



2 SOIL COMPOSITION

2.1 INTRODUCTION

Soils are naturally complex, multiphase materials. They are generally a matrix of an assortment of particles (solids), fluids, and gases. Each influences the behavior of the soil mass as a whole. Unless we understand the composition of a soil mass, we will be unable to estimate how it will behave under loads and how we can use it as a construction material.

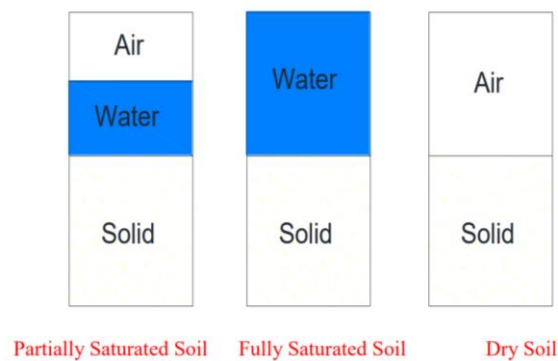
2.2 DEFINITIONS OF KEY TERMS

- Water content (ω) is the ratio of the weight of water to the weight of solids.
- Void ratio (e) is the ratio of the volume of void spaces to the volume of solids.
- Porosity (n) is the ratio of the volume of voids to the total volume of soil.
- Degree of saturation (S) is the ratio of the volume of water to the volume of voids.
- Density, ρ is the density, that is, the mass of a soil per unit volume
- Bulk unit weight (γ) is the unit weight, that is, the weight of a soil per unit volume.
- Saturated unit weight (γ_{sat}) is the unit weight of a saturated soil per unit volume.
- Dry unit weight (γ_d) is the unit weight of a dry soil per unit volume.
- Effective unit weight (γ') is the unit weight of a saturated soil submerged in water per unit volume.
- Liquid limit (LL) is the water content at which a soil changes from a plastic state to a liquid state.
- Plastic limit (PL) is the water content at which a soil changes from a semisolid to a plastic state.
- Shrinkage limit (SL) is the water content at which a soil changes from a solid to a semisolid state without further change in volume.
- Plasticity index (PI) is the range of water content for which a soil will behave as a plastic material (deformation without cracking).
- Liquidity index (LI) is a measure of soil strength using the Atterberg limits (liquid and plastic limits based on test data).
- Shrinkage index (SI) is the range of water content for which a soil will behave as a semisolid (deformation with cracking).

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2.3 PHASE RELATIONSHIPS

- Soil is composed of solids, liquids, and gases (Figure 2.1a).
- Naturally occurred soils always consist of solid particles, water, and air, so that soil has three phases: solid, liquid and gas.
 - a) Three Phase (Partially Saturated Soil).
 - b) Two Phase (Fully Saturated Soil).
 - c) Two Phase (Dry Soil).
- If all the voids are filled by water, the soil is saturated. Otherwise, the soil is unsaturated. If all the voids are filled with air, the soil is said to be dry.



2.4 Phase Diagram and Definitions

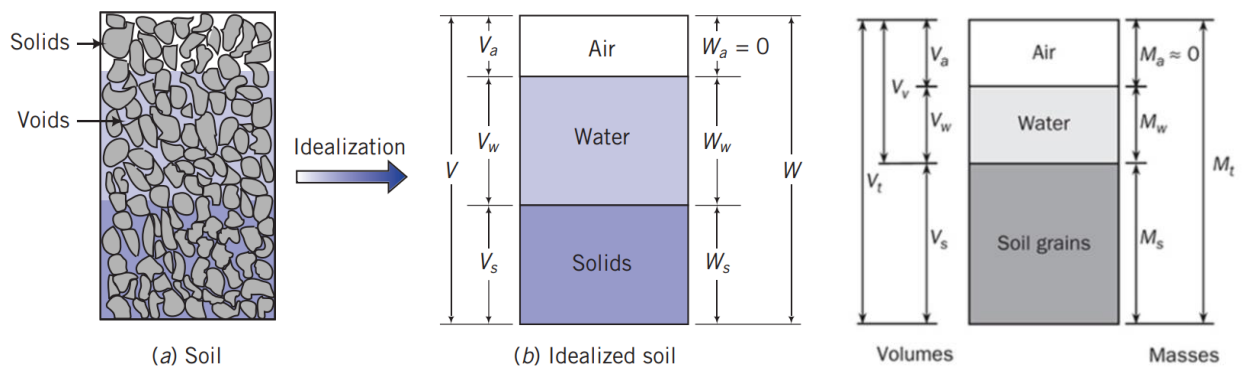


Figure 2.1 (soil phases)

- We can idealize the three phases of soil, as shown in Figure 2.1b. The physical parameters of soils are influenced by the relative proportions of each of these phases.
- The total volume of the soil is the sum of the volume of solids (V_s), volume of water (V_w), and volume of air (V_a); that is,

$$V = V_s + V_w + V_a = V_s + V_v$$

Where:

$$V_v = V_w + V_a \quad \text{is the volume of voids.}$$

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- The weight of the soil is the sum of the weight of solids (W_s) and the weight of water (W_w). The weight of air is negligible. Thus,

$$W = W_s + W_w$$

or

$$M = M_s + M_w$$

- Water content (w)** is the ratio, often expressed as a percentage, of the weight of water to the weight of solids: (It can be larger than 100%, especially in soft saturated clays which can have large void ratios as high as 2 or even more).

$$w = \frac{W_w}{W_s} \times 100\%$$

- Void ratio (e)** is the ratio of the volume of void space to the volume of solids. The void ratio is usually expressed as a decimal quantity:

$$e = \frac{V_v}{V_s}$$

- Porosity (n)** is the ratio of the volume of voids to the total volume. Porosity is usually expressed as a percentage:

$$n = \frac{V_v}{V}$$

Porosity and void ratio are related by the expression

$$n = \frac{e}{1 + e}$$

Where:

$$e = \frac{V_v}{V_s} = \frac{V_v}{V - V_v} = \frac{V_v/V}{1 - V_v/V} = \frac{n}{1 - n}$$

$$n = \frac{V_v}{V} = \frac{V_v}{V_s + V_v} = \frac{V_v/V_s}{1 + V_v/V_s} = \frac{e}{1 + e}$$

and is generally multiplied by 100 and expressed as a percentage. A soil with 25% porosity means that 25% of the total volume consists of voids. Based on the definition, **it can be seen that while the porosity must be less than 100%, the void ratio can exceed 1**. Apparently, for the same material we always have $e > n$.

- Degree of saturation (S)** is the ratio, often expressed as a percentage, of the volume of water to the volume of voids:

$$S = \frac{V_w}{V_v} \times 100\%$$



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S is expressed as a percentage in the range of 0–100%, with 0 for the dry soils and 100% for the saturated soils. A soil with 35% degree of saturation means that 35% of the voids are filled with water.

