Coulomb's Law قانون كولوم

In 1785, Coulomb established the fundamental law of *electric force* . between two stationary, charged particles. Experiments show that an electric force has the following properties:

في عام 1785 اجرى العالم كولوم مجموعة تجارب من اجل وضع القانون الاساسي للقوة الكهربائية بين

(1) The force is *inversely proportional* to the square of separation, *r2*, شحنتين ساكنتين حيث استنتج ثلاث نقاط : between the two charged particles.

$$F \propto \frac{1}{r^2}$$

(2) The force is *proportional* to the product of charge *q* \1 \ and the charge *q* \2 \ on the particles.

$$F \propto q_1 q_2$$

(3) The force is *attractive* if the charges are of opposite sign and *repulsive* if the charges have the same sign.

We conclude that the electric force between two electric charges is directly proportional to the product of the product the two charges are inversely with the square of the distance between them

نستنج ان القوة الكهربائية بين شحنتين كهربائيتين تتناسب طرديا مع حاصل ضرب الشحنتين وعكسيا مع مربع المسافة بينهما

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\therefore F = K \frac{q_1 q_2}{r^2}$$

where K is the coulomb constant = 9×10^9 P N.m^2 P/CP^2P.

The above equation is called *Coulomb's law*, which is used to calculate the force between electric charges. In that equation F is measured in Newton (N), q is measured in unit of coulomb (C) and r in meter (m).

The constant K can be written as where

$$K = \frac{1}{4\pi\varepsilon_o}$$

where ε_o is known as the *Permittivity constant of free space.*

$\varepsilon_o = 8.85 \times 10^{-12} C^2/N.m^2$

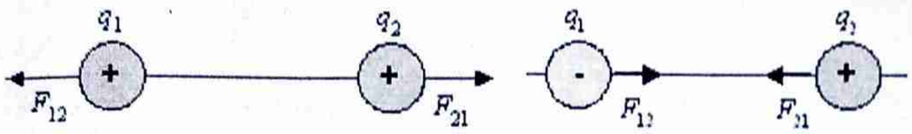
$$K = \frac{1}{4\pi\varepsilon_o} = \frac{1}{4\pi \times 8.85 \times 10^{-12}} = 9 \times 10^9 N m^2 / C^2$$

Calculation of the electric force

Electrical forces are caused by the effect of one charge on another, or from the effect of a particular distribution of several charges on For example, to calculate the electrical force affecting that charge, q1, we follow the following steps

القوى الكهربائية تكون ناتجة من تأثير شحنة على شحنة أخرى أو من تأثير توزيع معين لعدة شحنات على شحنة معينة q_1 على سبيل المثال، ولحساب القوة الكهرابائية المؤثرة على تلك الشحنة نتبع الخطوات التالية:-

Electric force between two electric charges



(a) (b)

The amount of electric force $$F_{12} = K\frac{q_1 q_2}{r^2} = F_{21}$$

And its direction $$\bar{F}_{12} = -\bar{F}_{21}$$

That is, the two forces are equal in magnitude and opposite in direction.

The same applies to the figure (b), which represents two different charges, where mutual force is attractive.

Example 1

Calculate the value of two equal charges if they repel one another with a force of 0.1N when situated 50cm apart in a vacuum.

Solution

$$F = K\frac{q_1 q_2}{r^2}$$

Since q_1=q_2

$$0.1 = \frac{9\times 10^{9}\times q^{2}}{(0.5)^{2}}$$

$q = 1.7x10^{-6}$ C = 1.7μC

Example 2

What must be the distance between point charge q1 = 26.0 μC and point charge q_2 = −47.0 μC for the electrostatic force between them to have a magnitude of 5.70 N?

