5 FLOW IN SOILS

5.1 Introduction

Soils are permeable due to the existence of interconnected voids through which water can flow from points of high energy to points of low energy. The study of the flow of water through permeable soil media is important in soil mechanics.

The flow of water has caused instability and failure of many geotechnical structures (e.g., roads, bridges, dams, and excavations). Therefore, you need to understand how water flows through soil and the stresses it induces.

Learning outcomes

When you complete this chapter, you should be able to do the following:

■ Determine the rate of flow of water through soils.

■ Determine the hydraulic conductivity of soils.

■ Calculate flow under earth structures.

5.2 Bernoulli's Equation

From fluid mechanics, we know that, according to Bernoulli's equation, the total head at a point in water under motion can be given by the sum of the pressure, velocity, and elevation heads, or

$$h = \underbrace{\frac{u}{\gamma_w}}_{\text{Pressure head}} + \underbrace{\frac{v^2}{2g}}_{\text{Velocity head}} + \underbrace{Z}_{\text{Elevation head}}$$

where h = total head
u = pressure
v = velocity
g = acceleration due to gravity
γ_w = unit weight of water

If Bernoulli's equation is applied to the flow of water through a porous soil medium, the term containing the velocity head can be neglected because the seepage velocity is small, and the total head at any point can be adequately represented by

$$h = \frac{u}{\gamma_w} + Z$$

Figure 5.1 shows the relationship among pressure, elevation, and total heads for the flow of water through soil. Open standpipes called piezometers are installed at points A and B. The levels to which water rises in the piezometer tubes situated at points A and B are known as the piezometric levels of points A and B, respectively.

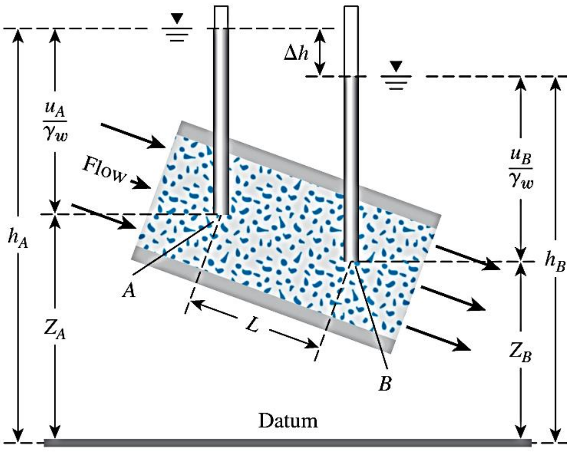


Figure 5.1: Pressure, elevation, and total heads for flow of water through soil.

The loss of head between two points, A and B, can be given by

$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right)$$

The head loss, h, can be expressed in a nondimensional form as

$$i = \frac{\Delta h}{L}$$

where i =hydraulic gradient, L distance between points A and B (that is, the length of flow over which the loss of head occurred)

In general, the variation of the velocity v with the hydraulic gradient i is as shown in Figure 5.2. This figure is divided into three zones:

1. Laminar flow zone (Zone I)
2. Transition zone (Zone II)
3. Turbulent flow zone (Zone III)

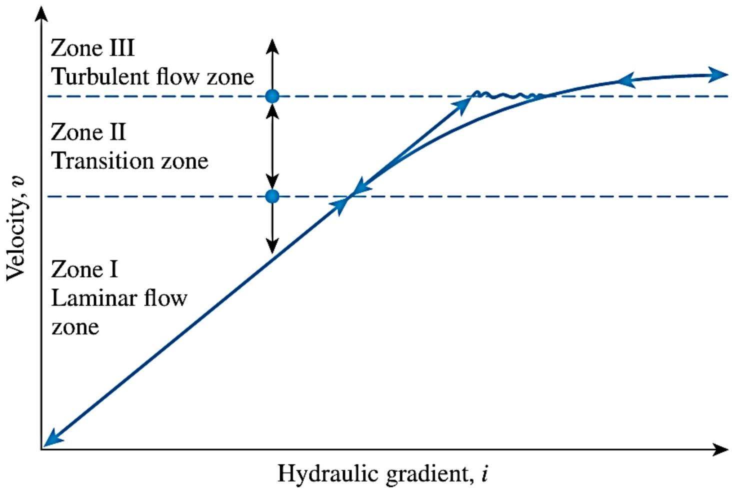


Figure 5.2: Nature of variation of v with hydraulic gradient, i.

5.3 Darcy's Law

In 1856, Darcy published an empirical relationship for the laminar flow of water through saturated porous media known as Darcy's Law:

$$v = ki$$
$$q = kiA$$

where

v=discharge velocity, which is the quantity of water flowing in unit time through a unit gross cross-sectional area of soil at right angles to the direction of flow

k = hydraulic conductivity (otherwise known as the coefficient of permeability)

q = flow rate

A = cross-sectional area

i = hydraulic gradient

v is the discharge velocity of water based on the gross cross-sectional area of the soil. However, the actual velocity of water (that is, the seepage velocity) through the void spaces is greater than v.

A relationship between the discharge velocity and the seepage velocity can be derived as follows:

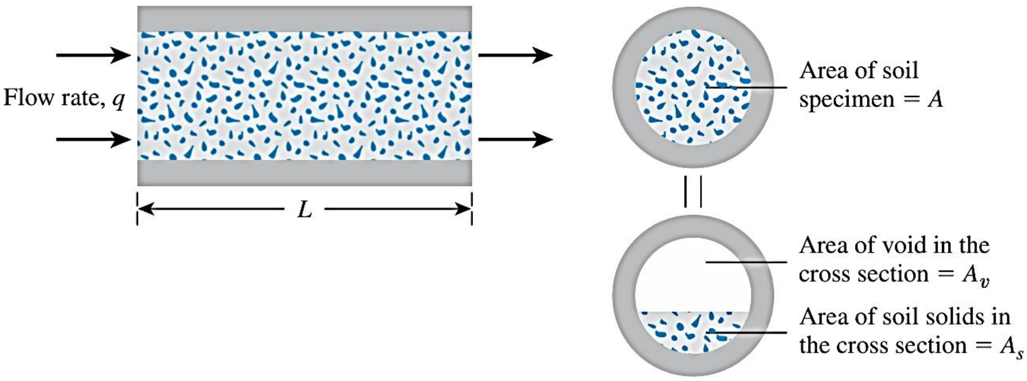


Figure 5.3: Discharge velocity and seepage velocity.

$$q = vA = A_v v_s$$

where v_s = seepage velocity, A_v = area of void in the cross section of the specimen.

$$A = A_v + A_s$$

where
As = area of soil solids in the cross section of the specimen.