- **Air content (A)** is the ratio of the volume of air to the total volume:

$$A = \frac{V_a}{V}$$
$$A = n(1 - S)$$

- **Densities and Unit Weights**

Density (ρ) is the ratio of mass to volume. Depending on whether the soil is dry, moist, or saturated, the density can be defined slightly differently. For example, **bulk density (ρ_m)** is defined considering the soil with all phases present and is defined as the ratio of the total mass to the total volume. Therefore,

$$\rho_m = \frac{M_t}{V_t}$$

It is also known as **moist, total, or wet density**.

Dry density (ρ_d) is defined as the ratio of the mass of the soil grains to the total volume, assuming there is no water in the voids, and the void volume remains the same. Therefore,

$$\rho_d = \frac{M_s}{V_t}$$

Saturated density (ρ_{sat}) assumes that the voids are filled with water. Therefore,

$$\rho_{sat} = \frac{M_t}{V_t}$$

Therefore, $\rho_{sat} \geq \rho_m \geq \rho_d$.

In submerged (and hence saturated) soils, the effective density, considering the buoyancy effects, is known as the submerged density or buoyant density (ρ'). It is defined as

$$\rho' = \rho_{sat} - \rho_w$$

The common units of density are g/cm^3 , Mg/m^3 , t/m^3 , and kg/m^3 . Density of water is 1.0 g/cm^3 , 1.0 Mg/m^3 , 1.0 t/m^3 , or 1000 kg/m^3 . Here, metric ton is denoted by t, which is 1000 kg.

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Unit weight (γ) is similar to density, where the mass (e.g., kg, g, t) is replaced by the weight (e.g., kN). It is the weight per unit volume.

The **bulk unit weight (γ_m)**, **dry unit weight (γ_d)**, **saturated unit weight (γ_{sat})**, and **submerged or buoyant unit weight (γ')** are defined similar to the way we defined densities

i- **Total unit weight, γ_t : (moist unit weight, γ_{moist}) (bulk unit weight, γ_{bulk}) (wet unit weight, γ_{wet})**

$$\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s[1 + (W_w/W_s)]}{V} = \frac{W_s(1 + w)}{V}$$

ii- **Dry unit weight γ_d :**

$$\gamma_d = \frac{W_s}{V} = \frac{\gamma_t}{1 + w} \quad \because (W_s + W_w) > W_s \quad \therefore \gamma_d < \gamma_t$$

iii- **Saturated unit weight γ_{sat} :** (when saturation, S=1)

$$\gamma_{sat} = \frac{W}{V} = \frac{W_s + W_w}{V}$$

iv- **Submerged unit weight, γ_{sub} : (buoyant unit weight, γ_b)**

$$\gamma_{sub} = \gamma_{sat} - \gamma_w$$

The unit weight is generally expressed in kN/m³.

Unit weight of water γ_w is **9.81 kN/m³**. From Newton's law, force = mass \times acceleration. Therefore, weight = mass \times gravitational acceleration (g), suggesting $\gamma = \rho g$.

$$\gamma(\text{kN/m}^3) = \frac{\rho(\text{kg/m}^3)}{1000} \times g$$

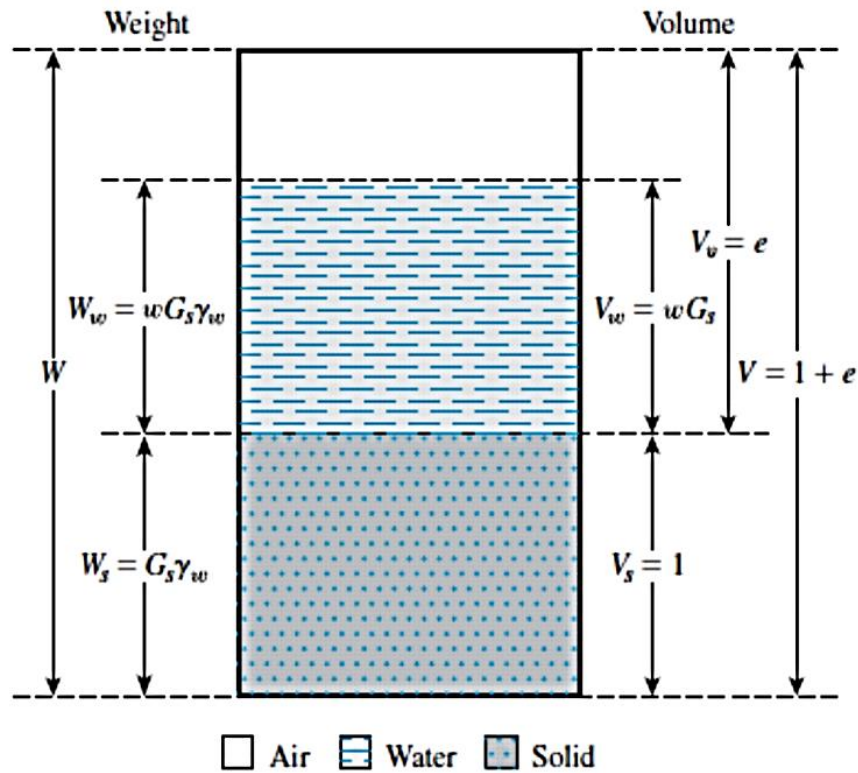
- **Specific Gravity (G_s)** of the soil grains says how much heavier the soil grains are compared to water. It is the ratio of the density of a soil grain (ρ_s) to density of water (ρ_w). Remember that the specific gravity of water is 1.0. Soil grains in general have G_s lying in a narrow range of **2.6–2.9**. Some organic soils and fly ash can have G_s significantly less.

$$G_s = \frac{\rho_s}{\rho_w} = \frac{M_s}{V_s \cdot \rho_w}$$

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{W_s}{V_s \cdot \gamma_w}$$

Soil Mechanics-Third Class**Phase Diagram for $V_s = 1$**

let us consider a small fraction of the soil where $V_s = 1$ (in any unit). From the definition of void, $V_v = e$. From the definition of degree of saturation, $V_w = Se$.



$$\gamma_s = G_s \gamma_w, \quad V_s = 1 \rightarrow W_s = G_s \gamma_w$$

$$W_w = w W_s = w G_s \gamma_w$$

$$\gamma_t = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + w G_s \gamma_w}{1 + e} = \frac{(1 + w) G_s \gamma_w}{1 + e}$$

and

$$\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w}{1 + e}$$

$$\gamma_w = \frac{W_w}{V_w} \rightarrow V_w = \frac{W_w}{\gamma_w} = \frac{w G_s \gamma_w}{\gamma_w} = G_s w$$

$$S = \frac{V_w}{V_v} = \frac{w G_s}{e}$$

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$$S \cdot e = w G_s$$

For saturated Soil

$$S = 1 \rightarrow e = w G_s$$

$$\gamma_{sat} = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{G_s \gamma_w + e \gamma_w}{1 + e} = \frac{(G_s + e) \gamma_w}{1 + e}$$

Several other forms of relationships that can be obtained for γ , γ_d , and γ_{sat} are given in Table below.

Table 2.1: Definitions of unit weight.