Solution

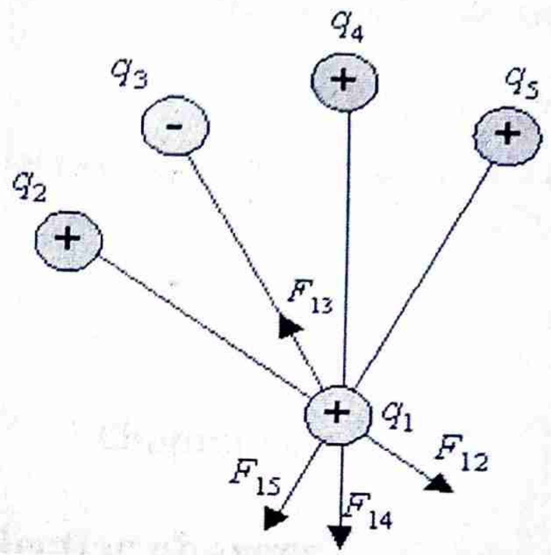
We are given the charges and the magnitude of the (attractive) force between them. We can use Coulomb's law to solve for r, the distance between the charges:

$$F = K\frac{q_1 q_2}{r^2}$$

$r_2 = k\,|q_1q_2|\,/\,F$

Electric force between more than two electric charges

In case of dealing with more than two charges, the total electrical forces affecting the Q1 charge are calculated as shown in the figure this force is F_1, which is the directional addition of all forces exchanged with the charge, meaning that:



في حالة التعامل مع أكثر من شحنتين والمراد حساب القوى الكهربائية الكلية المؤثرة على شحنة q_1 كما في الشكل فإن هذه القوة هي F_1 وهى الجمع الاتجاهي لجميع القوى المتبادلة مع الشحنة اي ان:

$$\bar{F}_1 = \bar{F}_{12} + \bar{F}_{13} + \bar{F}_{14} + \bar{F}_{15}$$

$$F_{12} = K\frac{q_1 q_2}{r^2}$$

$$F_{13} = K\frac{q_1 q_3}{r^2}$$

$$F_{14} = K\frac{q_1 q_4}{r^2}$$

The result of these forces is F1, but, as is evident from the figure, the line of action of the forces is different the method of analyzing vectors into two components is as follows

تكون محصلة هذه القوى هي F1 ولكن كما هو واضح على الشكل فإن خط عمل القوى مختلف ولذلك نستخدم طريقة تحليل المتجهات إلى مركبتين كما يلي:

$F_{1x}= F_{12x}+ F_{13x}+ F_{15x}$

$F_{1y}= F_{12y}+ F_{13y}+ F_{15y} + F_{14}$

The resultant of the forces

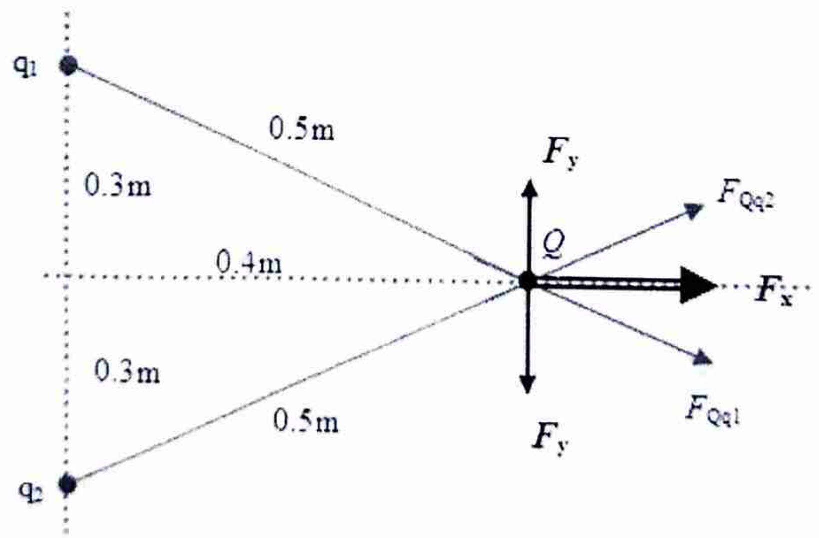
$$F_1=\sqrt{(F_x)^2+(F_y)^2}$$

and its direction

$$\theta = \tan^{-1}\frac{F_y}{F_x}$$

Example (1)

In figure, two equal positive charges $q=2\times10^{-6}$C interact with a third charge $Q=4\times10^{-6}$C. Find the magnitude and direction of the resultant force on Q.



Solution

To find the resultant of the electric forces affecting the charge Q, we apply Coulomb's law to calculate how much force each affects charge on charge Q.

Since the two charges q1 & q2 are equal and they are the same distance from charge Q, the two forces are equal in magnitude and value of force.

