Work, Energy, and Power

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Definition of Work

When a force acts upon an object to cause a displacement of the object, it is said that **work** was done upon the object. There are three key ingredients to work - force, displacement, and cause. For a force to qualify as having done work on an object, there must be a displacement and the force must cause the displacement.

Everyday life examples of Work

A horse pulling a plough through the field, a father pushing a grocery cart down the aisle of a grocery store, a freshman lifting a backpack full of books upon his shoulder, a weightlifter lifting a barbell above his head.

Example 1: Read the following five statements and determine whether or not they represent examples of work.

1. A teacher applies a force to a wall and becomes exhausted.
2. A book falls off a table and free falls to the ground.
3. A waiter carries a tray full of meals above his head by one arm straight across the room at constant speed.
4. A rocket accelerates through space.

Work Equation

Mathematically, work can be expressed by the following equation:

$$W = F \times d \times \cos\theta$$

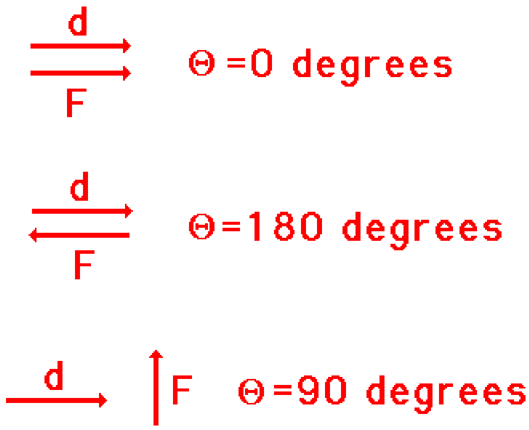
where F is the force, d is the displacement, and the angle θ (theta) is defined as the angle between the force and the displacement vectors.

Three Scenarios of θ

Scenario A: A force acts rightward upon an object as it is displaced rightward. In such an instance, the force vector and the displacement vector are in the same direction. Thus, the angle between F and d is 0 degrees.

Scenario B: A force acts leftward upon an object that is displaced rightward. In such an instance, the force vector and the displacement vector are in the opposite direction. Thus, the angle between F and d is 180 degrees.

Scenario C: A force acts upward on an object as it is displaced rightward. In such an instance, the force vector and the displacement vector are at right angles to each other. Thus, the angle between F and d is 90 degrees.



Units of Work

the standard metric unit is the Joule (abbreviated J). One Joule is equivalent to one Newton of force causing a displacement of one meter. In other words, the Joule is the unit of work.

1 Joule = 1 Newton * 1 meter

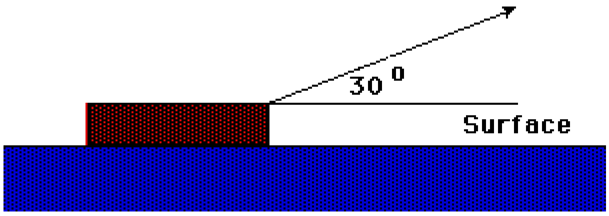
1 J = 1 N * m

Example 2: A roller coaster designer is considering three different incline angles at which to drag the 2000-kg car train to the top of the 60-meter-high hill. In each case, the force applied to the car will be applied parallel to the hill.

	Hill Angle	**Force**	**Distance**	**Work (J)**
a.	35°	1.12×10^4 N	105 m	?
b.	45°	1.39×10^4 N	84.9 m	?
c.	55°	1.61×10^4 N	73.2 m	?

Example 3: Ben carries a 200-N suitcase up three flights of stairs (a height of 10.0 m) and then pushes it with a horizontal force of 50.0 N at a constant speed of 0.5 m/s for a horizontal distance of 35.0 meters. How much work does Ben do on his suitcase during this entire motion?

Example 4: A force of 50 N acts on the block at the angle shown in the diagram. The block moves a horizontal distance of 3.0 m to the right. How much work is done by the applied force?

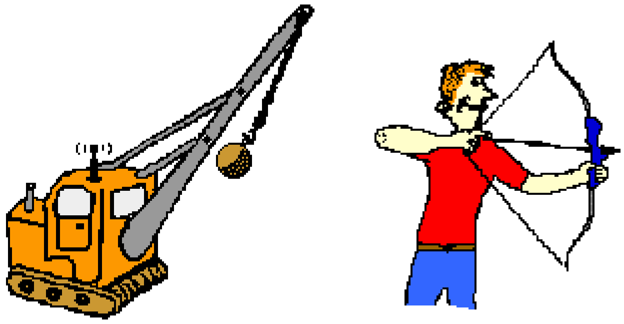


Example 5: A student with a mass of 80.0 kg runs up three flights of stairs in 12.0 sec. The student has gone a vertical distance of 8.0 m. Determine the amount of work done by the student to elevate his body to this height. Assume that his speed is constant.

