Ex.3: The Point $(+1,-2)$ is belong to the Parabola $x^2=4Py$ find the focus and directrix.

Sol. $x^2 = 4Py$, $\because (x,y)=(1,-2)$

the Parabola lie in y-axis (Positive)

$\therefore 1^2 = 4P(-2)$

$1 = -8P \Rightarrow P = -1/8$

The focus $F(0,-1/8)$, directrix is $y = 1/8$

Ex.4: Find the equation of Parabola if the vertix is $(-1,4)$ and Pass through $(\underset{\bar{x}}{4},\underset{\bar{y}}{-2})$ directrix this Parabola is parallel the y-axis

Sol: $\because$ The vertix is $(\underset{h}{-1},\underset{k}{4})$, from remark 2, to get:

$(y-k)^2 = 4P(x-h)$

$\therefore (-2-4)^2 = 4P(4+1)$

$(-6)^2 = 4P(5)$

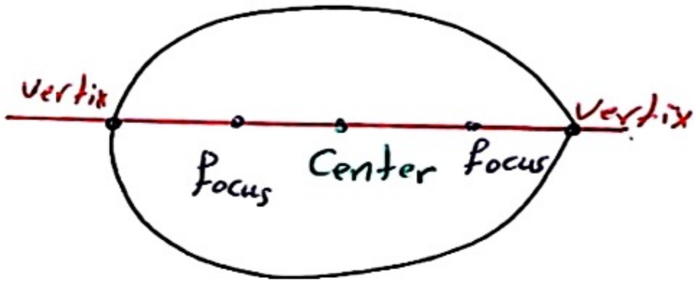
$36 = 20P \Rightarrow P = \frac{36}{20}$

$\therefore (y-k)^2 = 4P(x-h)$

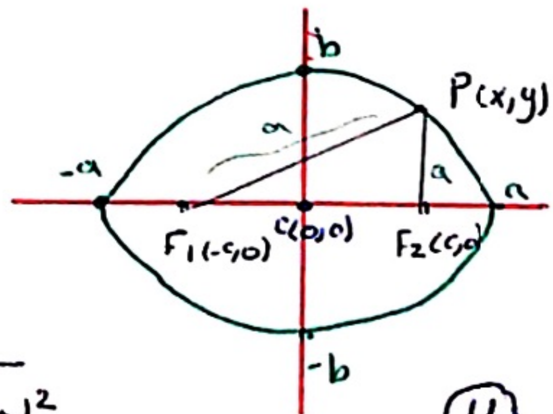
$(y-4)^2 = \cancel{4}^{1} \times \frac{36}{\cancel{20}_{5}}(x+1)$

$(y-4)^2 = \frac{36}{5}(x+1)$ the Parabola eq. where vertix is $(-1,4)$.

focal axis. The point of this line halfway between the foci is the ellipses center, the points where the focal axis and the ellipse cross are the ellipses vertices



to find the equation of ellipse, suppose that, the two fixed points are $F_1(c,0)$, $F_2(-c,0)$ and the sum of the distances $PF_1 + PF_2$ is denoted by $2a$, the coordinates of a point P on the ellipse satisfy:



$$PF_1 + PF_2 = 2a$$

$$\sqrt{(x+c)^2+(y-0)^2} + \sqrt{(x-c)^2+(y-0)^2} = 2a$$

or

$$\sqrt{(x+c)^2+(y-0)^2} = 2a - \sqrt{(x-c)^2+(y-0)^2}$$

Sequare two side and simplified, to get

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1$$, let $a^2 - c^2 = b^2$, to get

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

Remark :-

1. The standard form equation for ellipses Centered at the origin

A- Foci on the x-axis:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad , \quad a > b$$

- Center to focus distance $c = \sqrt{a^2 - b^2}$
- foci = $(\mp c, 0)$
- vertices = $(\mp a, 0)$

B- Foci on the y-axis:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad , \quad a < b$$

- Center to focus distance $c = \sqrt{a^2 - b^2}$
- Foci = $(0, \mp c)$
- Vertices = $(0, \mp a)$

In each cases, a is the Semimajor axis and b is the Seminnor axis.

