1-3. Classifying Conic Sections by Eccentricity.

Def:

1. The eccentricity of Parabola is $e = 1$.

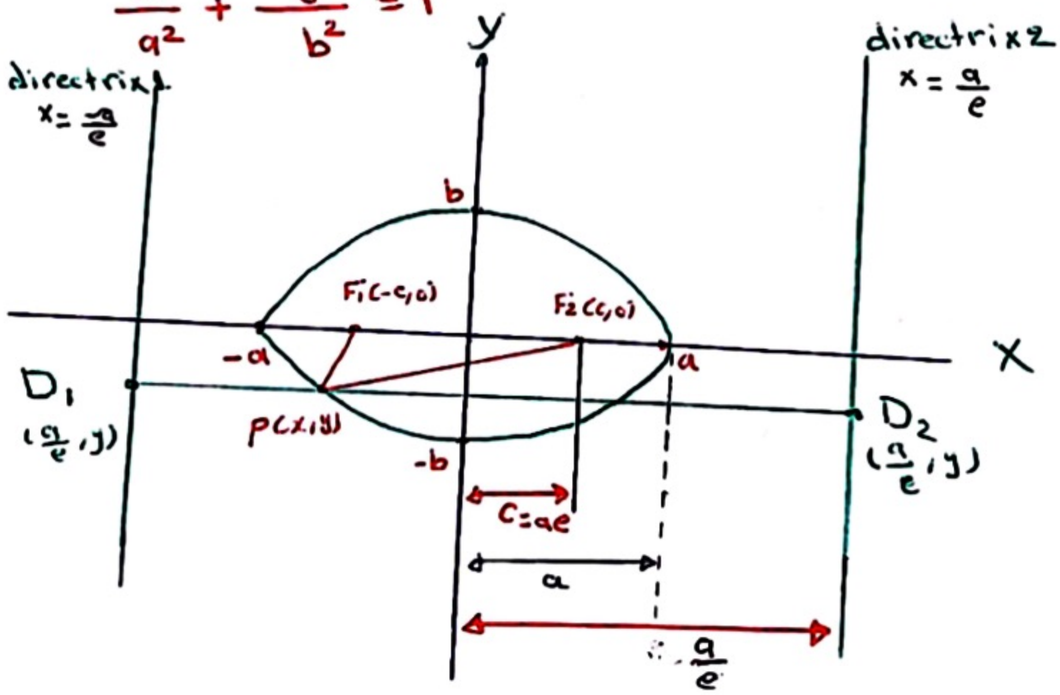
2. The eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $(a > b)$ is the number $e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} < 1$, $0 < e < 1$.

3. The eccentricity of the hyparabola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the number $e = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a}$, $e > 1$.

$$\text{Eccentricity} = \frac{\text{Distance between foci}}{\text{Distance between vertices}} = \frac{2c}{2a} = \frac{c}{a}$$