$$q = v(A_v + A_s) = A_v v_s$$

or

$$v_s = \frac{v(A_v + A_s)}{A_v} = \frac{v(A_v + A_s)\,L}{A_v L} = \frac{v(V_v + V_s)}{V_v}$$

where V_v = volume of voids in the specimen
V_s = volume of soil solids in the specimen

Equation (7.9) can be rewritten as

$$v_s = v\left[\frac{1 + \left(\frac{V_v}{V_s}\right)}{\frac{V_v}{V_s}}\right] = v\left(\frac{1 + e}{e}\right) = \frac{v}{n}$$

where e = void ratio
n = porosity

5.4 Coefficient of permeability (Hydraulic conductivity)

- Hydraulic conductivity is generally expressed in cm/sec or m/sec in SI units.
- The hydraulic conductivity of soils depends on several factors: fluid viscosity, pore size distribution, grain-size distribution, void ratio, roughness of mineral particles, and degree of soil saturation. In clayey soils, structure plays an important role in hydraulic conductivity.
- The value of hydraulic conductivity (k) varies widely for different soils. Some typical values for saturated soils are given in Table 5.1. The hydraulic conductivity of unsaturated soils is lower and increases rapidly with the degree of saturation.

Table 7.1Typical Values of Hydraulic Conductivity of Saturated Soils

Soil type	*k* cm/sec
Clean gravel	100–1.0
Coarse sand	1.0–0.01
Fine sand	0.01–0.001
Silty clay	0.001–0.00001
Clay	<0.000001

5.5 Laboratory determination of hydraulic conductivity

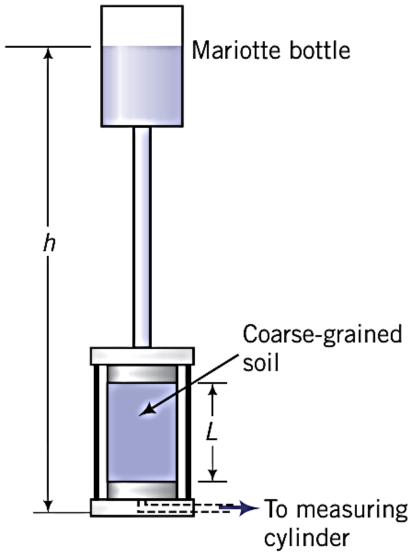
Two standard laboratory tests are used to determine the hydraulic conductivity.

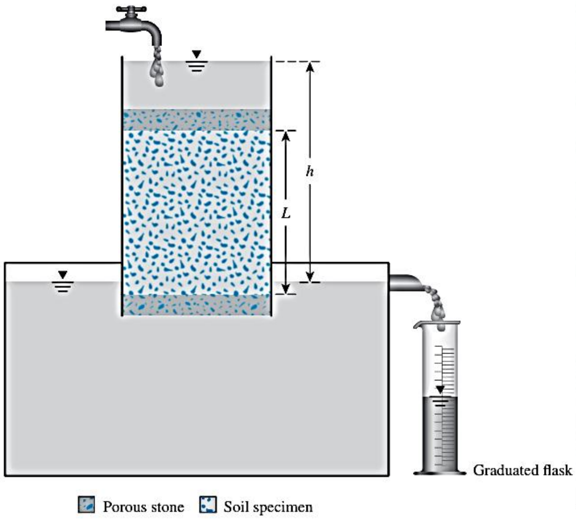
a) Constant head test

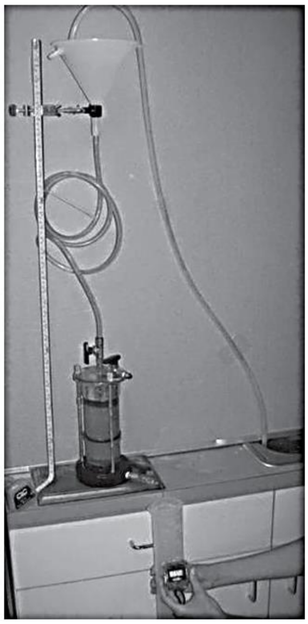
➢ The constant head test is used primarily for coarse-grained soils;

➢ The test is based on the assumption of laminar flow where k is independent of i;

➢ ASTM D 2434;







$$Q = Avt = A(ki)t$$

$$i = \frac{h}{L}$$

$$Q = A\left(k\frac{h}{L}\right)t \quad \Longrightarrow \quad k = \frac{QL}{Aht}$$

Q is volume of water collected; A is cross sectional area of soil specimen; t is duration of water collection and L is specimen length.

b) Falling head test (Variable head test)

➢ The falling head test is used both for coarse-grained soils as well as fine- textured soils;

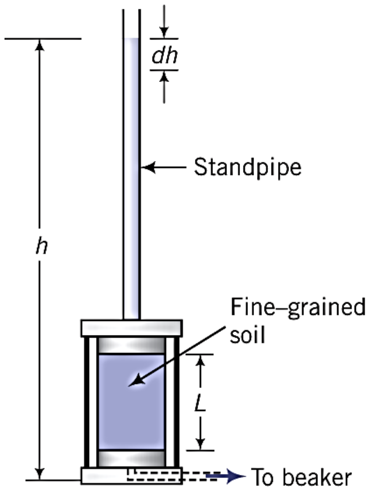
➢ Same procedure in constant head test except:

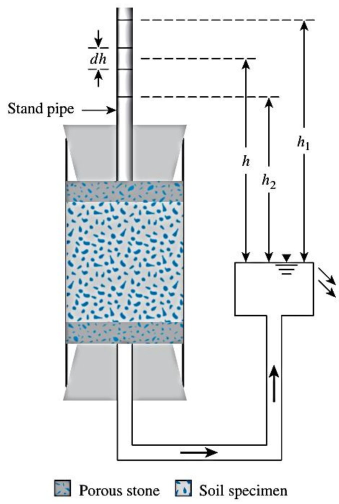
1) Record initial head difference, h1 at t = 0.

2) Allow water to flow through the soil specimen.

3) Record the final head difference, h2 at time t = t2.

4) Collect water at the outlet, Q (in ml) at time t ≈ 60 sec.





The rate of flow of the water through the specimen at any time t can be given by:

$$q = k\frac{h}{L}A = -a\frac{dh}{dt}$$

where
q = flow rate
a = cross-sectional area of the standpipe
A = cross-sectional area of the soil specimen.

$$dt = \frac{aL}{Ak}\left(-\frac{dh}{h}\right)$$

$$k = \frac{aL}{At}\ ln\left(\frac{h_1}{h_2}\right)$$

Example 5.1

Refer to the constant-head permeability test arrangement shown in the figure. A test gives these values:

- L = 30 cm
- A = area of the specimen = 177 cm^2
- Constant-head difference, h = 50 cm
- Water collected in aperiod of 5 min = 350 cm^3

Calculate the hydraulic conductivity in cm/sec.