لإيجاد محصلة القوى الكهربائية المؤثرة على الشحنة Q نطبق قانون كولوم لحساب مقدار القوة التي تؤثر بها كل شحنة على الشحنة Q . وبما أن الشحنتين q_1&q_2 متساويتان وتبعدان نفس المسافة عن الشحنة Q فإن القوتين متساويتان في مقدار وقيمة القوة

يتم استخراج ر من نظرية فيثاغورس وتكون النتيجة :. ر = 0.5

$$F_{Qq1} = K\frac{qQ}{r^2} = 9\times10^9\,\frac{(4\times10^{-6})(2\times10^{-6})}{(0.5)^2} = 0.29\,N = F_{Qq2}$$

By analyzing the force vector of two components, it produces:

$$F_x = F\cos\theta = 0.29\left(\frac{0.4}{0.5}\right) = 0.23\,N$$

$$F_y = -F\sin\theta = -0.29\left(\frac{0.3}{0.5}\right) = -0.17\,N$$

وبالمثل يمكن إيجاد القوة المتبادلة بين الشحنتين Q وهي F Qq_2 وبالتحليل الاتجاهي نلاحظ أن مركبتي Y متساويتان في المقدار ومتعاكستان في الاتجاه

$$\sum F_x = 2\times0.23 = 0.46\,N$$

$$\sum F_y = 0$$

توضيح نظرية فيثاغورس :-

$$r^2 = x^2 + y^2$$

$$r^2 = (0.3)^2 + (0.4)^2 = 0.09 + 0.16 = 0.25$$

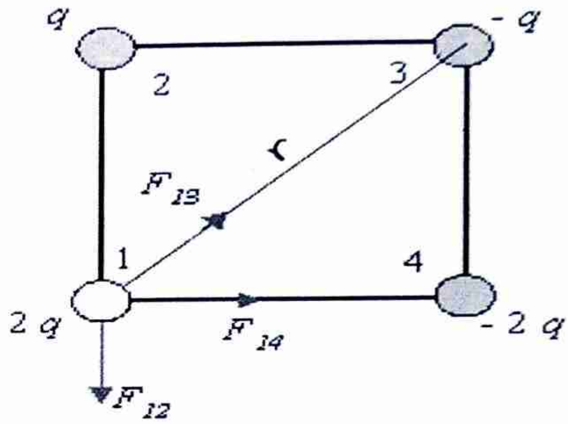
$$\therefore r = 0.5\,m$$

Example (2)

In this figure what is the resultant force on the charge in the lower left corner of the square? Assume that $q=1\times10^{-7}$ C and $a = 5$cm.

Solution

For simplicity we number the charges as shown in above figure, and then we determine the direction of the electric forces acted on the charge in the lower left corner of the square q_1



$$\bar{F}_1 = \bar{F}_{12} + \bar{F}_{13} + \bar{F}_{14}$$

$$r^2 = a^2 + a^2 = 2a^2$$

$$F_{12} = K\frac{2qq}{a^2}$$

نطبق القوانين

$$F_{13} = K\frac{2qq}{2a^2}$$

$$F_{14} = K\frac{2q2q}{a^2}$$

By substituting in the equations it results that:

***F*12 = 0.072 N**
***F*13 = 0.036 N**
***F*14 = 0.144 N**

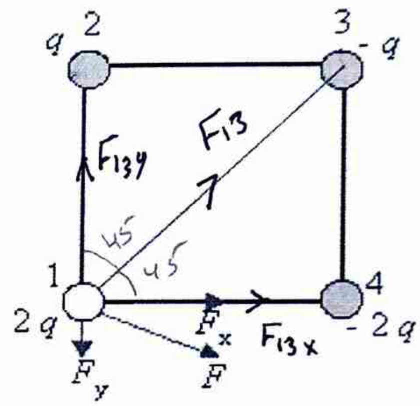
We notice here that we cannot directly add the three forces because the action line of the powers is different, so we calculate the outcome the two axes are orthogonal X and Y, and we analyze the forces that do not fall on these two axes, that is, the vector of F13 to become

