Finite and infinite set

Definition A set A is said to be $\boxed{\text{finite}}$ if $\exists$ bijective $f: A \longrightarrow B$ with $B \subseteq N$.

Definition A set A is said to be $\boxed{\text{infinite set}}$ if $\nexists$ bijective $f: A \longrightarrow B$ with $B \subseteq N$.

Remark

Let A be a set. The $\boxed{\text{cardinal numbers}}$ of A is the numbe of elements in the set A. ($\#(A)$ ويرمز له or $|A|$)

Remark 1. the set of natural numbers is infinite set.

2- $B = \{6, 8, 10, 12, \dots\}$ is infinite set

3- The empty set φ or $\{\}$ is finite set (because $\exists$ a bijective $g: \varphi \longrightarrow \{0\}$).

4- The set $A = \{a, b, c, d\}$ is finite set

Definition

① A set A is said to be Countable قابل العد if $\exists$ 1-1 function $f: A \longrightarrow N$.

② A set S is said to be Uncountable غير قابلة للعد if $\nexists$ 1-1 function $g: S \longrightarrow N$.

③ $\#(N) = \alpha_0$ (alpha-null). If $\# A$ or $|A| \leq \alpha_0$ then A is Countable.

④ A is Countable infinite if $|A| = \alpha_0$

⑤ Every subset of countable set is either finite or Countable set.

Infinite sets

A. Set A is said to be finite if there is a one-one corrospondence between N and a subset of N where $B = \{0, 1, 2, \dots, m\}$ $m \in N$. Note that $\#(A) = m$ (Cardinal number of A). otherwise, A is said to be infinite set.

Remark.

1- The following set is infinite set

ⓐ The set of natural number

ⓑ " " " integers

ⓒ $B = \{6, 8, 10, 12, \dots\}$

2. the set $A = \{a, b, c, d\}$ is finite set

Definition

① A set A is said to be Countable if there is an injective with a natural numbers, N. i.e $\exists$ injective $f: A \to N$

قابلة للعد

② A set A is said to be At most Countable if A is Countable or finite

③ A set A is said to be uncountable if it is not at most Countable.

④ Let $\#(N) = N_0$ (aleph-null). If $\#(A)$ or $|A| \le N_0$, then A is Countable.

⑤ A is Countably infinite if $|A| = N_0$

⑥ Every subset of Countable set is either finite or countable set.

Theorem For any set A, the following statments are equivalent: العبارات الاتية متكافئة

① A is Countable. (A مجموعة معدودة)

② $\exists$ bijective function $\alpha : A \longrightarrow N$

③ A either finite or Countably infinite.
(A إما منتهية أو معدودة غير منتهية)

Theorem For any set A, the following statments are equivalent العبارات الاتية متكافئة

① A is Countably infinite (A معدود غير منتهية)

② $\exists$ bijective $f : S \longrightarrow N$

③ The element of A can be arranged in an ifinite Sequence $a_0, a_1, \ldots, a$ where a_i is distanc from a_j for $i \neq j$.

(عناصر المجموعة A يمكن ترتيبها بمتتابعة غير منتهية من العناصر مختلفة a_i بحيث $i \neq j$)

Remarks

① If A and B are countable sets, then $A \times B$ is Countable set.

② If A and B are Countable sets, then $A \cup B$ is Countable.

③ The set of all finite subsets of N is Countable

Cantor's theorem مبرهنة كانتور

Let A be a set and $P(A)$ be a power set of A. then $\nexists$ surjective $f : A \longrightarrow P(A)$.

Theorem

The set $P(N)$ is uncountable

الجداء الديكارتي للأعداد الطبيعية مجموعة معدودة.

Theorem. The Cartiesian product of N is Countable.

Proof To prove $N \times N$ is Countable:

The elements of $N \times N$ Can be arrangment as a matrix

$$\begin{array}{ccccc}
(0,0) & (0,1) & (0,2) & (0,3) & \cdots \\
(1,0) & (1,1) & (1,2) & (1,3) & \cdots \\
(2,0) & (2,1) & (2,2) & (2,3) & \cdots \\
(3,0) & (3,1) & (3,2) & (3,3) & \cdots \\
\vdots & & & &
\end{array}$$

$\therefore N \times N = \{(0,0), (1,0), (0,1), (2,0), (1,1), (0,2), \ldots\}$

$= \{a_0, a_1, a_2, a_3, a_4, a_5, \ldots\}$

where $a_0 = (0,0)$, $a_1 = (1,0)$, $a_2 = (0,1)$, $a_3 = (2,0) \ldots$

The elements of $N \times N$ Can be numbering

هذا يعني بأننا تمكنا من ترتيب الاعداد الطبيعية على شكل عناصر مختلفة

للجداء الديكارتي $N \times N$

$\therefore N \times N$ is Countable