Solution:

$$k = \frac{QL}{Aht}$$

Given $Q = 350\ cm^3$, $L = 30$ cm, $A = 177\ cm^2$, $h = 50$ cm, and $t = 5$ min, we have

$$k = \frac{(350)(30)}{(177)(50)(5)(60)} = \mathbf{3.95 \times 10^{-3}\ cm/sec}$$

Example 5.2

For a falling-head permeability test, the following values are given:

- Length of specimen 200 mm.
- Area of soil specimen 1000 mm^2.
- Area of standpipe 40 mm^2.
- Head difference at time t 0 500 mm.
- Head difference at time t 180 sec 300 mm.

Determine the hydraulic conductivity of the soil in cm/sec.

Solution:

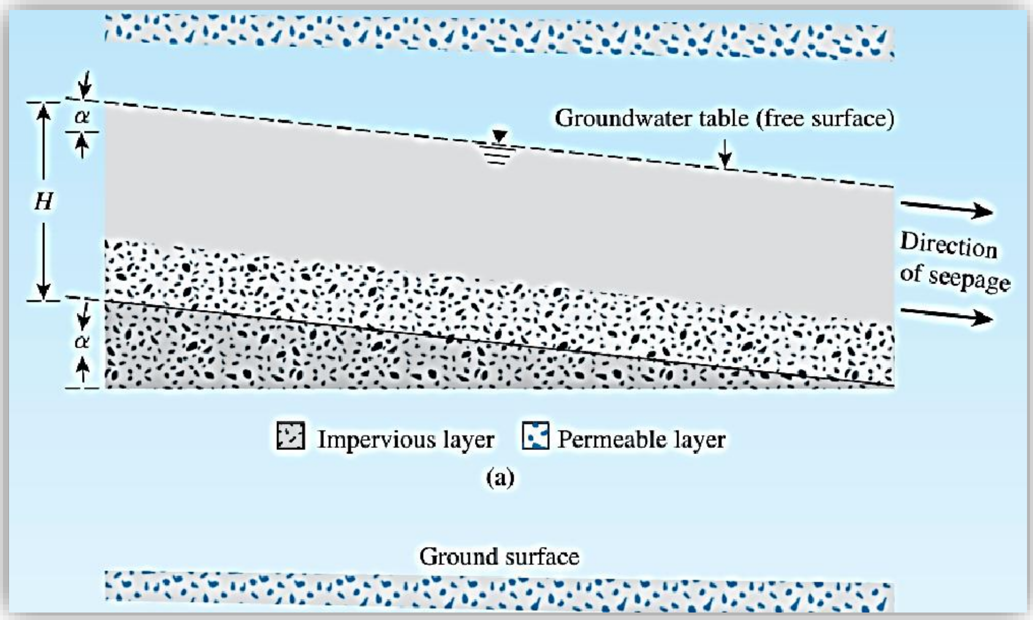
$$k = 2.303 \frac{aL}{At} \log_{10}\left(\frac{h_1}{h_2}\right)$$

We are given $a = 40$ mm^2, $L = 200$ mm, $A = 1000$ mm^2, $t = 180$ sec, $h_1 = 500$ mm, and $h_2 = 300$ mm,

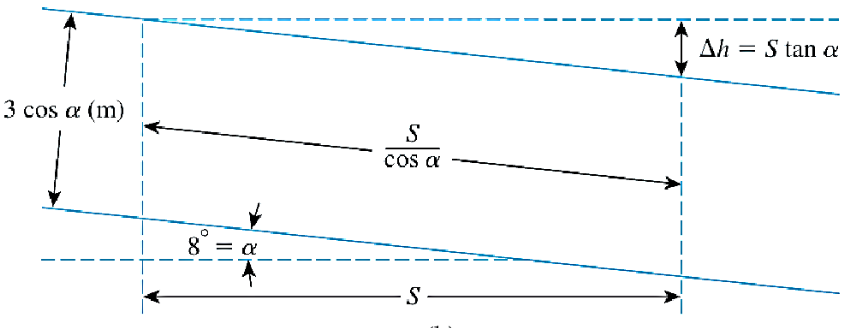
$$k = 2.303 \frac{(40)(200)}{(1000)(180)} \log_{10}\left(\frac{500}{300}\right)$$
$$= \mathbf{2.27 \times 10^{-2}\ cm/sec}$$

Example 5.3

A permeable soil layer is underlain by an impervious layer, as shown in the following figure. With k = 5.3 ×10^{-5} m/sec for the permeable layer, calculate the rate of seepage through it in m^3/hr/m width if H = 3 m and α= 8°.



Solution:



$$i = \frac{\text{head loss}}{\text{length}} = \frac{S \tan \alpha}{\left(\frac{S}{\cos \alpha}\right)} = \sin \alpha$$

$$q = kiA = (k)(\sin \alpha)(3 \cos \alpha)(1)$$

$$k = 5.3 \times 10^{-5} \text{ m/sec}$$

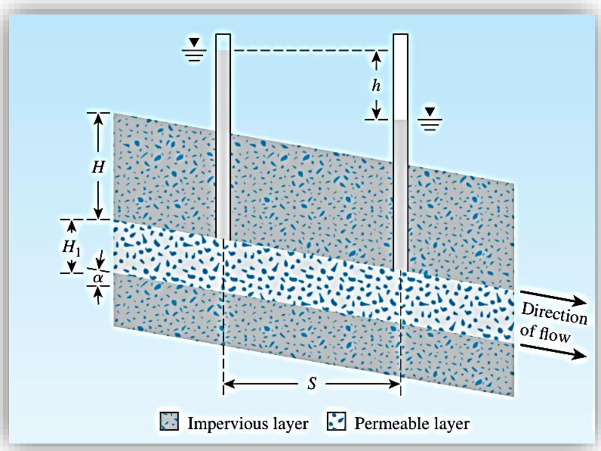
$$q = (5.3 \times 10^{-5})(\sin 8°)(3 \cos 8°)(3600) = \mathbf{0.0789\ m^3/hr/m}$$

↑

To change to

Example 5.4

Find the flow rate in m^3/sec/m length (at right angles to the cross section shown) through the permeable soil layer shown in the following figure given H = 8 m, H1= 3 m, h = 4 m, S = 50 m, α= 8°,and k = 0.08 cm/sec.