نلاحظ هنا أننا لا نستطيع جمع القوى الثلاث مباشرة لأن خط عمل القوى مختلف، ولذلك لحساب المحصلة نفرض محورين متعامدين X ,Y ونحلل القوى التي لا تقع على هذين المحورين أي متجه القوة F13 ليصبح

$\cos 45 = \frac{1}{\sqrt{2}} = 0.707 = \sin 45$

***F*13x = *F*13 sin 45 = 0.025 N &**

***F*13y = *F*13 cos 45 = 0.025 N**



***F*x = *F*13x + *F*14 = 0.025 + 0.144 = 0.169 N**
***F*y = *F*13y - *F*12 = 0.025 - 0.072 = -0.047 N**

The negative signal indicates that the direction of the force vehicle is in the direction of the y axis negative.

The resultant force equals

$$F_1 = \sqrt{(F_x)^2 + (F_y)^2} =$$

F$_1$ = 0.175 N

The direction with respect to the x-axis equals

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\frac{-0.047}{0.169}$$

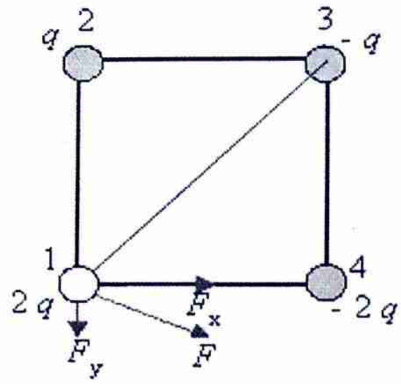
= -15.5°

axes are orthogonal X and Y, and we analyze the forces that do not fall on these two axes, that is, the vector of F13 to become

نلاحظ هنا أننا لا نستطيع جمع القوى الثلاث مباشرة لأن خط عمل القوى مختلف، ولذلك لحساب المحصلة نفرض محورين متعامدين X ,Y ونحلل القوى التي لا تقع على هذين المحورين أي متجه القوة F13 ليصبح

$F13x = F13 \sin 45 = 0.025$ N &

$F13y = F13 \cos 45 = 0.025$ N



$Fx = F13x + F14 = 0.025 + 0.144 = 0.169$ N
$Fy = F13y - F12 = 0.025 - 0.072 = -0.047$ N

The negative signal indicates that the direction of the force vehicle is in the direction of the y axis negative.

The resultant force equals

$$F_1 = \sqrt{(F_x)^2 + (F_y)^2}$$

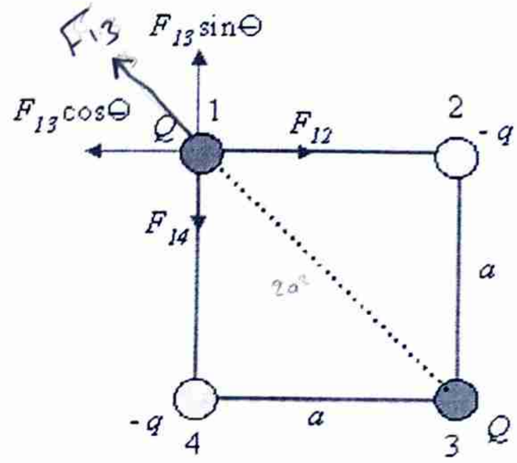
$F_1 = 0.175$ N

The direction with respect to the x-axis equals

$$\theta = \tan^{-1} \frac{F_y}{F_x} = -0.278$$

$$= -15.5^\circ$$

Example (3)



A charge Q is fixed at each of two opposite corners of a square as shown in figure is below. A charge q is placed at each of the other two corners. (a) If the resultant electrical force on Q is Zero, how are Q and q related.

Solution

نحدد اتجاهات القوى على الشكل بعد تحليل متجه القوة F_{13} نلاحظ أن هناك أربعة متجهات قوى متعامدة، كما هو موضح في الشكل أدناه،وبالتالي يمكن أن تكون محصلتهم تساوى صفرا إذا كانت محصلة المركبات الأفقية تساوى صفرا وكذلك محصلة المركبات الرأسية

