Convolution in Spatial Domain

Convolution can be described as a function that is the integral or summation of two component functions, and that measures the amount of overlap as one function is shifted over the other. Here is a simple example of convolution of 3x3 input signal and impulse response (kernel) in 2D spatial.

Local Neighborhood

F(i-1,j-1)	F(i-1,j)	F(i-1,j+1)
F(i,j-1)	F(i,j)	F(i,j+1)
F(i+1,j-1)	F(i+1,j)	F(i+1,j+1)

Mask

W(-1,-1)	W(-1,0)	W(-1,1)
W(0,-1)	W(0,0)	W(0,1)
W(1,-1)	W(1,0)	W(1,1)

Notice that the origin of impulse response is always centered. (w[0,0] is located at the center sample of kernel, not the first element).

Let's start calculate each sample of the output one by one. First, flip the kernel or the sample block of image, in both horizontal and vertical direction. Then, move the kernel over the input array.

If the kernel is centered (aligned) exactly at the sample that we are interested in, multiply the kernel data by the overlapped input data. The accumulation (adding these 9 multiplications) is the last thing to do to find out the output value.

Local Neighborhood

F(i-1,j-1)	F(i-1,j)	F(i-1,j+1)
F(i,j-1)	F(i,j)	F(i,j+1)
F(i+1,j-1)	F(i+1,j)	F(i+1,j+1)

Mask

W(1,1)	W1,0)	W(1,-1)
W(0,1)	W(0,0)	W(0,-1)
W(-1,1)	W(-1,0)	W(-1,-1)

Convolution Operation is represented by the Following Formula:

$$g(i,j) = \sum_{k=-1}^{1} \sum_{l=-1}^{1} w(k,l)\, f(i-k, j-l)$$

MATLAB Code

```
clc
clear all
close all
A=im2double(imread('cameraman.tif'));
[m n]=size(A);

% *to construct the gaussian function with standard deviation = S and size
% (mn x mn)
%*****************************************************************

mn=input('size of filter');
S=input('standared deviation')
G = fspecial('gaussian', [mn  mn], S);
FG=G(mn:-1:1,mn:-1:1);     % flip mask

%Padarray with zeros

B=padarray(A,[floor(mn/2),floor(mn/2)]);

    for i=1:size(A,1)
       for j=1:size(A,2)

        I(i,j)= sum(sum(FG.*B(i:i+(mn-1),j:j+(mn-1))));

    end
end
figure, subplot(1,2,1),imshow(A), title('the original image')
subplot(1,2,2),,imshow(I);title('convolved Image');
```