Solution: Hydraulic gradient (i)=$\frac{h}{\frac{S}{\cos \alpha}}$

$$q = kiA = k\left(\frac{h \cos \alpha}{S}\right)(H_1 \cos \alpha \times 1)$$

$$= (0.08 \times 10^{-2} \text{ m/sec})\left(\frac{4 \cos 8°}{50}\right)(3 \cos 8° \times 1)$$

$$= \mathbf{0.19 \times 10^{-3}\ m^3/sec/m}$$

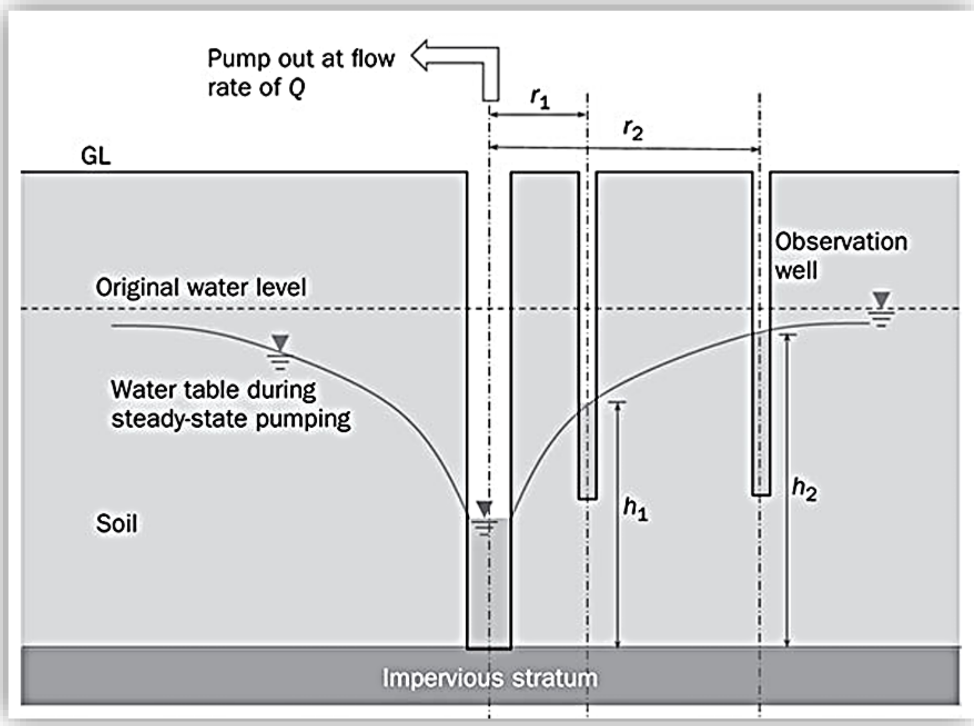
5.6 Permeability Test in the Field by Pumping from Wells

In the field, the average hydraulic conductivity of a soil deposit in the direction of flow can be determined by performing pumping tests from wells.

➢ During the test, water is pumped out at a constant rate from a test well that has a perforated casing.

➢ Several observation wells at various radial distances are made around the test well.

➢ The steady state is established when the water level in the test and observation wells becomes constant.



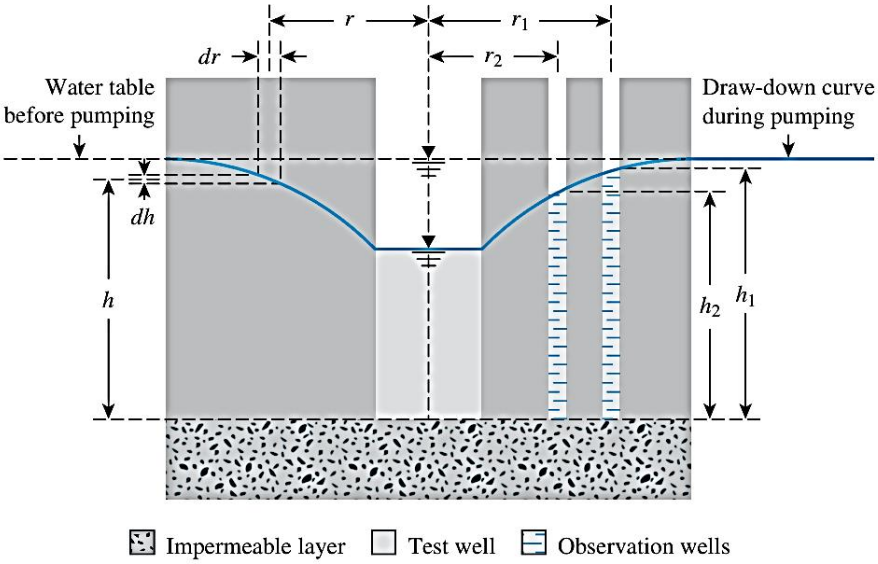
a) Unconfined aquifer

The expression for the rate of flow of groundwater into the well, which is equal to the rate of discharge from pumping, can be written as:

$$q = k\left(\frac{dh}{dr}\right) 2\pi r h$$

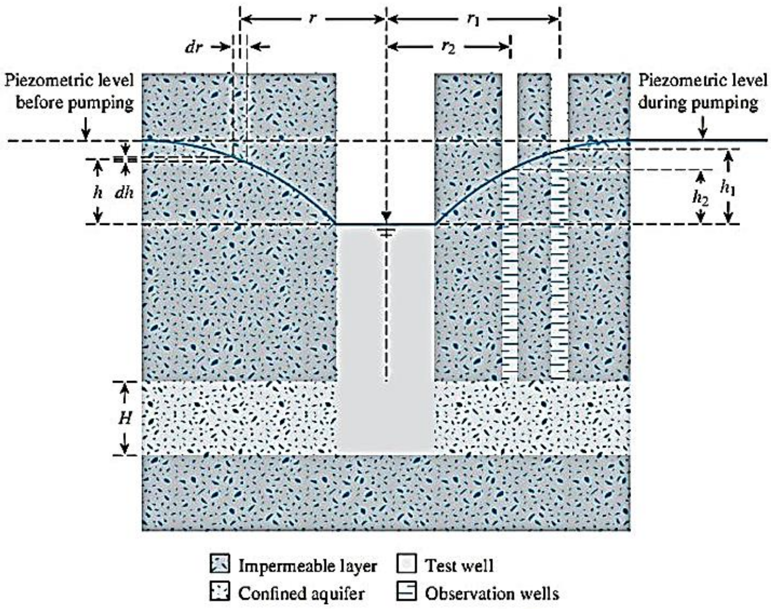
$$\int_{r_2}^{r_1} \frac{dr}{r} = \left(\frac{2\pi k}{q}\right) \int_{h_2}^{h_1} h\,dh$$

$$k = \frac{q}{\pi\left(h_1^2 - h_2^2\right)} \ln\left(\frac{r_1}{r_2}\right)$$



b) Confined aquifer

The average hydraulic conductivity for a confined aquifer can also be determined by conducting a pumping test from a well with a perforated casing that penetrates the full depth of the aquifer.