Example 6: Calculate the work done by a 2.0-N force (directed at a 30° angle to the vertical) to move a 500 gram box a horizontal distance of 400 cm across a rough floor at a constant speed of 0.5 m/s.

Potential Energy

It is the stored energy of position possessed by an object.



The massive ball of a demolition machine and the stretched bow possesses stored energy of position - potential energy.

Types of Potential Energy

There are two forms of potential energy - gravitational potential energy and elastic potential energy.

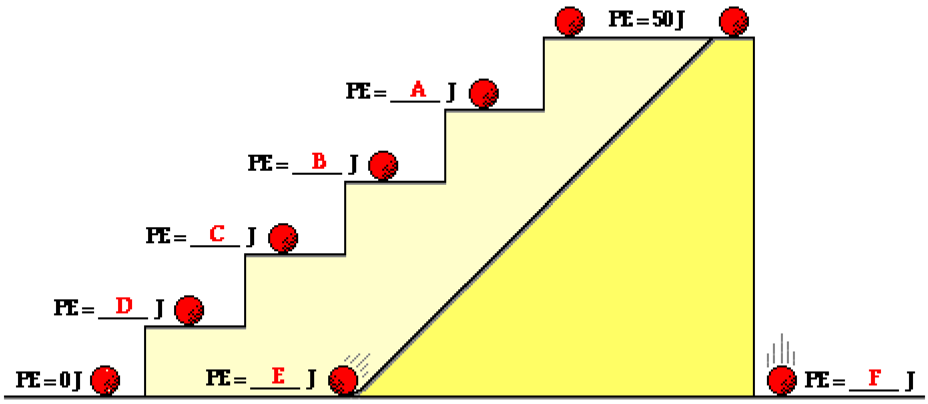
Gravitational Potential Energy

Gravitational potential energy is the energy stored in an object as the result of its vertical position or height. The energy is stored as the result of the gravitational attraction of the Earth for the object. The gravitational potential energy of the massive ball of a demolition machine is dependent on two variables - the mass of the ball and the height to which it is raised. These relationships are expressed by the following equation:

$$PE_{grav} = m \times g \times h$$

In the above equation, m represents the mass of the object, h represents the height of the object and g represents the gravitational field strength (9.8 N/kg on Earth).

Example 7: Use this principle to determine the blanks in the following diagram. Knowing that the potential energy at the top of the tall platform is 50 J, what is the potential energy at the other positions shown on the stair steps and the incline?



Elastic Potential Energy

Elastic potential energy is the energy stored in elastic materials as the result of their stretching or compressing. Elastic potential energy can be stored in rubber bands, bungee chords, trampolines, springs, an arrow drawn into a bow, etc. The amount of elastic potential energy stored in such a device is related to the amount of stretch of the device - the more stretch, the more stored energy.

Springs

A spring is an object that can be deformed by a force and then return to its original shape after the force is removed. Springs are a special instance of a device that can store elastic potential energy due to either compression or stretching. A force is required to compress a spring; the more compression there is, the more force that is required to compress it further. For certain springs, the amount of force F_{spring} is directly proportional to the amount of stretch or compression (x); the constant of proportionality is known as the spring constant (k) (in Newtons per meter (N/m)).

$$\boldsymbol{F_{spring} = k \times x}$$

Such springs are said to follow Hooke's Law.

Hooke's Law

Hooke's law, law of elasticity discovered by the English scientist Robert Hooke in 1660, which states that, for relatively small deformations of an object, the displacement or size of the deformation is directly proportional to the deforming force or load.

Spring Equilibrium Position

If a spring is not stretched or compressed, then there is no elastic potential energy stored in it. The spring is said to be at its equilibrium position. The equilibrium position is the position that the spring naturally assumes when there is no force applied to it. In terms of potential energy, the equilibrium position could be called the zero-potential energy position.

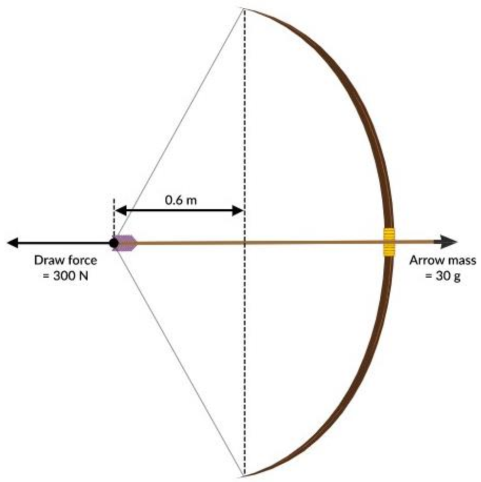
Elastic Potential Energy Equation

There is a special equation for springs that relates the amount of elastic potential energy to the amount of stretch (or compression) and the spring constant. The equation is:

$$PE_{spring} = \frac{k \times x^2}{2}$$

Example 8: A truck spring has a spring constant of 5×10^4 N/m. When unloaded, the truck sits 0.8 m above the road. When loaded with goods, it lowers to 0.7 m above the ground. How much potential energy is stored in the four springs?