Moist unit weight (γ)		Dry unit weight (γ_d)		Saturated unit weight (γ_{sat})	
Given	Relationship	Given	Relationship	Given	Relationship
w, G_s, e	$\frac{(1+w)G_s\gamma_w}{1+e}$	γ, w	$\frac{\gamma}{1+w}$	G_s, e	$\frac{(G_s+e)\gamma_w}{1+e}$
S, G_s, e	$\frac{(G_s+Se)\gamma_w}{1+e}$	G_s, e	$\frac{G_s\gamma_w}{1+e}$	G_s, n	$[(1-n)G_s+n]\gamma_w$
w, G_s, S	$\frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$	G_s, n	$G_s\gamma_w(1-n)$	G_s, w_{sat}	$\left(\frac{1+w_{sat}}{1+w_{sat}G_s}\right)G_s\gamma_w$
w, G_s, n	$G_s\gamma_w(1-n)(1+w)$	G_s, w, S	$\frac{G_s\gamma_w}{1+\left(\frac{wG_s}{S}\right)}$	e, w_{sat}	$\left(\frac{e}{w_{sat}}\right)\left(\frac{1+w_{sat}}{1+e}\right)\gamma_w$
S, G_s, n	$G_s\gamma_w(1-n) + nS\gamma_w$	e, w, S	$\frac{eS\gamma_w}{(1+e)w}$	n, w_{sat}	$n\left(\frac{1+w_{sat}}{w_{sat}}\right)\gamma_w$
		γ_{sat}, e	$\gamma_{sat} - \frac{e\gamma_w}{1+e}$	γ_d, e	$\gamma_d + \left(\frac{e}{1+e}\right)\gamma_w$
		γ_{sat}, n	$\gamma_{sat} - n\gamma_w$	γ_d, n	$\gamma_d + n\gamma_w$
		γ_{sat}, G_s	$\frac{(\gamma_{sat} - \gamma_w)G_s}{(G_s - 1)}$	γ_d, S	$\left(1 - \frac{1}{G_s}\right)\gamma_d + \gamma_w$
				γ_d, w_{sat}	$\gamma_d(1+w_{sat})$

Example 2.1

A 50-mm-diameter and 150-mm-long undisturbed soil sample was collected from a site. The sample has a mass of 480 g, which becomes 350 g when dried in the oven at 105°C for 24 h. The specific gravity of the soil grains is 2.71. Determine the moisture content, void ratio, degree of saturation, and the bulk density.

Soil Mechanics-Third Class**Solution**

$$\text{Volume of the sample } V_t = \frac{\pi}{4} \times 5^2 \times 15 = 294.5 \text{ cm}^3$$

$$M_t = 480 \text{ g}, M_s = 350 \text{ g} \rightarrow M_w = 480 - 350 = 130 \text{ g}$$

$$\text{Moisture content } w = \frac{M_w}{M_s} = \frac{130}{350} = 0.371 = \text{or } 37.1\%$$

$$\text{Dry density } \rho_d = \frac{M_s}{V_t} = \frac{350}{294.5} = 1.19 \text{ g / cm}^3$$

From Eq. (2.14),

$$\rho_d = \left(\frac{G_s}{1 + e} \right) \rho_w$$

$$1.19 = \left(\frac{2.71}{1 + e} \right) 1.0$$

$$e = 1.28$$

$$w = \frac{Se}{G_s}$$

$$S = \frac{wG_s}{e} = \frac{0.371 \times 2.71}{1.28} = 0.786 \text{ or } 78.6\%$$

$$\text{Bulk density } \rho_m = \frac{M_t}{V_t} = \frac{480}{294.5} = 1.63 \text{ g / cm}^3$$

Soil Mechanics-Third Class**Example 2.2**

The soil from a borrow area is excavated and used in a compacted fill for an embankment. The void ratio at the borrow area and the compacted fill are 0.96 and 0.62, respectively. What would be the volume of the excavation in the borrow area to construct 200,000 m³ of compacted fill?

Solution In the compacted fill (see Fig. 2.4),

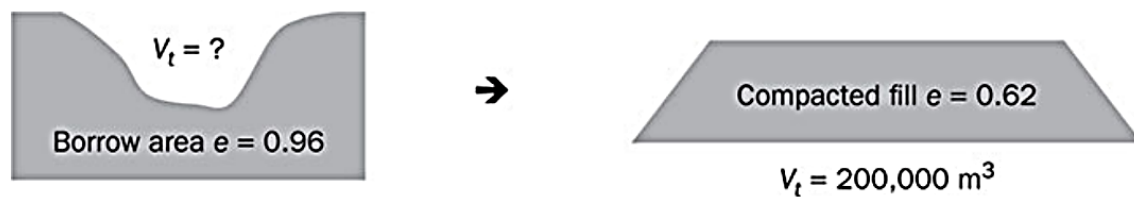


FIGURE 2.4 Borrow area and constructed embankment.

$$e = \frac{V_v}{V_s}$$

$$0.62 = \frac{200,000 - V_s}{V_s}$$

Therefore, $V_s = 123,456.8 \text{ m}^3$

Soil grains are incompressible and hence V_s (and M_s) remains the same in the borrow area and the compacted fill.

In the borrow area,

$$e = \frac{V_v}{V_s}$$

$$0.96 = \frac{V_v}{123456.8}$$

Therefore, $V_v = 118,518.5 \text{ m}^3$

Volume of the excavation $V_t = V_s + V_v = 123,456.8 + 118,518.5 =$
241,975.3 m³

Note that 241,975.3 m³ of soil is compacted into 200,000 m³.

Soil Mechanics-Third Class**Example 2.3**

A subbase for an airport runway, 100 m wide, 2000 m long, and 500 mm thick, is to be constructed out of a clayey sand excavated from a nearby borrow, where the in situ moisture content is 6%. This soil is being transported by trucks having a capacity of 8 m³, where each load weighs 13.2 metric tons (1 metric ton = 1000 kg). In the subbase course, the soil will be placed at moisture content of 14.2% to a dry density of 1.89 t/m³. Specific gravity of the soil grains is 2.72.

- How many truckloads will be required to complete the job?
- How many liters of water should be added to each truckload?
- If the subbase becomes saturated due to rain, what would be the new moisture content?

Solution Let us summarize the given data in Fig. 2.5.

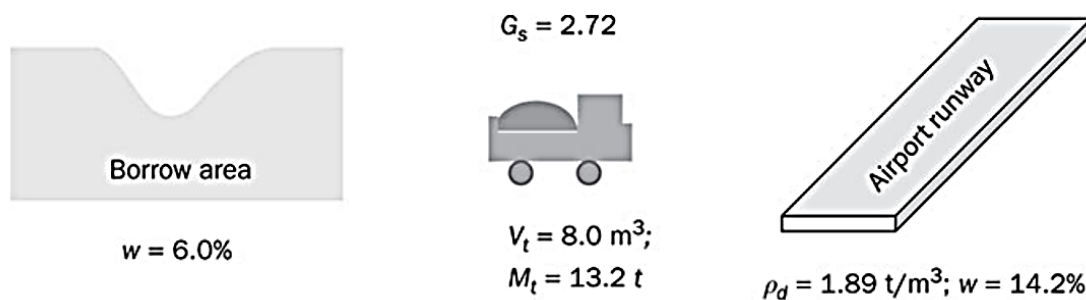


FIGURE 2.5 Borrow area, truck and the runway.