Because water can enter the test well only from the aquifer of thickness H, the steady state of discharge is:

$$q = k\left(\frac{dh}{dr}\right)\ 2\pi rH$$

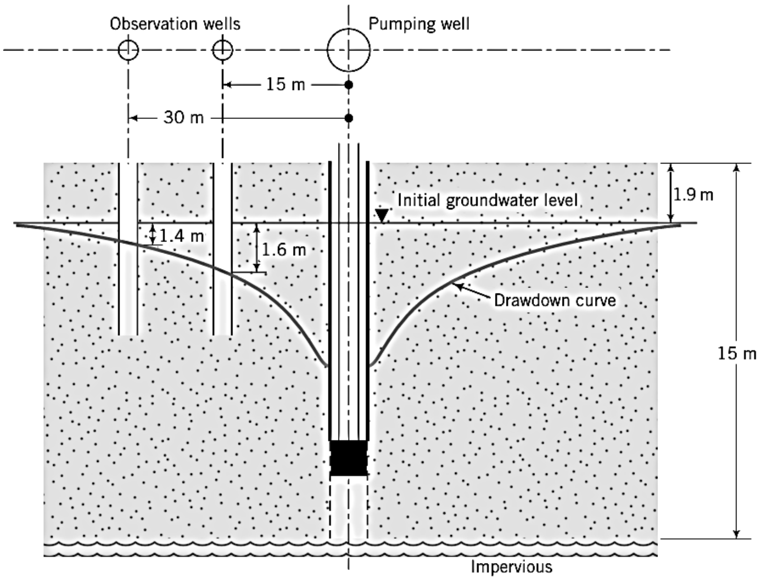
$$\int_{r_2}^{r_1} \frac{dr}{r} = \left(\frac{2\pi kH}{q}\right)\int_{h_2}^{h_1} dh$$

$$k = \frac{q}{2\pi H(h_1 - h_2)}\ \ ln\left(\frac{r_1}{r_2}\right)$$

Example 5.5

A pumping test was carried out in a soil bed of thickness 15 m and the following measurements were recorded. Rate of pumping was 10.6×10^{-3} m^3/s; drawdowns in observation wells located at 15 m and 30 m from the center of the pumping well were 1.6 m and 1.4 m, respectively, from the initial groundwater level. The initial groundwater level was located at 1.9 m below ground level. Determine k.

Solution:



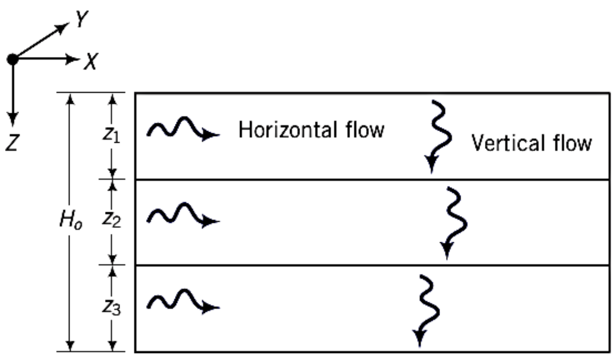
$$r_2 = 30\text{ m},\quad r_1 = 15\text{ m},\quad h_2 = 15 - (1.9 + 1.4) = 11.7\text{ m}$$

$$h_1 = 15 - (1.9 + 1.6) = 11.5\text{ m}$$

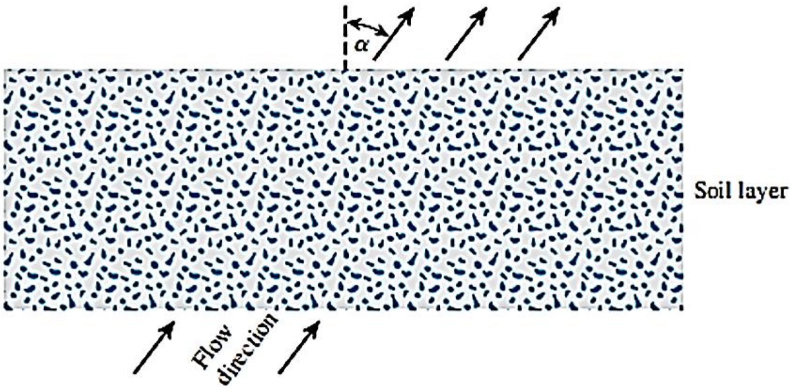
$$k = \frac{q_z \ln(r_2/r_1)}{\pi(h_2^2 - h_1^2)} = \frac{10.6\times 10^{-3}\ln(30/15)}{\pi(11.7^2 - 11.5^2)10^4} = 5.0\times 10^{-2}\text{ cm/s}$$

5.7 Equivalent Hydraulic Conductivity in Stratified Soil

- In a stratified soil deposit where the hydraulic conductivity for flow in a given direction changes from layer to layer.



- Generally, the hydraulic conductivity is the largest in the horizontal direction (k_h) and the smallest in the vertical direction (k_v).
- Heterogeneity is the variation with respect to position. Anisotropy is the variation with respect to direction.



$$r_k = k_H/k_V$$

where r_k is the anisotropy ratio.

The equivalent hydraulic conductivity for flow parallel and normal to soil layers is:

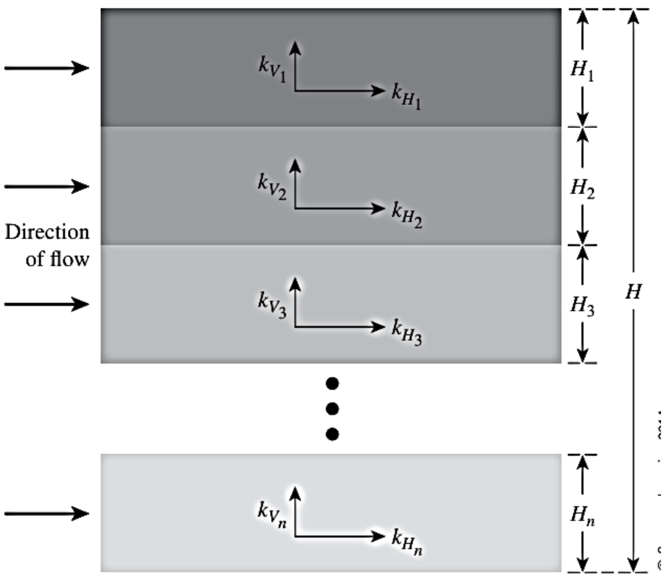
$$k_{eq} = \sqrt{k_H k_V}$$

- The following derivations relate to the equivalent hydraulic conductivities for flow in vertical and horizontal directions through multilayered soils with horizontal stratification.

a) Horizontal Flow

Assume n layers of soil with flow in the horizontal direction (horizontal flow & constant hydraulic gradient).

