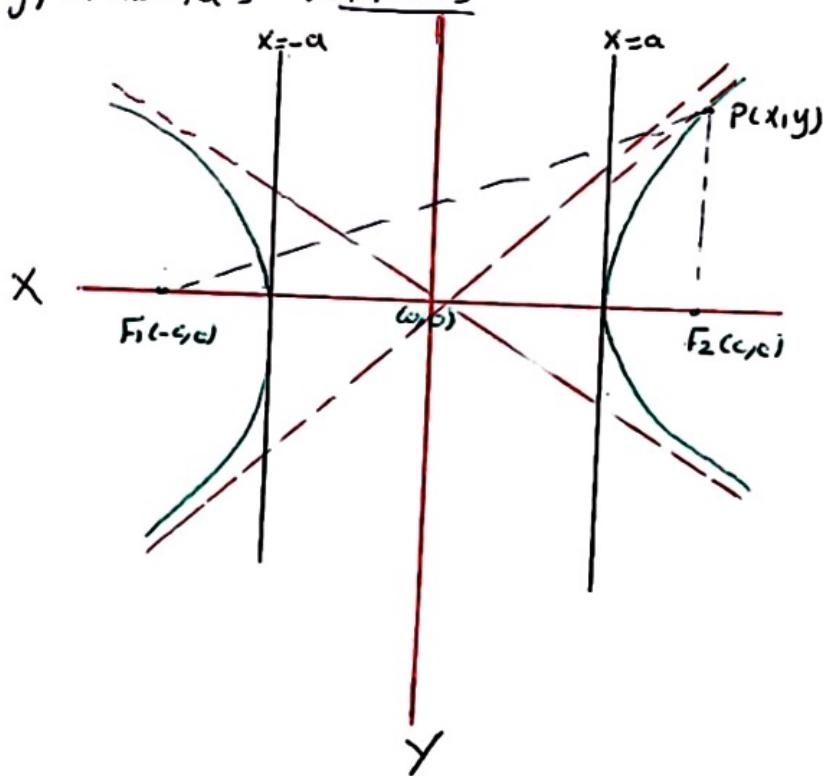
Def: A Hy Perabola is a set of Points in a plane whose distances from two fixed Points in the Plane have a constant difference.

The Two fixed Points are the foci of the hyperabola.

The line through the foci of a Hy Perabola is the focal axis.

The point on this line halfway between the foci is the hyperabola's

Center. The Points where the hyperabola and focal axis cross are the hyperabola's vertices.



If the foci are  $F_1(-c, 0)$ ,  $F_2(c, 0)$  and the constant difference is  $2a$ , then a point  $(x, y)$  lies on the hyperabola if and only if

$$PF_1 - PF_2 = 2a$$

$$\frac{x^2}{a^2} - \frac{y^2}{c^2 - a^2} = 1, \quad c > a, \quad \text{let } c^2 - a^2 = b^2$$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , the equation of hyperbola where the foci belong to the x-axis

**Remark:**

1- The standard equations for hyperbolas centered at the origin.

A. Focus on the x-axis:  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

- Center to focus distance  $c = \sqrt{a^2 + b^2}$

- Foci:  $(c, 0), (-c, 0)$

- Vertices:  $(a, 0), (-a, 0)$

- Asymptotes: we can find the asymptotes by the following:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0 \Rightarrow \frac{y^2}{b^2} = \frac{x^2}{a^2} \Rightarrow \boxed{y = \pm \frac{b}{a} x}$$
 the equation of asymptotes

B. focus on the y-axis:  $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

- Center to foci distance:  $c = \sqrt{a^2 + b^2}$

- Foci:  $(0, c), (0, -c)$

- Vertices:  $(0, a), (0, -a)$

- Asymptotes: we can find the asymptotes by the following

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 0 \Rightarrow \frac{y^2}{a^2} = \frac{x^2}{b^2} \Rightarrow \boxed{y = \pm \frac{a}{b} x}$$