2- The standard form equation for ellipses centered at the Point (h,k)

A- foci on the x-axis

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \quad , a>b$$

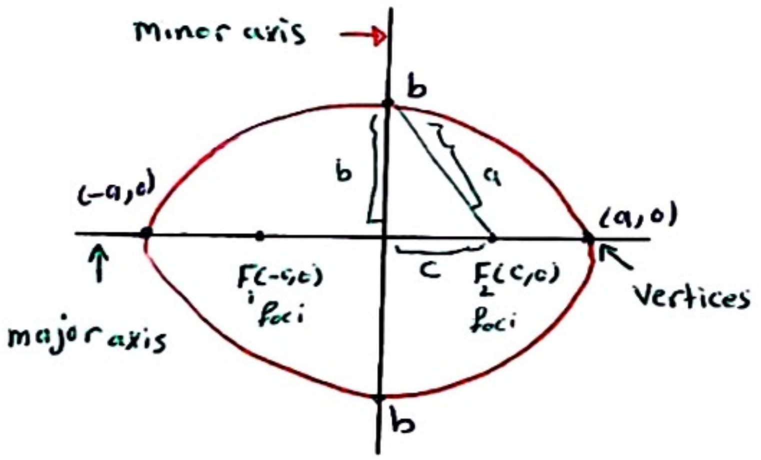
-the foci is: $(h+c,k)$, $(h-c,k)$

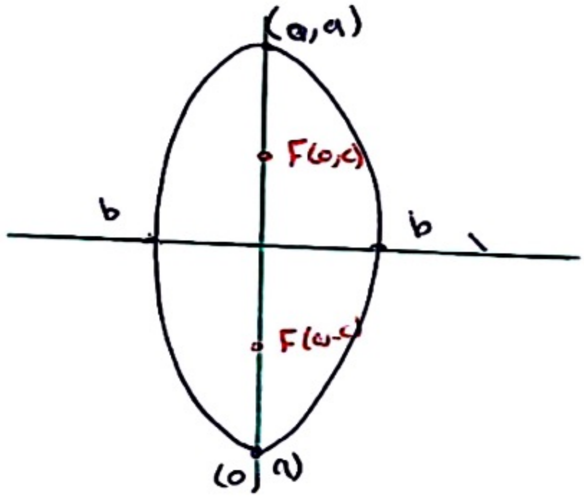
B- foci on the y-axis

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1 \quad , a<b$$

the foci is: $(h,k+c)$, $(h,k-c)$

The Length of Major axis $2a$, the length of minor axis is $2b$.





the standard form of ellipses equation on x-axis

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

the standard form of of ellipses equation on y-axis

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

and foci, and the Center to the following ellipses equation.

1. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Sol

The semimajor axis: $a = \sqrt{16} = 4$,

The semi minor axis: $b = \sqrt{9} = 3$

The center of the focus: $c = \sqrt{a^2 - b^2} = \sqrt{16-9} = \sqrt{7}$

$\therefore$ the foci is: $(\sqrt{7}, 0), (-\sqrt{7}, 0)$

The length of major axis $= 2a = 8$

The length of minor axis $= 2b = 6$, The vertices is $(4,0), (-4,0)$

2. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

Sol

The semi major axis: $a = \sqrt{25} = 5$

The semi ~~maj~~ minor axis : $b = \sqrt{9} = 3$

The center of the focus: $c = \sqrt{a^2 - b^2} = \sqrt{25-9} = \sqrt{16} = 4$

$\therefore$ The foci is $(0,4), (0,-4)$

The vertices is $(0,5), (0,-5)$

The length of major axis $= 2a = 10$

the length of minor axis $= 2b = 6$

Ex: find the equation of ellipse where the center is $(0,0)$ and $(0,3)$ is focus and the half length of the major axis is 5.

Sol

the focus is $(0,3)$, $c=3$

the half length of the major axis is 5 .i.e. $a=5$

$\therefore$ vertix is $(0,5)$, $(0,-5)$

$$c=\sqrt{a^2-b^2} \Rightarrow c^2=a^2-b^2$$

$$9=25-b^2 \Rightarrow b^2=25-9=16$$

$\therefore b=4$, $(4,0)$, $(-4,0)$

$$\therefore \frac{x^2}{16}+\frac{y^2}{25}=1$$

Ex: find the equation of ellipse

$$4x^2+9y^2-48x+72y+144=0$$

Sol

$$(4x^2-48x)+(9y^2+72y)+144=0$$

$$4(x^2-12x)+9(y^2+8y)+144=0$$

$$4(x^2-12x+36-36)+9(y^2+8y+16-16)+144=0$$

$$4(x^2-12x+36)-144+9(y^2+8y+16)\cancel{-144}\cancel{+144}=0$$

$$4(x-6)^2+9(y+4)^2=144 \quad]\div 144$$

$$\frac{(x-6)^2}{36}+\frac{(y+4)^2}{16}=1$$