The total flow through the cross section in unit time can be written as



written as

$$q = v \cdot 1 \cdot H$$
$$= v_1 \cdot 1 \cdot H_1 + v_2 \cdot 1 \cdot H_2 + v_3 \cdot 1 \cdot H_3 + \ldots + v_n \cdot 1 \cdot H_n \quad (7.39)$$

where v = average discharge velocity
$v_1, v_2, v_3, \ldots, v_n$ = discharge velocities of flow in layers denoted by the subscripts

If $k_{H_1}, k_{H_2}, k_{H_3}, \ldots, k_{H_n}$ are the hydraulic conductivities of the individual layers in the horizontal direction and $k_{H(eq)}$ is the equivalent hydraulic conductivity in the horizontal direction, then, from Darcy's law,

$$v = k_{H(eq)} i_{eq}; \quad v_1 = k_{H_1} i_1; \quad v_2 = k_{H_2} i_2; \quad v_3 = k_{H_3} i_3; \quad \ldots \quad v_n = k_{H_n} i_n;$$

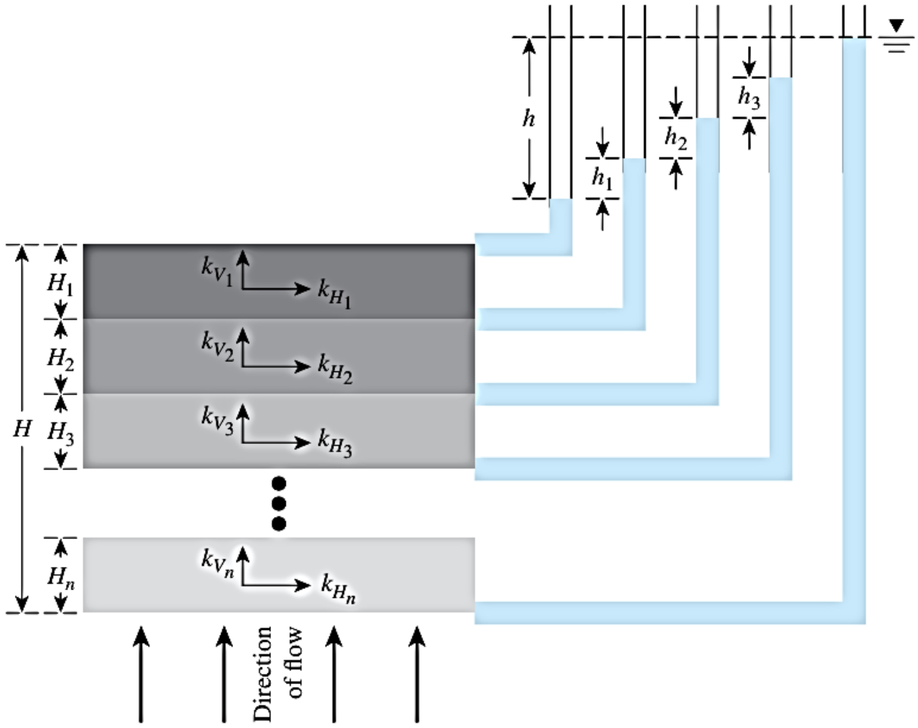
Substituting the preceding relations for velocities into Eq. (7.39) and noting that $i_{eq} = i_1 = i_2 = i_3 = \ldots = i_n$ results in

$$k_{H(eq)} = \frac{1}{H}(k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3 + \ldots + k_{H_n}H_n)$$

b) Vertical Flow

Assume n layers of soil with flow in the vertical direction (vertical flow & constant flow velocity). In this case, the velocity of flow through all the layers is the same. However, the total head loss, h, is equal to the sum of the head losses in all layers. Thus

$$v = v_1 = v_2 = v_3 = \cdots = v_n$$



and

$$h = h_1 + h_2 + h_3 + \cdots + h_n \tag{7.42}$$

Using Darcy's law, we can rewrite Eq. (7.41) as

$$k_{V(\text{eq})}\left(\frac{h}{H}\right) = k_{V_1}i_1 = k_{V_2}i_2 = k_{V_3}i_3 = \cdots = k_{V_n}i_n \tag{7.43}$$

where $k_{V_1}, k_{V_2}, k_{V_3}, \ldots, k_{V_n}$ are the hydraulic conductivities of the individual layers in the vertical direction and $k_{V(\text{eq})}$ is the equivalent hydraulic conductivity.

Again, from Eq. (7.42),

$$h = H_1 i_1 + H_2 i_2 + H_3 i_3 + \cdots + H_n i_n \tag{7.44}$$

Solving Eqs. (7.43) and (7.44) gives

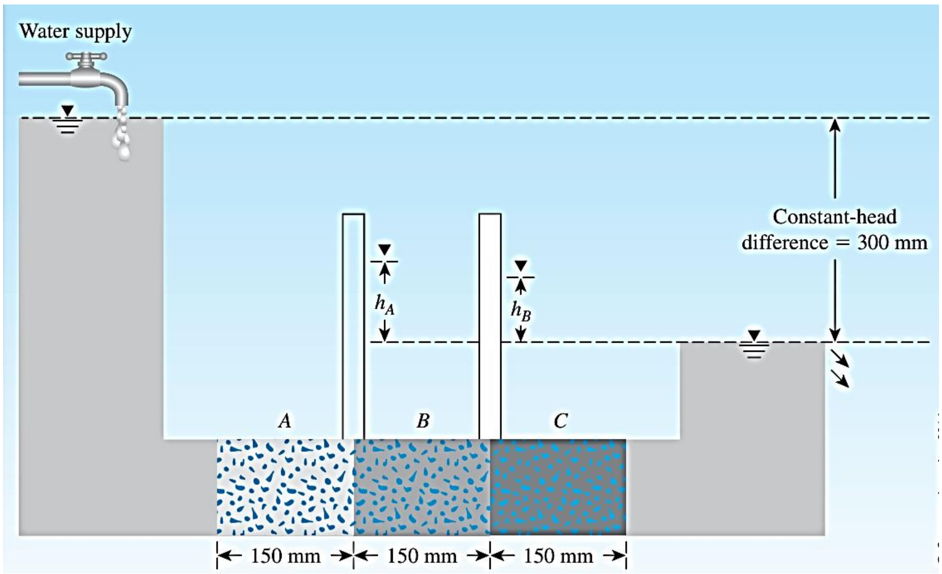
$$k_{v(\text{eq})} = \frac{H}{\left(\frac{H_1}{k_{V_1}}\right) + \left(\frac{H_2}{k_{V_2}}\right) + \left(\frac{H_3}{k_{V_3}}\right) + \cdots + \left(\frac{H_n}{k_{V_n}}\right)} \tag{7.45}$$

Example 5.6

The following figure shows three layers of soil in a tube that is 100 mm×100 mm in cross section. Water is supplied to maintain a constant-head difference of 300 mm across the sample. The hydraulic conductivities of the soils in the direction of flow through them are as follows:

A	10^{-2}
B	3×10^{-3}
C	4.9×10^{-4}

Find the rate of water supply in cm^3/hr.



