(9) $\dfrac{z_1+z_2}{z_3} = \dfrac{z_1}{z_3} + \dfrac{z_2}{z_3}$; $z_3 \neq 0$ and

$$\frac{z_1 z_2}{z_3 z_4} = \left(\frac{z_1}{z_3}\right)\left(\frac{z_2}{z_4}\right) \;;\; z_3 \neq 0,\ z_4 \neq 0,\ z_3 z_4 \neq 0$$

(10) If $z_1 z_2 = 0$, either $z_1 = 0$ or $z_2 = 0$

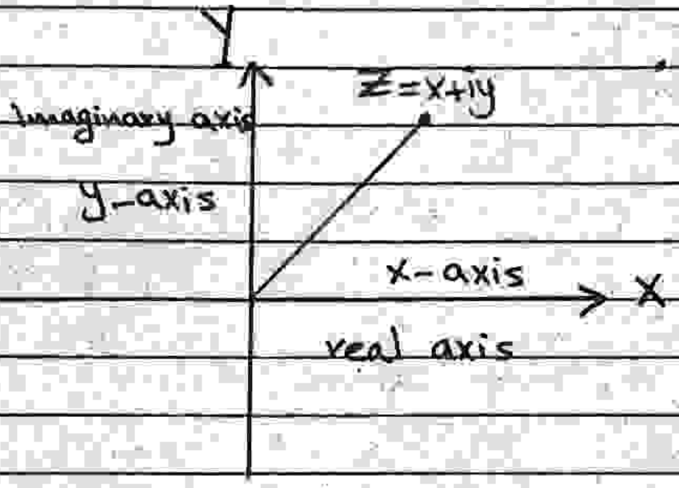
Geometric Representation of $\mathbb{C}$

التمثيل الهندسي للأعداد العقدية

Complex numbers can be represented as points in the plane, using the correspondence $x+iy \longleftrightarrow (x,y)$. The represention is known as the Argand diagram or Complex plane. The real Complex numbers lie on the x-axis, which is then Called the real axis, while the imaginary numbers lie on the y-axis, which is known as the imaginary axis. The complex numbers with positive imaginary part lie in the upper half plane, while those with negative imaginary part lie in the lower half plane.

اي كل عدد عقدي $z = x+iy$ يقابله نقطة واحدة في المستوي xy وهي النقطة (x,y).
وكذلك يمكن اعتبار العدد العقدي z هو قطعة مستقيم

متجه من نقطة الاصل الى النقطة (x,y) ويسمى المستوي بالمستوي العقدي حيث يمثل فيه x الاحداثي الحقيقي و y المحور الخيالي .



Z plane or xy plane

Example: The Complex number $Z = 1 - 2i$ Correspondence the point $(1,-2)$ in the Z plane and the Complex number $Z = 0 + 0i = (0,0)$ Correspondence the origin point.

Now, we have the following examples:

Example(1): Find the value of the following problem:

$$\frac{(1+i)(-1+2i)+(2-i)}{2-3i} - 2i$$

Solution: $\frac{(1+i)(-1+2i)+(2-i)}{(2-3i)} - 2i = \frac{(-3+2)}{2-3i} - 2i = \frac{-1}{2-3i} - 2i$

$= \frac{-1-2i(2-3i)}{2-3i} = \frac{-1-4i-6}{2-3i} = \frac{-(7+4i)}{2-3i} \cdot \frac{2+3i}{2+3i} = \frac{-(14-12)-(8i+21i)}{4+9}$

$= \frac{-2}{13} - \frac{29}{13} i$

Example (1) : Find the value of $\left(\frac{1}{2-3i} \cdot \frac{1}{1+i}\right)$

solution :

let $Z_1 = 2-3i$ and $Z_2 = 1+i$

Then we want to find $\frac{1}{Z_1} \cdot \frac{1}{Z_2} = Z_1^{-1} \cdot Z_2^{-1}$

Now, $Z_1^{-1} = \left(\frac{2}{13} + \frac{3}{13}i\right) \cdot \left(\frac{1}{2} - \frac{1}{2}i\right) = \frac{5}{26} + \frac{1}{26}i$

Example (2) : Show that if the product Z_1Z_2 is Zero then so is at least one of the factors Z_1 and Z_2 .

Solution: let $Z_1Z_2 = 0$ and $Z_1 \neq 0$ where

$Z_1 = (x_1, y_1)$, $Z_2 = (x_2, y_2)$

(Since $\because Z_1 \neq 0$, then either $x_1 \neq 0$ or $y_1 \neq 0$)

we take $x_1 \neq 0$

Now, $(x_1, y_1) \cdot (x_2, y_2) = (x_1x_2 - y_1y_2 , x_1y_1 + x_2y_1) = (0,0)$

$\Leftarrow x_1x_2 - y_1y_2 = 0$ and $x_1y_2 + x_2y_1 = 0 \to$ ②

Then $x_1x_2 - y_1y_2 = 0 \Longrightarrow x_2 = \frac{y_1y_2}{x_1} \Longrightarrow$ by subsit in ② ,

we get $x_1y_2 + \frac{y_1y_2}{x_1}y_1 = 0 \Longrightarrow x_1y_2 + \frac{y_1^2y_2}{x_1} = 0$

$\Longrightarrow \frac{y_2(x_1^2 + y_1^2)}{x_1} = 0 \Longrightarrow y_2 = 0$ and from ① we obtian

$x_2 = 0$ and hence $z_2 = 0$

Problems : (H.W)

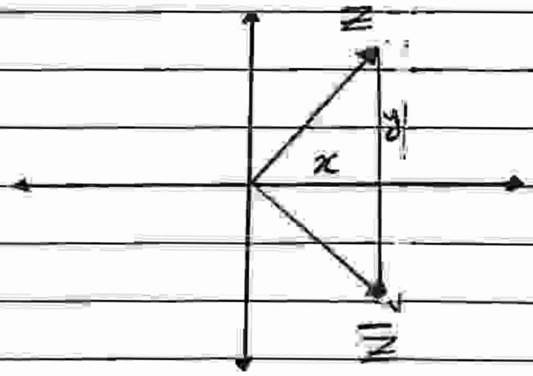
1. Solve the equation $z^2 + z + 1 = 0$

2. Show that $\frac{5}{(1-i)(2-i)(3-i)} = \frac{1}{2} i$

Complex Conjugate المرافق المعقد

Definition If $z = x + iy$, the complex conjugate of z is the complex number defined $\bar{z} = x - iy$.

Geometrically, the complex conjugate of z is obtained by reflecting z in the real axis, see the following figure.



The following properties of the complex conjugate are easy to verify :

1. $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$;