- The eccentricity in ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



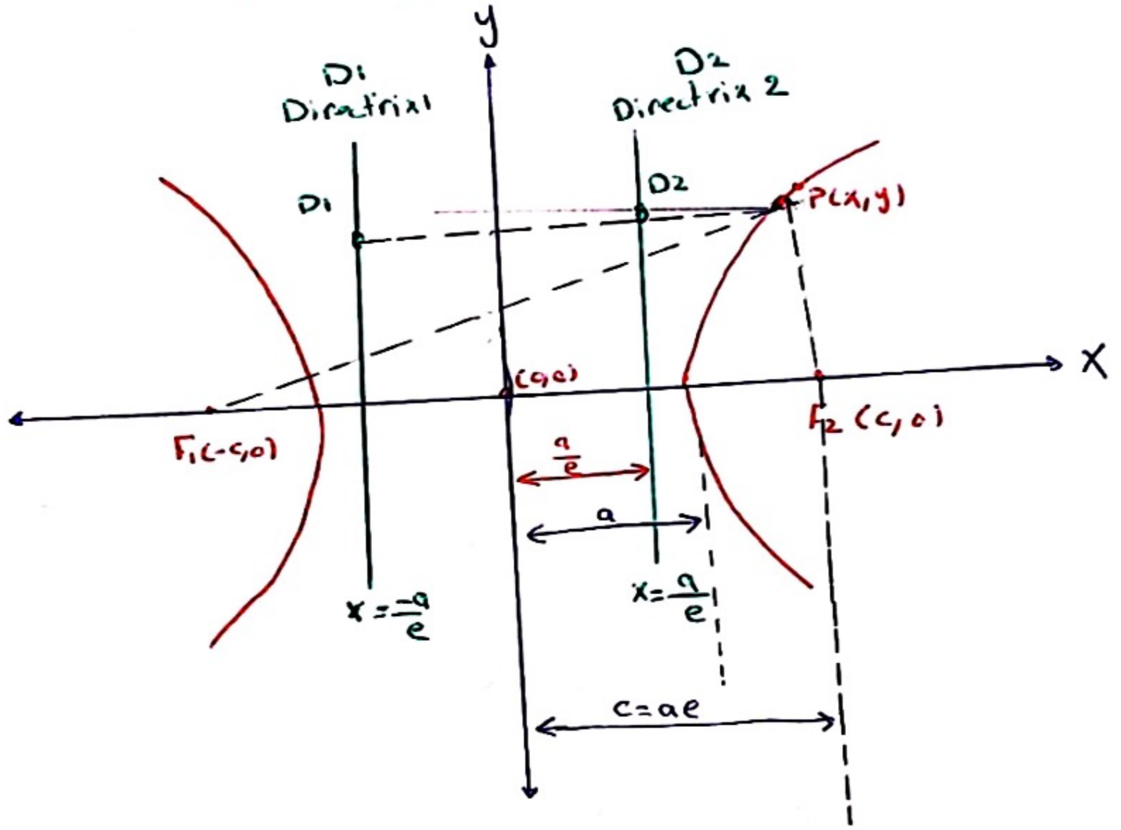
$PF_1 = e\,PD_1$, $PF_2 = e\,PD_2$

where F_1, F_2 are Focus , P is a Point belong to ellipse

D_1 is a point belong to directrix $x = \frac{-a}{e}$

D_2 is a point belong to directrix $x = \frac{a}{e}$

- The eccentricity in hyperabola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



$PF_1 = e\,PD_1$, $PF_2 = e\,PD_2$

where F_1, F_2 are Foci , $e = \frac{c}{a}$ is eccentricity ,

P is a point belong to hyperabola , D_1 is a point in directrix $x = \frac{-a}{e}$, D_2 is a point belong to directrix $x = \frac{a}{e}$

Ex: Find the eccentricity of the hyperabola

$$9x^2 - 16y^2 = 144.$$

sol:

$$9x^2 - 16y^2 = 144 \quad] \div 144$$

$$\frac{9x^2}{144} - \frac{16y^2}{144} = 1 \Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$$

with $a^2 = 16 \Rightarrow a = 4$, $(4,0), (-4,0)$ is the vertices

and $b^2 = 9 \Rightarrow b = \pm 3$

The foci $c = \sqrt{a^2 + b^2} = \sqrt{16+9} = \sqrt{25} = 5$

$$e = \frac{c}{a} = \frac{5}{4}.$$

H.W: Find the eccentricity and directrix for:

1. $2x^2 - y^2 = 6$

2. $144y^2 - 25x^2 = 3600$

1.2. General Quadratic Equation and Rotations

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Theorem: General Quadratic Equation

Consider a second degree equation:

$$\boxed{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0} \quad \cdots (*)$$

The general quadratic eq.

where A, B, C, D, E, F are coefficients then the discriminant at eq. (*) is $\boxed{B^2 - 4AC}$

1. if $B^2 - 4AC = 0$ The eq (*) represented Perabola.
2. if $B^2 - 4AC < 0$ The eq (*) represented ellipse.
3. if $B^2 - 4AC > 0$ The eq (*) represented hyperabola.

Ex: show that the following equation represented Parabola, ellipse, hyPerabola.

1. $2x^2 + 3xy + 2y^2 - x - 6 = 0$

sol:

Comparing with: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

then $A = 2$, $B = 3$, $C = 2$, $D = -1$, $E = -6$, $F = 0$

$$B^2 - 4AC = (3)^2 - 4 * 2 * 2$$
$$= 9 - 16 = -7 < 0$$

this equation represented ellipse eq.

2. $x^2+6xy+y^2-1=0$

Sol: Comparing with $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=1$, $B=6$, $C=1$, $D=0$, $E=0$, $F=-1$

discriminant is $B^2-4AC = 36-4*1*1 = 32 > 0$

This eq. represented by hy perabola eq.

3. $x^2-2xy+y^2+x-y+3=0$

Sol:

Comparing with $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$

$A=1$, $B=-2$, $C=1$, $D=1$, $E=-1$, $F=3$

discriminant is $B^2-4AC=4-4=0$

This eq. represented by perabola eq.

H-W:

show that the following equation represented Parabola, ellipse, and hy perabola:

1. $3x^2-9xy+2y^2+2=0$

2. $4x^2-2xy+y^2-5x-1=0$

3. $x^2+2xy+y^2+x-y=0$