Solution:

$$k_{V(\text{eq})} = \frac{H}{\left(\frac{H_1}{k_1}\right) + \left(\frac{H_2}{k_2}\right) + \left(\frac{H_3}{k_3}\right)} = \frac{450}{\left(\frac{150}{10^{-2}}\right) + \left(\frac{150}{3 \times 10^{-3}}\right) + \left(\frac{150}{4.9 \times 10^{-4}}\right)}$$

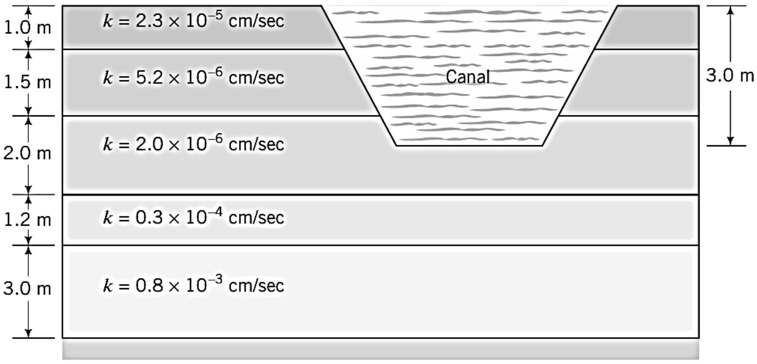
$$= 0.001213 \text{ cm/sec}$$

$$q = k_{V(\text{eq})} i A = (0.001213)\left(\frac{300}{450}\right)\left(\frac{100}{10} \times \frac{100}{10}\right)$$

$$= 0.0809 \text{ cm}^3\text{/sec} = \mathbf{291.24\ cm^3/hr}$$

Example 5.7

A canal is cut into a soil with a stratigraphy shown in the following figure. Assuming that flow takes place laterally and vertically through the sides of the canal and vertically below the canal, determine the equivalent hydraulic conductivity in the horizontal and vertical directions. The vertical and horizontal hydraulic conductivities for each layer are assumed to be the same. Calculate the ratio of the equivalent horizontal hydraulic conductivity to the equivalent vertical hydraulic conductivity for flow through the sides of the canal.



Solution:

Step 1: Find $k_{x(eq)}$ and $k_{z(eq)}$ for flow through the sides of the canal.

$$H_o = 3 \text{ m}$$

$$k_{x(eq)} = \frac{1}{H_o}(z_1 k_{x1} + z_2 k_{x2} + \cdots + z_n k_{xn})$$

$$= \frac{1}{3}(1 \times 0.23 \times 10^{-6} + 1.5 \times 5.2 \times 10^{-6} + 0.5 \times 2 \times 10^{-6})$$

$$= 3 \times 10^{-6} \text{ cm/s}$$

$$k_{z(eq)} = \frac{H_o}{\frac{z_1}{k_{z1}} + \frac{z_2}{k_{z2}} + \cdots + \frac{z_n}{k_{zn}}}$$

$$= \frac{3}{\frac{1}{10^{-6}}\left(\frac{1}{0.23} + \frac{1.5}{5.2} + \frac{0.5}{2}\right)} = 0.61 \times 10^{-6} \text{ cm/s}$$

Step 2: Find the $k_{x(eq)}/k_{z(eq)}$ ratio.

$$\frac{k_{x(eq)}}{k_{z(eq)}} = \frac{3 \times 10^{-6}}{0.61 \times 10^{-6}} = 4.9$$

Step 3: Find $k_{z(eq)}$ below the bottom of the canal.

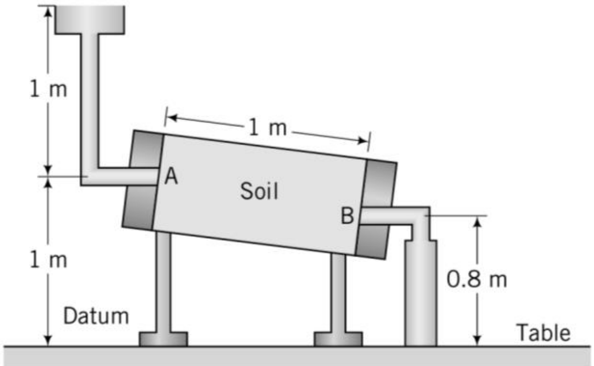
$$H_o = 1.5 + 1.2 + 3.0 = 5.7 \text{ m}$$

$$k_{z(eq)} = \frac{H_o}{\frac{z_1}{k_{z1}} + \frac{z_2}{k_{z2}} + \cdots + \frac{z_n}{k_{zn}}} = \frac{5.7}{\frac{1.5}{2 \times 10^{-6}} + \frac{1.2}{30 \times 10^{-6}} + \frac{3}{800 \times 10^{-6}}}$$

$$= 7.2 \times 10^{-6} \text{ cm/s}$$

Example 5.8

A soil sample 10 cm in diameter is placed in a tube 1 m long. A constant supply of water is allowed to flow into one end of the soil at A, and the outflow at B is collected by a beaker. The average amount of water collected is 1 cm^3 for every 10 seconds. The tube is inclined as shown in the following figure. Determine the (a) hydraulic gradient, (b) flow rate, (c) average velocity, (d) seepage velocity if e = 0.6, and (e) hydraulic conductivity.



Step 1: Define the datum position. Select the top of the table as the datum.