Part a At the runway, $V_t = 100 \times 2000 \times 0.5 = 100,000 \text{ m}^3$

$$M_s = 1.89 \times 100,000 = 189,000 \text{ tonnes}$$

$$M_w = 0.142 \times 189,000 = 26,838 \text{ tonnes}$$

In the truck, $V_t = 8.0 \text{ m}^3$

$$M_t = 13.2 \text{ tonnes}$$

Assuming the same 6% moisture content for the soil in the truck as in the borrow,

$$0.06 = \frac{13.2 - M_s}{M_s}$$

$$M_s = 12.45 \text{ tonnes}$$

Therefore, the number of truckloads required = $189,000/12.45 = \mathbf{15,181}$.

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Part b For each truckload, the moisture content will be increased from 6% to 14.2%. Therefore, the mass of water to add per truckload = $12.45(0.142 - 0.06) = 1.0209$ tonnes or **1020.9 L**.

Part c

$$\rho_d = \frac{G_s \rho_w}{1 + e}$$

$$e = \frac{2.721.0}{1.89} - 1 = 0.439$$

$$w = \frac{Se}{G_s}$$

If the subbase becomes saturated, $S = 1$ and hence

$$w = \frac{1 \times 0.439}{2.72} = 0.161 \text{ or } 16.1\%$$

There will be a slight increase in the moisture content.

Example 2.4

It is required to carry out a compaction test on a soil, which will be placed into a cylindrical mold and compacted at moisture content of 10.0%. The current moisture content of the soil is 2.5%. How much water would you add to 5.0 kg of the above soil to bring the moisture content to 10.0%?

The specific gravity of the soil grains is 2.68. If the soil at moisture content of 10% is compacted to a bulk unit weight of 17.5 kN/m^3 , determine the void ratio and degree of saturation.

Solution Let the mass of soil grains in the 5 kg of soil be x .

$$0.025 = \frac{5 - x}{x}$$

Therefore, $x = M_s = 4.878 \text{ kg}$ and hence $M_w = 5 - 4.878 = 0.122 \text{ kg}$

At $w = 10\%$, $M_w = 0.10 \times 4.878 = 0.488 \text{ kg}$

Therefore, the water to be added = $488 - 122 = 366 \text{ g}$

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When compacted to $w = 10\%$,

$$w = \frac{Se}{G_s}$$

$$Se = 0.10 \times 2.68 = 0.268$$

$$\gamma_m = \left(\frac{G_s + Se}{1 + e} \right) \gamma_w$$

$$17.5 = \left(\frac{2.68 + 0.268}{1 + e} \right) 9.81$$

Therefore, $e = 0.653$

$$S = \frac{wG_s}{e} = \frac{0.10 \times 2.68}{0.653} = 0.410 \text{ or } 41.0\%$$

Example 2.5

Some perfectly spherical grains of the same diameter d were stacked on top of each other as shown in Fig. 2.6. This gives the loosest possible packing. What is the void ratio? Volume of a sphere = $\pi d^3/6$.

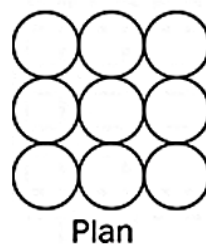
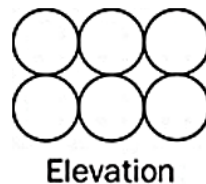


FIGURE 2.6 Elevation and plan views of the grain packing.

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Solution Considering a cube of width d ,

$$V_t = d^3$$
$$V_s = \pi d^3 / 6 = 0.5236 d^3$$

Therefore, $V_v = d^3 - \pi d^3 / 6 = 0.4764 d^3$

$$e = \frac{V_v}{V_s} = \frac{0.4764}{0.5236} = 0.91$$

Example 2.6

A saturated clay specimen with volume of 162.1 cm^3 has a mass of 295.3 g . When dried in the oven for 24 h at 105°C , the mass is 209.6 g . Determine the specific gravity of the soil grains.

Solution

The mass of water in the sample = $295.3 - 209.6 = 85.7 \text{ g}$

Volume of water (i.e., voids too) in the sample = 85.7 cm^3

Volume of the soil grains = $162.1 - 85.7 = 76.4 \text{ cm}^3$

Mass of soil grains = 209.6 g

Therefore, $G_s = 209.6 / 76.4 = 2.74$

Soil Mechanics-Third Class**Example 2.7**

A 3-m-thick saturated soft clay deposit is currently at natural moisture content of 115.0%. The site is surcharged with some fill placed at the ground level, and the clay layer settled by 500 mm while remaining saturated. What would be the new moisture content? Assume specific gravity of the soil grains as 2.75.

Solution

$$S = 1, w = 1.15, G_s = 2.75$$

Therefore, the initial void ratio is given by

$$e = 1.15 \times 2.75 = 3.16$$

Let us consider 1.0 m² plan area, where $V_t = 1 \times 3 = 3.0 \text{ m}^3$

$$e = 3.16 = \frac{V_v}{V_s} = \frac{3.0 - V_s}{V_s}$$

$$V_s = 0.721 \text{ m}^3$$

After settling by 0.5 m to 2.50 m, from 3.00 m to 2.50 m,

$$V_t = 2.5 \text{ m} \times 1 \text{ m}^2 = 2.5 \text{ m}^3$$

Therefore, the new volume of voids is given by $V_v = 2.5 - 0.721 = 1.779 \text{ m}^3$

The new void ratio is

$$e = \frac{1.779}{0.721} = 2.467$$

The new moisture content is $2.467/2.75 = \mathbf{0.897}$ or **89.7%**.

Soil Mechanics-Third Class**Example 2.8**

The soil at a borrow area is at moisture content of 10.5% and bulk unit weight of 18.5 kN/m^3 . The specific gravity of the soil grains is 2.72. This soil is taken in a dump truck to the site for making a compacted fill, carrying 90 kN per load. The fill at the site is compacted to dry unit weight of 18.5 kN/m^3 at moisture content of 15%. What would be the volume of the compacted fill from 100 truckloads? What would be the volume of the hole in the borrow area for the 100 truckloads? Assume that the moisture content of the soil in the truck is the same as that in the borrow area. What is the degree of saturation of the compacted fill?

