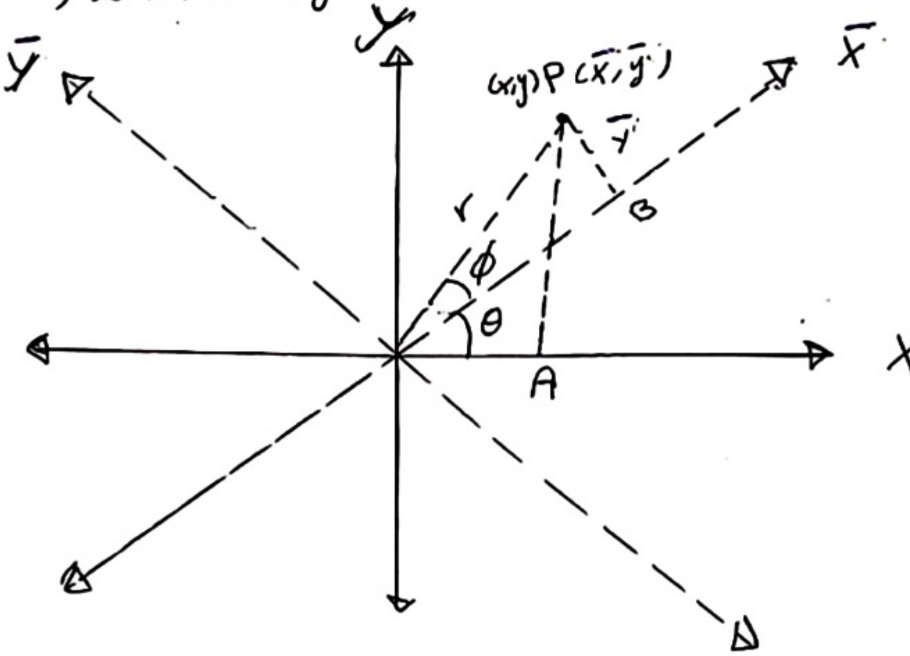
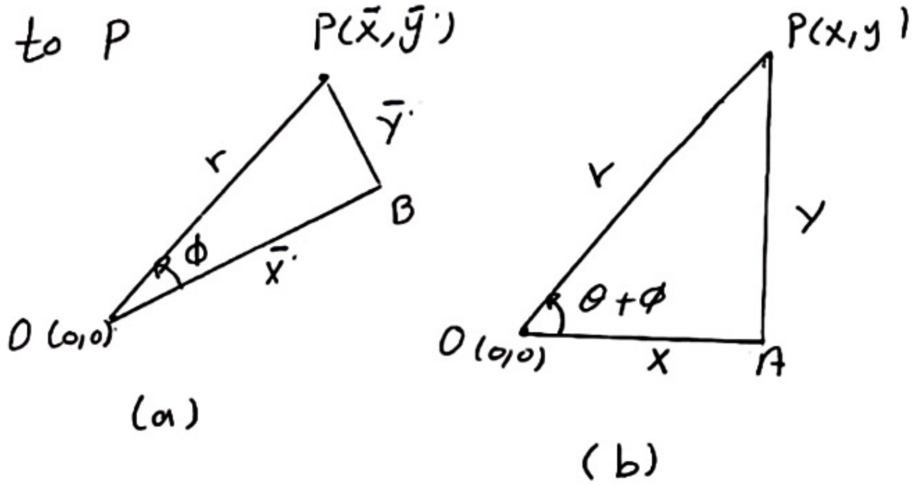
- Rotation Quadratic Equation. دوران المعادلات التربيعية

If a rectangular xy- Coordinate system is a rotated through an angle θ to form an $\bar{x}\bar{y}$-Coordinate system, then a point $P(x,y)$ will have Coordinates $P(\bar{x},\bar{y})$ in the new system, where (x,y) and $(\bar{x},\bar{y})$ related by:



First introduce a new pair of variables r and ϕ, represented respectively, the distance from P to origin and the angle fromed by the $\bar{x}$-axis and the line Connecting the origin to P



Ex: suppose that axis xy- Coordinate system are related through an angle of $\theta = 45^\circ$ to obtain an. $\bar{x}\bar{y}$- Coordinate system. Find the equation of the Curve $x^2 - xy + y^2 - 6 = 0$ in $\bar{x}\bar{y}$ coordinate.