← the standard form equation for hyperbolas centered at the point  $(h, k)$

A- Focus on the x-axis:  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

- Foci:  $(h+c, k), (h-c, k)$

- Asymptotes:  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 0 \Rightarrow \frac{(y-k)^2}{b^2} = \frac{(x-h)^2}{a^2}$

$$\Rightarrow \boxed{\frac{y-k}{b} = \pm \frac{x-h}{a}}$$

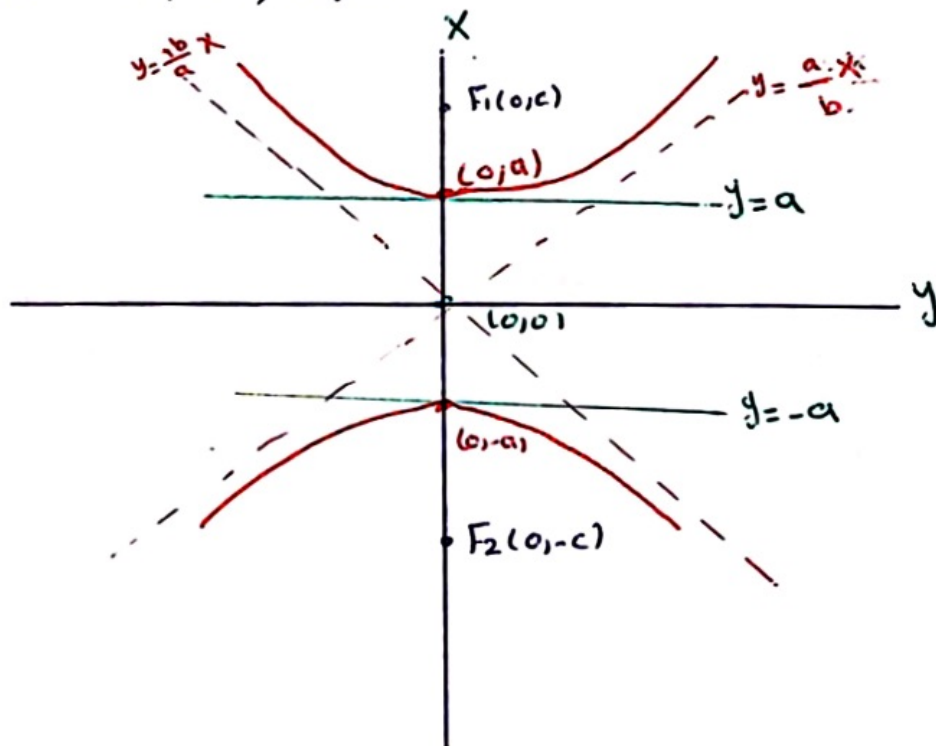
- Vertices:  $(a, 0), (-a, 0)$

B- Focus on the y-axis:  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

- Foci:  $(h, k+c), (h, k-c)$

- Asymptotes:  $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 0 \Rightarrow \boxed{\frac{y-k}{a} = \pm \frac{x-h}{b}}$

- Vertices:  $(0, a), (0, -a)$



Example:  $\frac{x^2}{4} - \frac{y^2}{5} = 1$ , The equation of hyperbola where the foci in x-axis.

Sol.

with  $a^2=4$ ,  $b^2=5$ , and Center  $(0,0)$ , we have:

$$a^2=4 \Rightarrow a = \pm 2, (2,0), (-2,0)$$

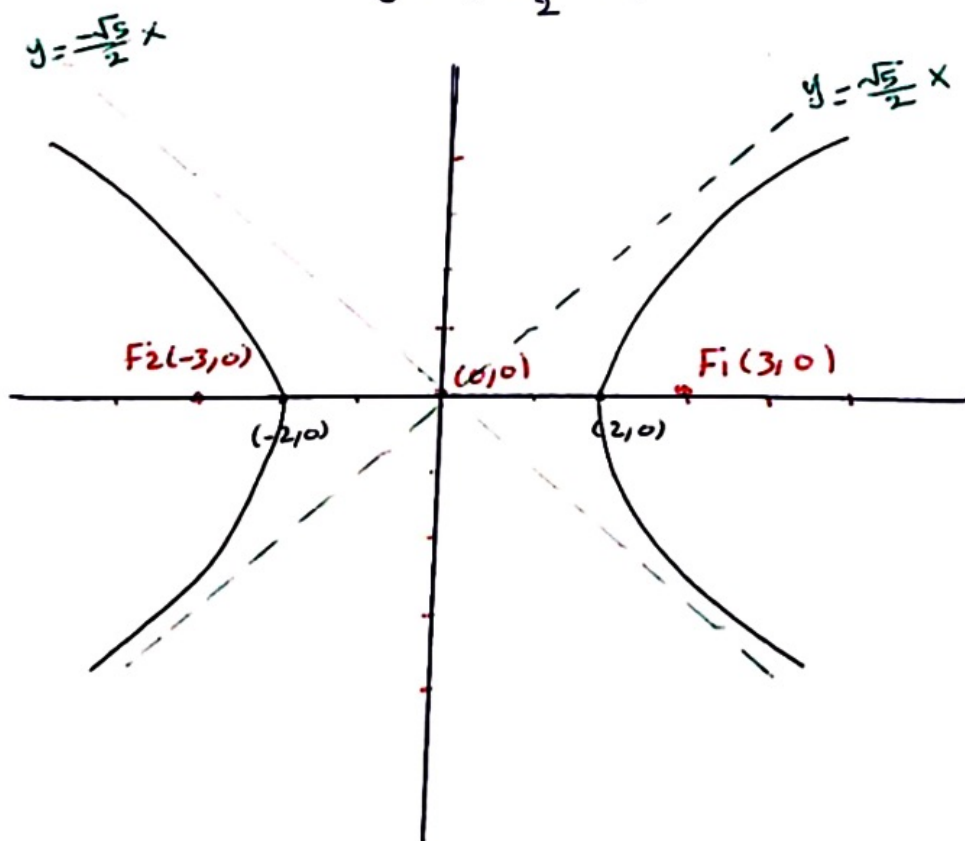
$$b^2=5 \Rightarrow b = \pm \sqrt{5}$$

$$c = \sqrt{a^2+b^2} = \sqrt{4+5} = \sqrt{9} = \pm 3$$

The foci:  $(3,0), (-3,0)$

The Asymptotes eq.  $y = \pm \frac{b}{a} x$

$$y = \pm \frac{\sqrt{5}}{2} x$$





represented a hyperbola, find the center, vertices, and asymptotes.

Sol  $(x^2 - 4y^2 + 2x + 8y - 7 = 0$

$$(x^2 + 2x) - (4y^2 - 8y) - 7 = 0$$

$$(x^2 + 2x + 1 - 1) - 4(y^2 - 2y + 1 - 1) - 7 = 0$$

$$(x+1)^2 - 1 - 4(y-1)^2 + 4 - 7 = 0$$

$$(x+1)^2 - 4(y-1)^2 - 4 = 0$$

$$(x+1)^2 - 4(y-1)^2 = 4 \quad ] \div 4$$

$$\frac{(x+1)^2}{4} - \frac{(y-1)^2}{1} = 1$$

Ex: Find the center and vertices and foci, asymptotes of the equation:  $9y^2 - 16x^2 = 144$

Sol:

$$9y^2 - 16x^2 = 144 \quad \div 144$$

$$\frac{y^2}{16} - \frac{x^2}{9} = 1, \text{ the eq. of hyperbola with origin}$$

Center on y-axis.

$$\text{Vertices: } a^2 = 16 \Rightarrow a = \pm 4, (0, 4), (0, -4)$$

$$b^2 = 9 \Rightarrow b = \pm 3$$

$$c = \sqrt{16+9} = \sqrt{25} = \pm 5, \text{ foci is } (0, 5), (0, -5)$$

$$\text{asymptotes: } y = \mp \frac{a}{b} x$$

$$y = \mp \frac{4}{3} x$$

H-w: Find the foci, center, vertices, and asymptotes of the following eq.

$$1. \frac{x^2}{9} - \frac{y^2}{4} = 1$$

$$2. \frac{y^2}{16} - x^2 = 1$$

$$3. x^2 - y^2 = 1$$

$$4. \frac{(x+2)^2}{9} - \frac{(y-1)^2}{16} = 1.$$