Solution

The weight of soil grains carried per truckload = $90.0 / (1 + 0.105) = 81.45 \text{ kN}$

Weight of the soil grains at the compacted fill = $100 \times 81.45 = 8145 \text{ kN}$

Volume of the compacted fill = $8145 / 18.5 = 440.27 \text{ m}^3$

Dry unit weight of the soil at the borrow = $18.5 / (1 + 0.105) = 16.74 \text{ kN/m}^3$

Therefore, the volume of the hole = $8145 / 16.74 = 486.56 \text{ m}^3$

At the compacted fill,

$$\gamma_d = \frac{G_s \gamma_w}{1 + e}$$

Therefore,

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{2.72 \times 9.81}{18.5} - 1 = 0.442$$

Since

$$w = \frac{Se}{G_s}$$

$$S = \frac{0.15 \times 2.72}{0.442} = 0.923 \text{ or } 92.3\%$$



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2.5 Main Points

1. The computations of masses and volumes of the different phases and the parameters such as moisture content, void ratio, and densities are an integral part of most laboratory tests. These computations can be carried out in terms of (a) masses and densities or (b) weights and unit weights—never mix them.
2. The definitions of void ratio, moisture content, degree of saturation, specific gravity, and the different densities are logical and make sense.
3. G_s varies in a narrow range of 2.6–2.9 for most soils. In the absence of laboratory-determined values, it is possible to assume a value for G_s .
4. With the different variables ($w, e, n, S, \rho_m, \rho_d, \rho_{sat}, \rho'$) discussed in this chapter, it is possible to develop more than a dozen equations relating each of them in terms of few other variables. The equations presented in this chapter are adequate for all computations.

2.6 Consistency of soil

Soil consistency describes the degree and kind of cohesion and adhesion between the soil particles as related to the resistance of the soil to deform or rupture.

- Since the consistency varies with moisture content, the consistency can be described as dry consistency and moist consistency.
- Consistency largely depends on soil minerals and the water content.

Cohesion & Adhesion

- Cohesion is the attraction of one water molecule to another resulting from hydrogen bonding (water-water bond).
- Adhesion is the attraction of a water molecule to a non-water molecule (water-solid bond).

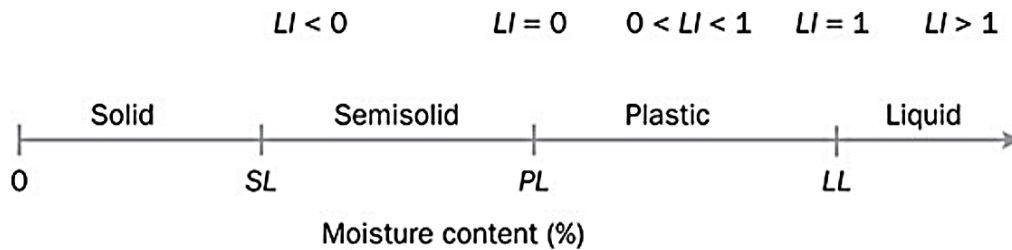
2.7 Atterberg's limits

- Atterberg limits were originally developed by a Swedish soil scientist Albert Atterberg, working in the ceramics industry in the early 1900s.
- Casagrande (1932) modified them to suit geotechnical applications. When the moisture content of a fine grained soil is increased from zero, its consistency (“how it feels between your fingers”) changes gradually, going through different states—solid, semisolid, plastic, and liquid.



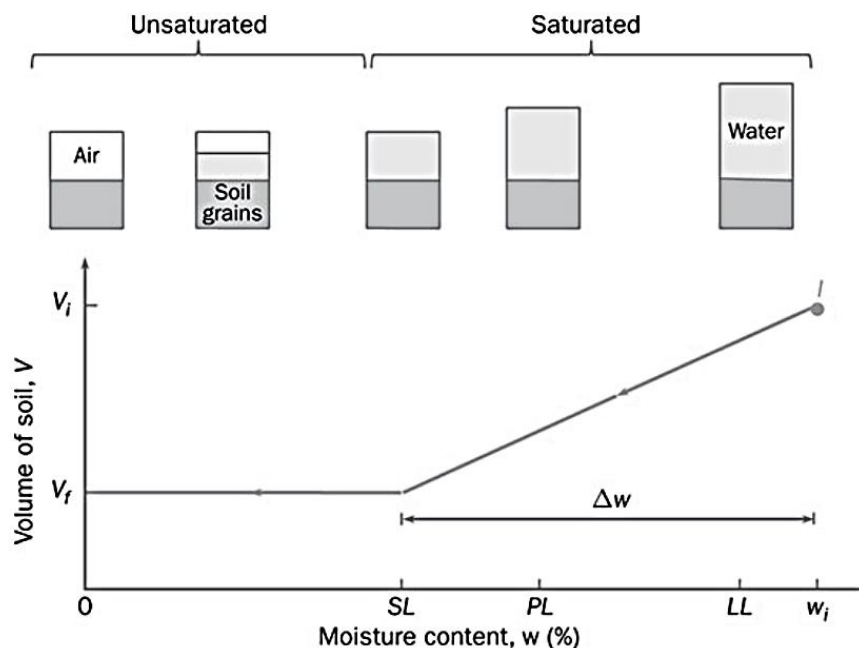
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The three border-line moisture contents separating the four states are known as the Atterberg limits. They are, shrinkage limit (SL or w_s), plastic limit (PL or w_p), and liquid limit (LL or w_L).



- The physical and mechanical behavior of fine-grained soils is linked to four distinct states: solid, semisolid, plastic, and liquid, in order of increasing water content.
- Shrinkage limit (SL) is the highest moisture content below which there will be no reduction in volume due to loss of moisture. It is also the lowest moisture content a saturated clay can have.
- Above SL, soil remains saturated while the moisture content decreases; the reduction in volume is due to the reduction in voids.
- Below SL, there is no reduction in voids when moisture is lost; there is only reduction in water within the voids, and the degree of saturation decreases.
- Shrinkage limit is the limiting water content between solid and semisolid states, where the soil volume is the minimum and it is saturated.
- This minimum void ratio occurring at the shrinkage limit, assuming $S = 100\%$, is given by:

$$Se = wGs \quad \text{where } (w = SL, S = 1) \Rightarrow e_{SL} = SL \times Gs$$





Soil Mechanics-Third Class

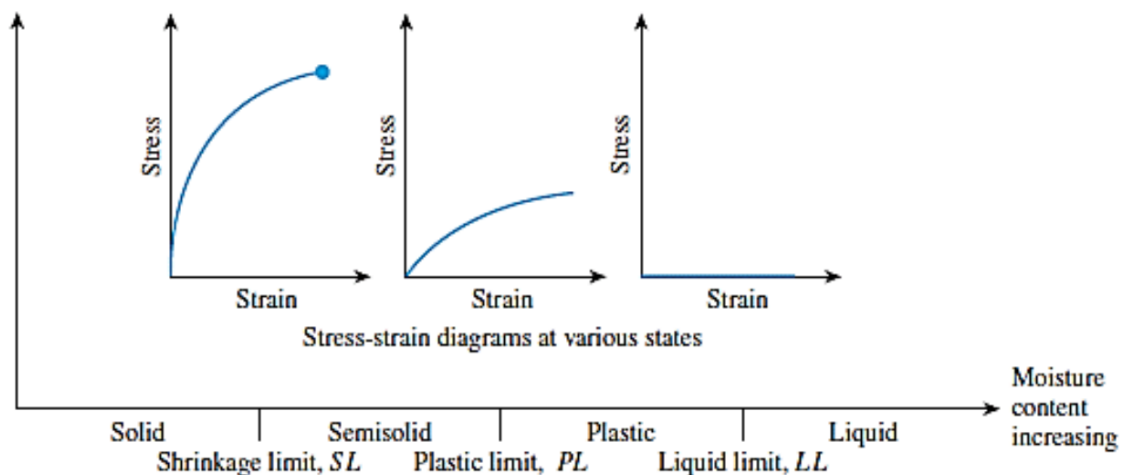
Atterberg's limits are the limits of water content and important to describe the consistency of fine textured soils:

Liquid Limit (LL) is the water content at which soil begins to behave as a liquid material and begins to flow (normally below 100).