Sol

$$x = \bar{x} \cos \frac{\pi}{4} - \bar{y} \sin \frac{\pi}{4} = \frac{\bar{x}}{\sqrt{2}} - \frac{\bar{y}}{\sqrt{2}}$$

$$y = \bar{x} \sin \frac{\pi}{4} + \bar{y} \cos \frac{\pi}{4} = \bar{x} \cdot \frac{1}{\sqrt{2}} + \bar{y} \cdot \frac{1}{\sqrt{2}} = \frac{\bar{x}}{\sqrt{2}} + \frac{\bar{y}}{\sqrt{2}}$$

substituting thes expressions in to the eq. $x^2 - xy + y^2 - 6 = 0$

$$\left(\frac{\bar{x}}{\sqrt{2}} - \frac{\bar{y}}{\sqrt{2}}\right)^2 - \left(\frac{\bar{x}}{\sqrt{2}} - \frac{\bar{y}}{\sqrt{2}}\right)\left(\frac{\bar{x}}{\sqrt{2}} + \frac{\bar{y}}{\sqrt{2}}\right) + \left(\frac{\bar{x}}{\sqrt{2}} + \frac{\bar{y}}{\sqrt{2}}\right)^2 - 6 = 0$$

$$\frac{\bar{x}^2}{2} - \cancel{2} \cdot \frac{\bar{x}\bar{y}}{\cancel{2}} + \frac{\bar{y}^2}{2} - \left(\frac{\bar{x}^2}{2} + \cancel{\frac{\bar{x}\bar{y}}{2}} - \cancel{\frac{\bar{x}\bar{y}}{2}} - \frac{\bar{y}^2}{2}\right) + \frac{\bar{x}^2}{2} + \cancel{2}\frac{\bar{x}\bar{y}}{\cancel{2}} + \frac{\bar{y}^2}{2} - 6 = 0$$

$$\cancel{\frac{\bar{x}^2}{2}} - \cancel{\bar{x}\bar{y}} + \frac{\bar{y}^2}{2} - \cancel{\frac{\bar{x}^2}{2}} + \frac{\bar{y}^2}{2} + \frac{\bar{x}^2}{2} + \cancel{\bar{x}\bar{y}} + \frac{\bar{y}^2}{2} - 6 = 0$$

$$\frac{\bar{x}^2}{2} + \frac{3\bar{y}^2}{2} = 6 \quad] \div 6$$

$$\boxed{\frac{\bar{x}^2}{12} + \frac{\bar{y}^2}{4} = 1}$$ the ellipse equation

From fig.(a) the triangle OBP, we see that:

$$\boxed{\bar{x} = r\cos\phi} \quad , \text{ and } \quad \boxed{\bar{y} = r\sin\phi}$$

and From fig(b) the triangle OAP, we see that:

$x = r\cos(\phi+\theta)$ and $y = r\sin(\phi+\theta)$

$x = r\cos(\phi+\theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta$

$y = r\sin(\phi+\theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta$

since $\bar{x} = r\cos\phi$ and $\bar{y} = r\sin\phi$, we have the following result:

$$\left\{\begin{array}{lll} x = \bar{x}\cos\theta - \bar{y}'\sin\theta & \text{and} & y = \bar{x}\sin\theta + \bar{y}\cos\theta \\ \text{and} & & \bar{y} = -x\sin\theta + y\cos\theta \\ \bar{x} = x\cos\theta + y\sin\theta & \text{and} & \end{array}\right\}$$

Ex: suppose that axis xy- Coordinate system are related through an angle of $\theta = 45^\circ$ to obtain an. $\bar{x}\bar{y}$- Coordinate system. Find the equation of the Curve $x^2 - xy + y^2 - 6 = 0$ in $\bar{x}\bar{y}$ coordinate.

<u>sol</u>

$$x = \bar{x}\cos\frac{\pi}{4} - \bar{y}\sin\frac{\pi}{4} = \frac{\bar{x}}{\sqrt{2}} - \frac{\bar{y}}{\sqrt{2}}$$

$$y = \bar{x}\sin\frac{\pi}{4} + \bar{y}\cos\frac{\pi}{4} = \bar{x}\cdot\frac{1}{\sqrt{2}} + \bar{y}\cdot\frac{1}{\sqrt{2}} = \frac{\bar{x}}{\sqrt{2}} + \frac{\bar{y}}{\sqrt{2}}$$

substituting thes expressions in to the eq. $x^2 - xy + y^2 - 6 = 0$

$$\left(\frac{\bar{x}}{\sqrt{2}} - \frac{\bar{y}}{\sqrt{2}}\right)^2 - \left(\frac{\bar{x}}{\sqrt{2}} - \frac{\bar{y}}{\sqrt{2}}\right)\left(\frac{\bar{x}}{\sqrt{2}} + \frac{\bar{y}}{\sqrt{2}}\right) + \left(\frac{\bar{x}}{\sqrt{2}} + \frac{\bar{y}}{\sqrt{2}}\right)^2 - 6 = 0$$