$F_x = 0 \Rightarrow F_{12} - F_{13x} = 0$

Then

$F_{12} = F_{13}\cos 45$

$$K\frac{Qq}{a^2} = K\frac{QQ}{2a^2}\frac{1}{\sqrt{2}} \quad\Rightarrow\quad q = \frac{Q}{2\sqrt{2}}$$

$$F_y = 0 \Rightarrow \quad F_{13y} - F_{14} = 0$$

$$F_{13}\sin 45 = F_{14}$$

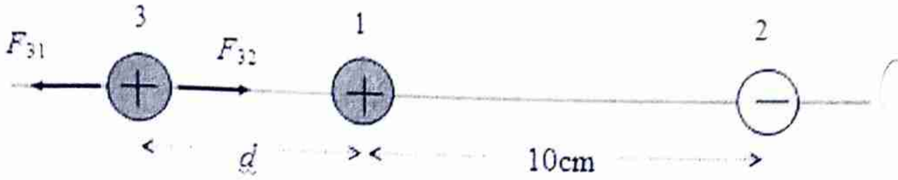
$$K\frac{QQ}{2a^2}\frac{1}{\sqrt{2}} = K\frac{Qq}{a^2} \quad\Rightarrow\quad q = \frac{Q}{2\sqrt{2}}$$

$$\longrightarrow Q = 2\sqrt{2}\,q$$

So: $\quad Q = -2\sqrt{2}\ q$

Example (4)

Two fixed charges, 1 μ C and -3 μ C are separated by 10cm as shown in figure, (a) where may a third charge be located so that no force acts on it? (b) Is the equilibrium stable or unstable for the third charge?



Solution

المطلوب من السؤال هو أين يمكن وضع شحنة ثالثة بحيث تكون محصلة القوى الكهربائية المؤثرة عليها تساوى صفراً أي أن تكون في وضع اتزان equilibrium. حتى يتحقق هذا فإنه يجب أن تكون القوى المؤثرة متساوية في المقدار ومتعاكسة في الاتجاه .وحتى يتحقق هذا الشرط فإن الشحنة الثالثة يجب أن توضع خارج الشحنتين وبالقرب من الشحنة الأصغر .لذلك نفرض شحنة موجبة q_3 كما في الرسم ونحدد اتجاه القوى المؤثرة عليها.

***F*31 = *F*32**

$$k\frac{q_3 q_1}{r_{31}^2} = k\frac{q_3 q_2}{r_{32}^2}$$

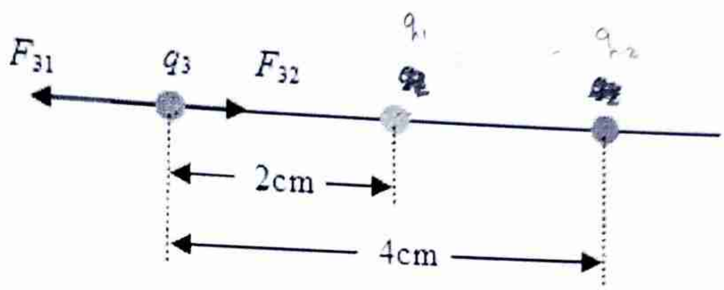
$$\frac{1\times10^{-6}}{d^2} = \frac{3\times10^{-6}}{(d+10)^2}$$

$\times10^{-2}$

نحل هذه المعادلة ونوجد قيمة d .

Example (5)

Two charges are located on the positive x-axis of a coordinate system, as shown in below figure. Charge $q1$=2nC is 2cm from the origin, and charge $q2$=-3nC is 4cm from the origin. What is the total force exerted by these two charges on a charge $q3$=5nC located at the origin?



Solution

The total force on q3 is the vector sum of the forces due to q1 and q2 individually.

$$F_{31} = \frac{(9\times10^{9})(2\times10^{-9})(5\times10^{-9})}{(0.02)^2} = \overset{+}{2.25}\times10^{-4}\,N$$

$$F_{32} = \frac{(9\times10^{9})(3\times10^{-9})(5\times10^{-9})}{(0.04)^2} = \overset{-}{0.84}\times10^{-4}\,N$$

$$F_3 = F_{31} + F_{32}$$

$-2.25\times10^{-4} + 0.84\times10^{-4} = -1.41\times10^{-4}\,N$

$$\therefore F_3 = 0.84\times10^{-4} - 2.25\times10^{-4} = -1.41\times10^{-4}\,N$$

The total force is directed to the left, with magnitude 1.41×10^{-4}N.