Example 9: A trained archer has the ability to draw a longbow with a force of up to 300 N, extending the string back by 0.6 m. Assuming the bow behaves like an ideal spring, what spring constant would allow the archer to make use of his full strength? And what potential energy is stored in the bow when it is drawn?



Kinetic Energy

Kinetic energy is the energy of motion. An object that has motion - whether it is vertical or horizontal motion - has kinetic energy.

Forms of Kinetic Energy

There are many forms of kinetic energy - vibrational (the energy due to vibrational motion), rotational (the energy due to rotational motion), and translational (the energy due to motion from one location to another).

Translational Kinetic Energy

The amount of translational kinetic energy that an object has depends upon two variables: the mass (m in kg) of the object and the speed (v in m/s) of the object. The following equation is used to represent the kinetic energy (KE) of an object:

$$\mathbf{KE} = \frac{m \times v^2}{2}$$

Like work and potential energy, the standard metric unit of measurement for kinetic energy is the Joule. As might be implied by the above equation, that:

1 Joule = 1 kg • m^2/s^2

Example 10: Determine the kinetic energy of a 625-kg roller coaster car that is moving with a speed of 18.3 m/s. If the roller coaster car was moving with twice the speed, then what would be its new kinetic energy?

Example 11: A platform diver had a kinetic energy of 12 000 J just prior to hitting the bucket of water. If her mass is 40 kg, then what is her speed?

Example 12: A 900-kg compact car moving at 60 mi/hr has approximately 320 000 Joules of kinetic energy. Estimate its new kinetic energy if it is moving at 30 mi/hr.

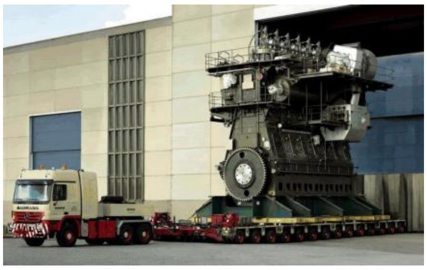
Power

Power ($\boldsymbol{P}$) is the rate at which work (W) is done. It is the work/time ratio. Mathematically, it is computed using the following equation:

$$\boldsymbol{P = \frac{W}{t}}$$

The standard metric unit of power is the **Watt**. As is implied by the equation for power, a unit of power is equivalent to a unit of work divided by a unit of time. Thus, a Watt is equivalent to a Joule/second.

For historical reasons, the *horsepower* (HP) is occasionally used to describe the power delivered by a machine. One horsepower is equivalent to approximately 750 Watts. Most machines are designed and built to do work on objects. All machines are typically described by a power rating. The power rating indicates the rate at which that machine can do work upon other objects. Thus, the power of a machine is the work/time ratio for that particular machine. A car engine is an example of a machine that is given a power rating. The power rating relates to how rapidly the car can accelerate the car. Suppose that a 40-HP engine could accelerate the car from 0 mi/hr to 60 mi/hr in 16 seconds. If this were the case, then a car with four times the horsepower could do the same amount of work in one-fourth the time. That is, a 160-HP engine could accelerate the same car from 0 mi/hr to 60 mi/hr in 4 seconds. For the record, the biggest engine in the world can produce 109,000 horsepower as shown in photo:



Example 13: Suppose that Ben elevates his 80-kg body up the 2.0-meter stairwell in 1.8 seconds. What is his power?

Since the expression for velocity is displacement/time, the expression for power can be rewritten once more as force*velocity. This is shown below:

$$\boldsymbol{P = \frac{W}{t} = \frac{F \times d}{t}}$$

And because $\boldsymbol{v = \frac{d}{t}}$

Then

$$\boldsymbol{P = F \times v}$$

Example 14: Two students, Will and Ben, are in the weightlifting room. Will lifts the 100-pound barbell over his head 10 times in one minute; Ben lifts the 100-pound barbell over his head 10 times in 10 seconds. Which student does the most work? _______________ Which student delivers the most power? _______________

Example 15: During a physics lab, Jack and Jill ran up a hill. Jack is twice as massive as Jill; yet Jill ascends the same distance in half the time. Who did the most work? _______________ Who delivered the most power? _______________

Example 16: An escalator is used to move 20 passengers every minute from the first floor of a department store to the second. The second floor is located 5.20 meters above the first floor. The average passenger's mass is 54.9 kg. Determine the power requirement of the escalator in order to move this number of passengers in this amount of time.