

Newton Raphson Method

The **Newton-Raphson method** (also known as Newton's method) is a way to quickly find a good approximation for the root of a real-valued function $f(x) = 0$. It uses the idea that a continuous and differentiable function can be approximated by a straight line tangent to it.

How it Works

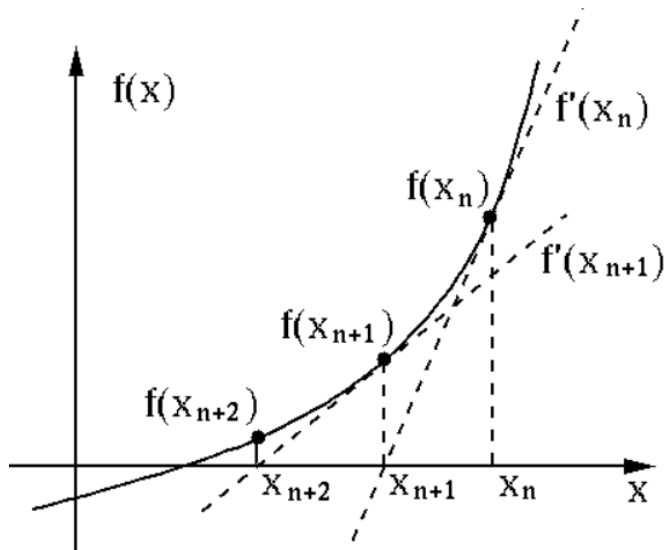
Suppose you need to find the root of a continuous, differentiable function $f(x)$, and you know the root you are looking for is near the point $x=x_0$. Then Newton's method tells us that a better approximation for the root is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

This process may be repeated as many times as necessary to get the desired accuracy.

In general, for any x -value

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$



Algorithm:

Step-1: Find an initial point x_n

Step-2: Find $f(x_n)$ and $f'(x_n)$

$$x_{n+1} = x_n - f(x_n)/f'(x_n)$$

Step-3: If $f(x_{n+1})=0$ then x_{n+1} is an exact root,
else $x_n=x_{n+1}$

Step-4: Repeat steps 2 to 4 until $f(x_i)=0$ or $|x_{n+1} - x_n| \leq \text{Accuracy}$.

EX1: Find the root of $x^2 + x - 2 = 0$ using **NEWTON RAPHSON** Method. Let the initial approximation at $x=0.1$.

```
clc
clear all

f=@(x) x^2 + x -2;
df=@(x) 2*x + 1 ;

% input the initial value
x1=input('initial'); % x1=0.1
fprintf('itr      x      err \n')

n=0;
ERR=inf;

while ERR>10^-5
n=n+1;

x2=(x1-(f(x1)/df(x1)));
ERR=abs(x2-x1);
fprintf('%3d  %13.9f  %12.8f \n', n, x2,ERR )
x1=x2;

end
```

itr	x	err
1	1.675000000	1.57500000
2	1.104741379	0.57025862
3	1.003418232	0.10132315
4	1.000003886	0.00341435
5	1.000000000	0.00000389