Plastic Limit (PL) is the water content at which soil begins to behave as a plastic material (normally below 40).

Shrinkage Limit (SL) is the water content at which no further volume change occurs with further reduction in water content.

- Four states are used to describe the soil consistency; solid, semi-solid, plastic and liquid.



- At moisture content greater than the liquid limit (LL), the clay behaves like a liquid with negligible strength. Between PL and LL, in the plastic range, the soil behaves like a plastic solid.
- At moisture content less than PL, the soil is no more plastic. It is in a semisolid state.
- The range of moisture content, over which the fine-grained soil remains plastic, is called plasticity index (PI), and is defined as

$$PI = LL - PL$$

- Strength decreases as water content increases;
- Soils swell-up when water content increases.

- Plasticity index typically lies in the range of 0–50, with larger values for some special clays such as montmorillonites. Burmister (1949) classified fine-grained soils based on plasticity index as shown in the following Table. Pure silts are nonplastic, with $PI \approx 0$.

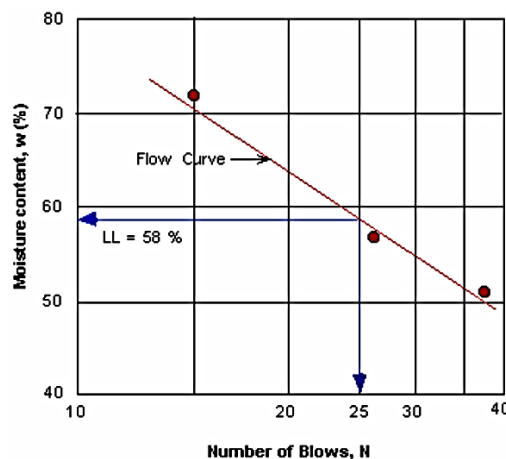
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PI	Classification
0	Nonplastic
1-5	Slightly plastic
5-10	Low plastic
10-20	Medium plastic
20-40	High plastic
>40	Very high plastic

a) Liquid limit

In the lab, the LL is defined as the water content required closing a 2mm wide groove in a soil sample for a distance of 0.5 in long after 25 blows.

- ASTM D 4318.
- Equipment: Casagrande liquid limit device.



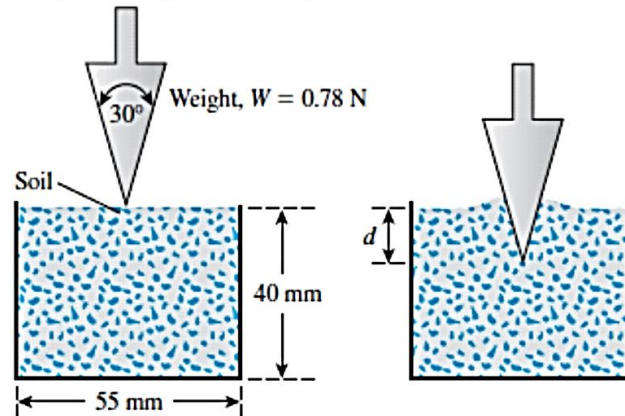
The slope of the flow line is defined as the flow index (I_F) and may be written as:

$$I_F = \frac{\omega_1 - \omega_2}{\log \frac{N_2}{N_1}}$$

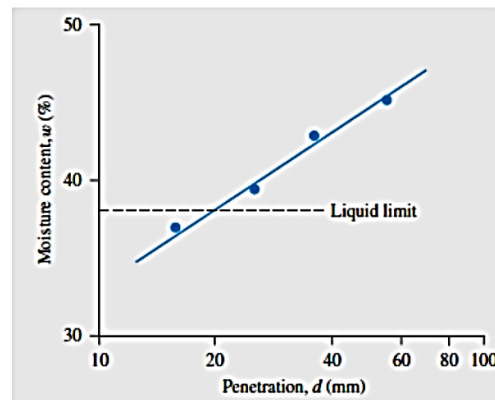
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Another method of determining liquid limit is the fall cone method (British Standard - BS1377).

- In this test, the liquid limit is defined as the moisture content at which a standard cone of apex angle 30° and weight of 0.78 N will penetrate a distance $d = 20\text{ mm}$ in 5 seconds when allowed to drop from a position of point contact with the soil surface.



- Due to the difficulty in achieving the liquid limit from a single test, four or more tests can be conducted at various moisture contents to determine the fall cone penetration, d .
- A semilogarithmic graph can then be plotted with moisture content (w) versus cone penetration d . The plot results in a straight line. The moisture content corresponding to $d = 20\text{ mm}$ is the liquid limit.



b) Plastic limit

In the lab, the plastic limit (PL) is defined as the water content at which the soil when rolled into threads of 3.2 mm ($1/8\text{ in}$) in diameter, will crumble.

- ASTM D-4318.



Soil Mechanics-Third Class**c) Shrinkage limit**

In the lab, the moisture content, in percent, at which the volume of the soil mass ceases to change.

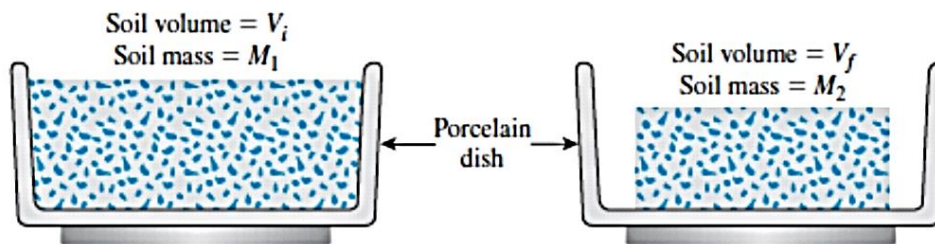
- ASTM Test Designation D-427;
- The procedure of test is:
 - 1) Prepare the soil sample;
 - 2) Prepare a porcelain dish about 44 mm (1.75 in.) in diameter and about 12.7 mm (0.5 in.) high. The inside of the dish is coated with petroleum jelly and is then filled completely with wet soil;
 - 3) The mass of the wet soil inside the dish is recorded;
 - 4) The soil pat in the dish is then oven-dried;
 - 5) The volume of the oven-dried soil pat is determined by the displacement of mercury.

$$SL = w_i(\%) - \Delta w (\%)$$

where

w_i = initial moisture content when the soil is placed in the shrinkage limit dish;

Δw = change in moisture content (that is, between the initial moisture content and the moisture content at the shrinkage limit).



$$w_i(\%) = \frac{M_1 - M_2}{M_2} \times 100$$

where

M_1 = mass of the wet soil pat at the beginning of the test (g).