$$\frac{\bar{x}^2}{2} - \cancel{2}\cdot\frac{\bar{x}\bar{y}}{\cancel{2}} + \frac{\bar{y}^2}{2} - \left(\frac{\bar{x}^2}{2} + \cancel{\frac{\bar{x}\bar{y}}{2}} - \cancel{\frac{\bar{x}\bar{y}}{2}} - \frac{\bar{y}^2}{2}\right) + \frac{\bar{x}^2}{2} + \cancel{2}\frac{\bar{x}\bar{y}}{\cancel{2}} + \frac{\bar{y}^2}{2} - 6 = 0$$

$$\cancel{\frac{\bar{x}^2}{2}} - \cancel{\bar{x}\bar{y}} + \frac{\bar{y}^2}{2} - \cancel{\frac{\bar{x}^2}{2}} + \frac{\bar{y}^2}{2} + \frac{\bar{x}^2}{2} + \cancel{\bar{x}\bar{y}} + \frac{\bar{y}^2}{2} - 6 = 0$$

$$\frac{\bar{x}^2}{2} + \frac{3\bar{y}^2}{2} = 6 \quad] \div 6$$

$$\boxed{\frac{\bar{x}^2}{12} + \frac{\bar{y}^2}{4} = 1}$$ the ellipse equation

Ex: Find the new coordinates of the point (2,4) if the coordinate axes are retated through the angle $\theta = 30^\circ$.

Sol.

$\bar{x} = x\cos\theta + y\sin\theta = 2\cos 30 + 4\sin 30$

$= \cancel{2} * \frac{\sqrt{3}}{\cancel{2}} + 4 * \frac{1}{2} = 2 + \sqrt{3}$

$\bar{y} = -x\sin\theta + y\cos\theta = -2 * \sin 30 + 4 * \cos 30$

$= \frac{-2}{2} + 4 * \frac{\sqrt{3}}{2} = -1 + 2\sqrt{2}$

$\therefore (2+\sqrt{3}, -1+2\sqrt{2})$ is a point in $\bar{x}\bar{y}$ - coordinate system

H.w: Show that the graph of the equation $xy = 1$ is a Hyperabola by rolated the xy - axes through an angle of $\pi/4$.

If we apply the rotation equations

$$x = \bar{x}\cos\theta - \bar{y}\sin\theta$$

$$y = \bar{x}\sin\theta + \bar{y}\cos\theta$$

to the general quadratic equation (*), we obtain a new quadratic equation. $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 \quad \ldots (A)$

$$A(\bar{x}\cos\theta - \bar{y}\sin\theta)^2 + B(\bar{x}\cos\theta - \bar{y}\sin\theta)(\bar{x}\sin\theta + \bar{y}\cos\theta)$$

$$+ C(\bar{x}\sin\theta + \bar{y}\cos\theta)^2 + D(\bar{x}\cos\theta - \bar{y}\sin\theta) + E(\bar{x}\sin\theta +$$

$$\bar{y}(\cos\theta) + F = 0$$

then

$$\bar{x}^2\left(A(\cos\theta)^2 + B(\sin\theta\cos\theta) + C(\sin\theta)^2\right) +$$

$$\bar{x}\bar{y}\left[-2A\cos\theta\sin\theta + B((\cos\theta)^2 - (\sin\theta)^2) + 2C\sin\theta\cos\theta\right]$$

$$+ \bar{y}^2\left(A(\sin\theta)^2 - B(\sin\theta\cos\theta) + C(\cos\theta)^2\right) +$$

$$\bar{x}(D\cos\theta + E\sin\theta) + \bar{y}(-D\sin\theta + E\cos\theta) + F = 0$$

The new coefficients are related to the old ones by the equations:

$$\bar{A} = A\cos^2\theta + B\cos\theta\sin\theta + C\sin^2\theta$$

$$\bar{B} = B\cos 2\theta + (C-A)\sin 2\theta$$

$$\bar{C} = A\sin^2\theta - B\sin\theta\cos\theta + C\cos^2\theta$$

$$\bar{D} = D\cos\theta + E\sin\theta$$

$$\bar{E} = -D\sin\theta + E\cos\theta \qquad \cdots\cdots (7)$$

$$\bar{F} = F$$

To find θ, Put $\bar{B}=0$ in the second equation in (7) and

Solve the resulting equation

$B\cos 2\theta + (C-A)\sin 2\theta = 0$, Then find θ from:

$$\left.\begin{aligned} &\operatorname{Cot} 2\theta = \frac{A-C}{B} \quad \text{or} \quad \tan 2\theta = \frac{B}{A-C} \\ &\text{or} \quad \frac{\cos 2\theta}{\sin 2\theta} = \frac{A-C}{B} \end{aligned}\right\} \cdots (**)$$