# 2D Fourier Transform

- For *M x N* matrix, forward and inverse fourier transforms can be written

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\exp\left[-2\pi i\left(\frac{xu}{M}+\frac{yv}{N}\right)\right].$$

$$f(x,y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u,v)\exp\left[2\pi i\left(\frac{xu}{M}+\frac{yv}{N}\right)\right].$$

where

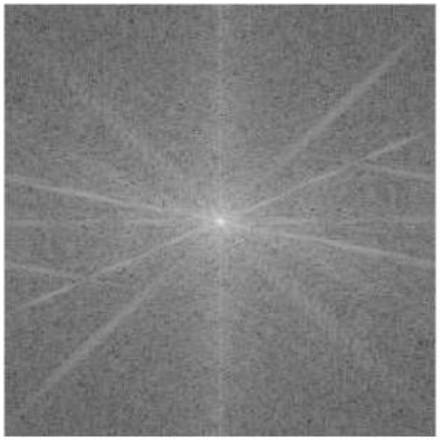
- *x* indices go from *0... M – 1 (x* cycles over distance *M)*
- *y* indices go from *0... N – 1 (*y cycles over distance *N)*

- $F(u,v)$ is complex in general,

$$F(u,v) = F_{\mathrm{R}}(u,v) + jF_{\mathrm{I}}(u,v)$$

- $|F(u,v)|$ is the magnitude spectrum
- $\arctan(F_{\mathrm{I}}(u,v)/F_{\mathrm{R}}(u,v))$ is the phase angle spectrum.

Image



DFT

## 2-DFT in MATLAB

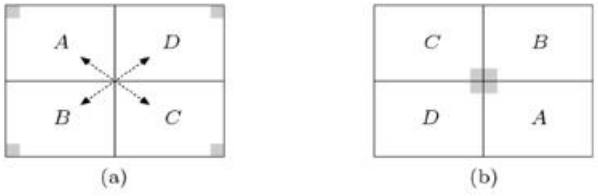
```
a= imread('cameraman.tif');

figure,subplot(2,2,1),imshow(a) ,title(' the origional image')
```


```
% fast fourier transform ************************************

  fb=fftshift(fft2(a));
```

**fft2:** is two dimensional fast Fourier transform.
**fftshift** : is used to shift the DC (high frequencies) component in center.

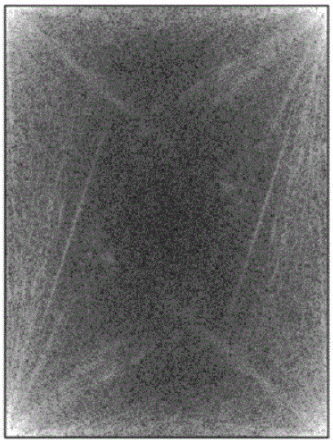
## Centering DFT Spectrum

- *F(0,0)* at top left corner
- For display, convenient to have DC component in center
- Just swap four quadrants of Fourier transform

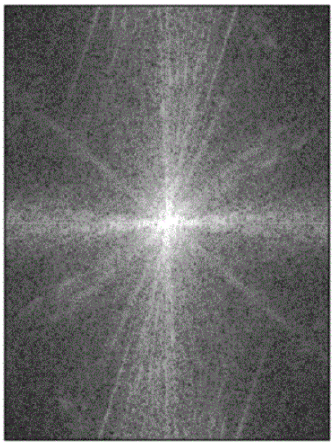



Swap 4 quadrants to center DC component

fft2 without fftshift



fftshift (fft2)



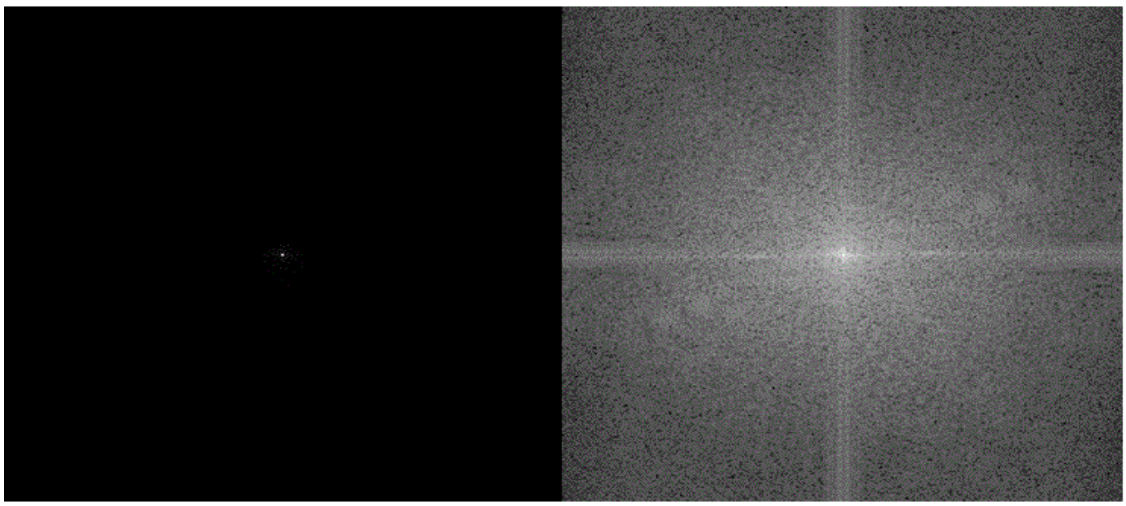
```
% magnitude ***************************************************
  M=abs(fb);
```

$$M = |F(u,v)| = \sqrt{R^2 + I^2}$$

```
% log transform ************************************************
  Mg=log(M);
```

M

Mg



```
% the phase ************************************************

  PH=angle(fb);
```

$$\emptyset = \tan^{-1}\frac{I}{R}$$

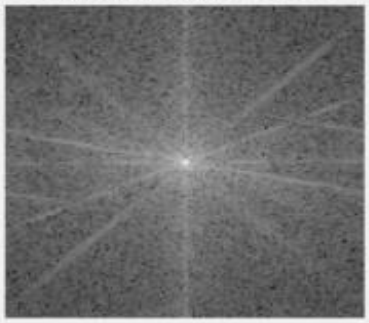
```
% invers fourier transform ***********************************

  BB=ifft2(fftshift(fb));
```

## Matlab code

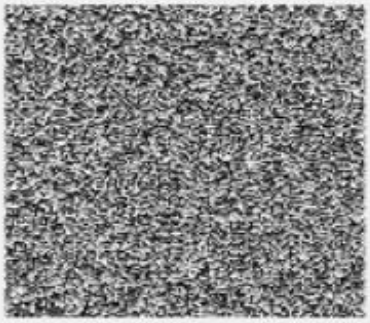
```
clc
clear all
a= imread('cameraman.tif');
figure,subplot(2,2,1),imshow(a),title('the origional image')


% fast fourier transform ************************************

fb=fftshift(fft2(a));

% magnitude *************************************************

M=abs(fb);

% log transform *********************************************

Mg=log(M);

subplot(2,2,2),imshow(Mg, [ ]),title(' the magnetude')

% the phase *************************************************

PH=angle(fb);

subplot(2,2,3),imshow(PH, [ ]), title(' the phase')

% invers fourier transform **********************************

BB=ifft2(fftshift(fb));
subplot(2,2,4),imshow(BB,[]), title(' the invers image ')
```

the origional image

the magnetude



the phase



the invers image