M_2 = mass of the dry soil pat (g).

Also,

$$\Delta w(\%) = \frac{(V_i - V_f)\rho_w}{M_2} (100)$$

where

V_i = initial volume of the wet soil pat (that is, inside volume of the dish, cm^3).

V_f = volume of the oven-dried soil pat (cm^3).

ρ_w = density of water (g/cm^3).

$$SL = \frac{M_1 - M_2}{M_2} (100) - \frac{V_i - V_f}{M_2} (\rho_w)(100)$$



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2.8 Liquidity Index and Consistency Index

The in-place natural moisture content (w_n) of a fine-grained soil can be compared to the moisture content scale, to know how the soil will behave. This is quantified through liquidity index (LI or IL) defined as

$$LI = \frac{w_n - PL}{LL - PL}$$

- A fine-grained soil with $LI > 1$ has moisture content larger than the liquid limit and flows like a liquid.
- The position of current moisture content in relation to the liquid and plastic limits better defines the state of the fine grained soil than simply the moisture content alone.

(For example, clay A at moisture content of 40% ($PL = 35$ and $LL = 80$) will be stiffer and display higher strength than clay B at the same moisture content of 40% ($PL = 17$ and $LL = 42$). Here, A is very close to the plastic limit and B is very close to the liquid limit and would be at the verge of flowing like a liquid).

Values of LI	Description of soil strength
$LI < 0$	Semisolid state: high strength, brittle (sudden) fracture is expected
$0 < LI < 1$	Plastic state: intermediate strength, soil deforms like a plastic material
$LI > 1$	Liquid state: low strength, soil deforms like a viscous fluid

Another index that is commonly used for engineering purposes is the consistency index (CI), which may be defined as:

$$CI = \frac{LL - w_n}{LL - PL}$$

Description	CI
Very soft (ooze out of finger when squeezed)	< 0.25
Soft (easily molded by finger)	0.25–0.50
Firm or medium (can be molded using strong finger pressure)	0.50–0.75
Stiff (finger pressure dents soil)	0.75–1.00
Very stiff (finger pressure barely dents soil, but soil cracks under significant pressure)	> 1

2.9 Activity

Alec Skempton (1953) showed that for soils with a particular mineralogy, the plasticity index is linearly related to the amount of the clay fraction. He coined a term called activity (A) to describe the importance of the clay fractions on the plasticity index. The equation for A is:

$$A = \frac{PI}{\text{Clay fraction (\%)}}$$



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The clay fraction in the equation is the amount of particles less than $2\mu\text{m}$. You should recall that ASTM-USCS delineates clay as less than $5\mu\text{m}$. Activity is one of the factors used in identifying expansive or swelling soils. Typical values of activity are given in the following table.

Description	Activity, <i>A</i>
Inactive	<0.75
Normal	0.75–1.25
Active	1.25–2
Very (highly) active (e.g., montmorillonite or bentonite)	>6
Minerals	
Kaolinite	0.3–0.5
Illite	0.5–1.3
Na-montmorillonite	4–7
Ca-montmorillonite	0.5–2.0

Key points

1. Fine-grained soils can exist in one of four states: solid, semisolid, plastic, or liquid.
2. Water is the agent that is responsible for changing the states of soils.
3. A soil gets weaker if its water content increases.
4. Three limits are defined based on the water content that causes a change of state. These are the liquid limit—the water content that caused the soil to change from a liquid to a plastic state; the plastic limit—the water content that caused the soil to change from a plastic to a semisolid; and the shrinkage limit—the water content that caused the soil to change from a semisolid to a solid state. Water contents at approximately these limits are found from laboratory tests.
5. The plasticity index defines the range of water content for which the soil behaves like a plastic material.
6. The liquidity index gives a qualitative measure of strength.
7. The soil strength is lowest at the liquid state and highest at the solid state.

2.10 Plasticity chart

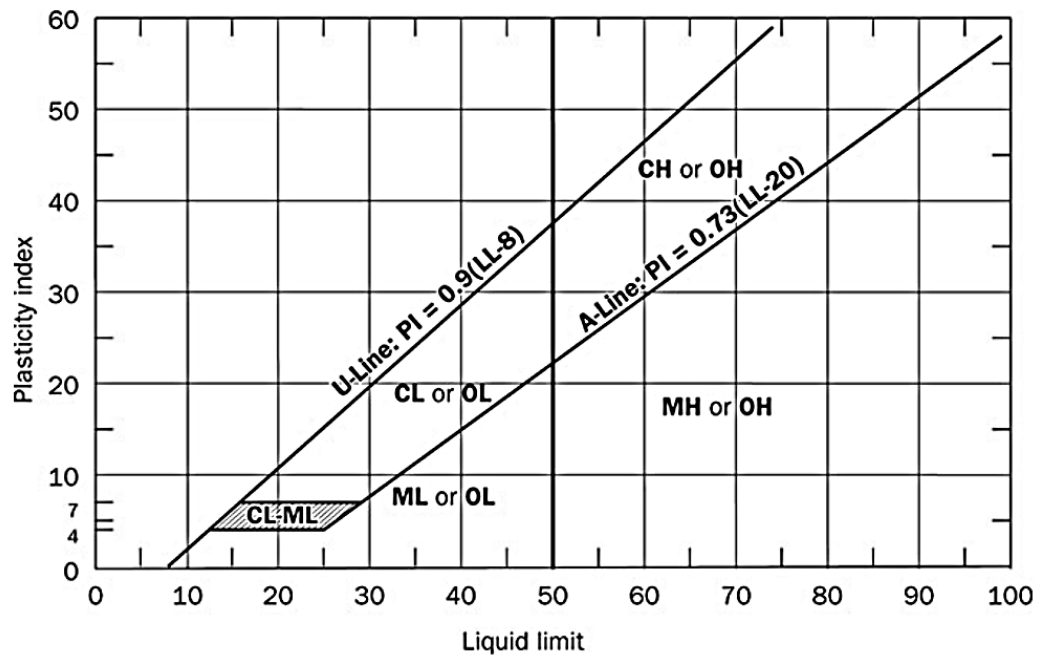
Experimental results from soils tested from different parts of the world were plotted on a graph of plasticity index (ordinate) versus liquid limit (abscissa). It was found that clays, silts, and organic soils lie in distinct regions of the graph. A line defined by the equation:

$$PI = 0.73(LL - 20), \quad PI \geq 4$$

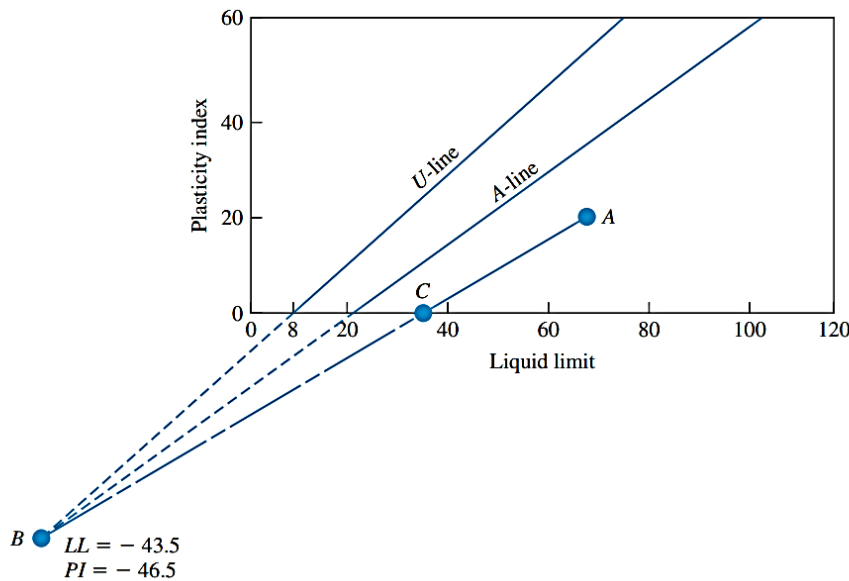
called the “A-line,” delineates the boundaries between clays (above the line) and silts and organic soils (below the line), as shown in the following figure. A second line, the U-line, expressed as $PI = 0.9(LL - 8) \%$, defines the upper limit of the correlation between plasticity index and liquid limit. If the results of your soil tests fall above the U-line, you should be suspicious of your results and repeat your tests.



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Casagrande has suggested that the SL of a soil can be approximately determined if its PI and LL are known



- 1) Plot the PI against the LL of a given soil such as point A in on the plasticity chart;
- 2) Project the A-line and the U-line downward to meet at point B. Point B will have the coordinates of $LL=43.5$ and $PI=46.4$;
- 3) Join points B and A with a straight line. This will intersect the liquid limit axis at point C. The point C is the estimated shrinkage limit.

Or by using the following equation:

$$SL = 46.4 \left(\frac{LL + 43.5}{PI + 46.4} \right) - 43.5$$

**Soil Mechanics-Third Class****EXAMPLE 2.9**

Calculations of Plasticity Index, Liquidity Index, and Activity A fine-grained soil has a liquid limit of 300% and a plastic limit of 55%. The natural water content of the soil in the field is 80% and the clay content is 60%. Determine the plasticity index, the liquidity index, and the activity.

$$PI = LL - PL = 300 - 55 = 245\%$$

$$LI = \frac{w - PL}{PI} = \frac{80 - 55}{245} = 0.1$$

$$A = \frac{PI}{\text{Clay fraction (\%)}} = \frac{245}{60} = 4.1$$

EXAMPLE 2.10 Interpreting Liquid Limit Data from Casagrande's Cup Device

A liquid limit test, conducted on a soil sample in the cup device, gave the following results:

Number of blows	10	19	23	27	40
Water content (%)	60.0	45.2	39.8	36.5	25.2

Two determinations for the plastic limit gave water contents of 20.3% and 20.8%. Determine (a) the liquid limit and plastic limit, (b) the plasticity index, (c) the liquidity index if the natural water content is 27.4%, (d) the void ratio at the liquid limit if $G_s = 2.7$ and (e) estimate the shrinkage limit. If the soil were to be loaded to failure, would you expect a brittle failure?

Strategy To get the liquid limit, you must make a semi-logarithmic plot of water content versus number of blows. Use the data to make your plot; then extract the liquid limit (water content on the liquid state line corresponding to 25 blows). Two determinations of the plastic limit were made, and the differences in the results were small. So, use the average value of water content as the plastic limit.

Step 1: Plot the data.

See Figure E2.10.

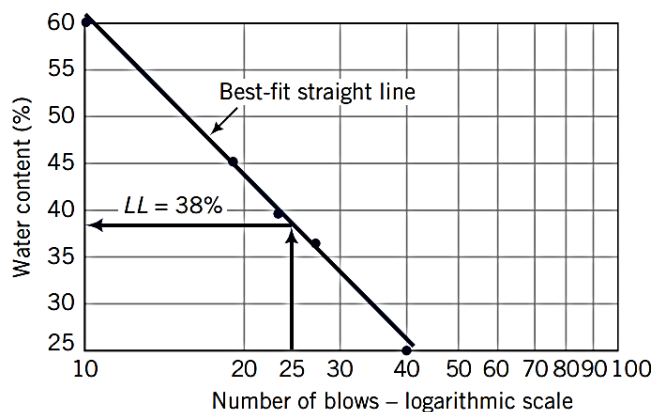


Figure E2.10

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Step 2: Extract the liquid limit.

The water content on the liquid state line corresponding to a terminal blow of 25 gives the liquid limit.

$$LL = 38\%$$

Step 3: Calculate the plastic limit.

The plastic limit is

$$PL = \frac{20.3 + 20.8}{2} = 20.6\%$$

Step 4: Calculate PI .

$$PI = LL - PL = 38 - 20.6 = 17.4\%$$

The LL , PL , and PI are reasonable for typical soils (Table 2.5).

Step 5: Calculate LI .

$$LI = \frac{(w - PL)}{PI} = \frac{27.4 - 20.6}{17.4} = 0.39$$

Step 6: Calculate the void ratio.

Assume the soil is saturated at the liquid limit. For a saturated soil, $e = wG_s$. Thus,

$$e_{LL} = LLG_s = 0.38 \times 2.7 = 1.03$$

Step 7: Estimate the shrinkage limit.

$$SL = 46.4 \left(\frac{LL + 45.5}{PI + 46.4} \right) - 43.5 = 46.4 \left(\frac{38 + 45.5}{17.4 + 46.4} \right) - 43.5 = 17.2\%$$

Step 8: Estimate type of failure.

Brittle failure is not expected, as the soil is in a plastic state ($0 < LI < 1$).

Example 2.11 A clay sample has dry density of 1.85 g/cm^3 and $G_s = 2.71$. Estimate the shrinkage limit of the clay. State your assumptions

Solution

$$\rho_d = \frac{G_s \rho_w}{1 + e}$$

Therefore,

$$e = \frac{G_s \rho_w}{\rho_d} - 1 = \frac{2.71 \times 1.0}{1.85} - 1 = 0.460$$

Assuming that the soil has dried to the void ratio at shrinkage limit, with no further reduction in void ratio during drying, $e_{SL} = 0.460$.

Assuming the clay to be saturated at shrinkage limit,

$$w = \frac{Se}{G_s} = \frac{0.460}{2.71} = 0.17 \text{ or } 17\%$$