Step 2: Find the total heads at A (inflow) and B (outflow).

$$H_A = (h_p)_A + (h_z)_A = 1 + 1 = 2 \text{ m}$$

$$H_B = (h_p)_B + (h_z)_B = 0 + 0.8 = 0.8 \text{ m}$$

Step 3: Find the hydraulic gradient.

$$\Delta H = H_A - H_B = 2 - 0.8 = 1.2 \text{ m}$$

$$L = 1 \text{ m}; \quad i = \frac{\Delta H}{L} = \frac{1.2}{1} = 1.2$$

If you were to select the outflow, point B, as the datum, then $H_A = 1 \text{ m} + 0.2 \text{ m} = 1.2 \text{ m}$ and $H_B = 0$. The head loss is $\Delta H = 1.2$ m, which is the same value we obtained using the table's top as the datum. It is often simpler, for calculation purposes, to select the exit flow position as the datum.

If you were to select the outflow, point B, as the datum, then $H_A = 1\text{ m} + 0.2\text{ m} = 1.2\text{ m}$ and $H_B = 0$. The head loss is $\Delta H = 1.2$ m, which is the same value we obtained using the table's top as the datum. It is often simpler, for calculation purposes, to select the exit flow position as the datum.

Step 4: Determine the flow rate.

Volume of water collected, $Q = 1\text{ cm}^3$, $t = 10$ seconds

$$q_z = \frac{Q}{t} = \frac{1}{10} = 0.1\text{ cm}^3/\text{s}$$

Step 5: Determine the average velocity.

$$q_z = A\text{v}$$

$$A = \frac{\pi \times (\text{diam})^2}{4} = \frac{\pi \times 10^2}{4} = 78.5\text{ cm}^2$$

$$\text{v} = \frac{q_z}{A} = \frac{0.1}{78.5} = 0.0013\text{ cm/s}$$

Step 6: Determine seepage velocity.

$$\text{v}_s = \frac{\text{v}}{n}$$

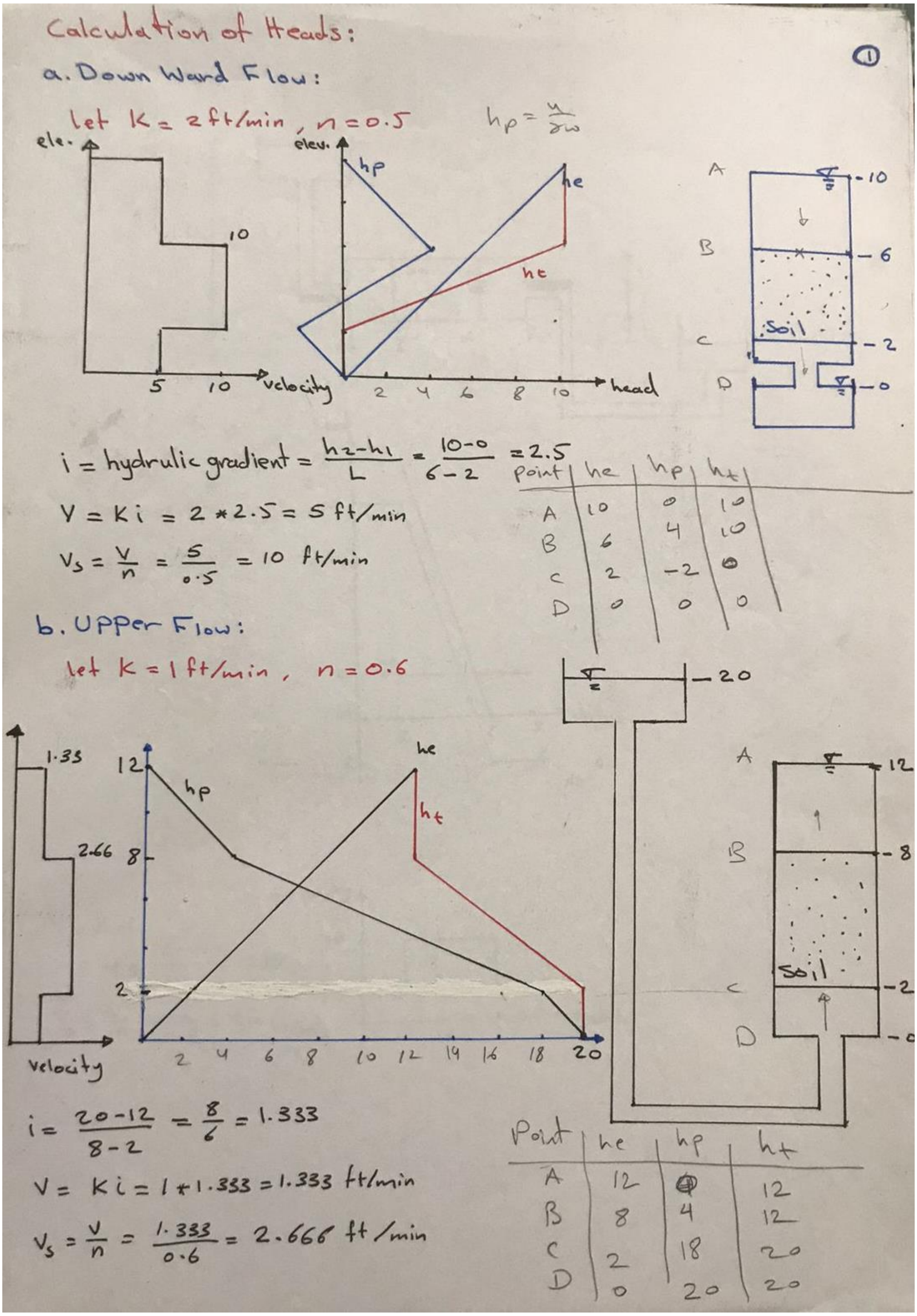
$$n = \frac{e}{1+e} = \frac{0.6}{1+0.6} = 0.38$$

$$\text{v}_s = \frac{0.0013}{0.38} = 0.0034\text{ cm/s}$$

Step 7: Determine the hydraulic conductivity. From Darcy's law, $\text{v} = k_z i$.

$$\therefore k_z = \frac{\text{v}}{i} = \frac{0.0013}{1.2} = 10.8 \times 10^{-4}\text{ cm/s}$$

Example 5.9



Calculation of Heads:

a. Down Ward Flow:

let K = 2 ft/min, n = 0.5 $\quad h_p = \frac{u}{\gamma_w}$

i = hydrulic gradient = $\frac{h_2 - h_1}{L} = \frac{10-0}{6-2} = 2.5$

$V = Ki = 2 * 2.5 = 5$ ft/min

$V_s = \frac{V}{n} = \frac{5}{0.5} = 10$ ft/min

point	he	hp	ht
A	10	0	10
B	6	4	10
C	2	-2	0
D	0	0	0

b. Upper Flow:

let K = 1 ft/min, n = 0.6

$i = \frac{20-12}{8-2} = \frac{8}{6} = 1.333$

$V = Ki = 1 * 1.333 = 1.333$ ft/min

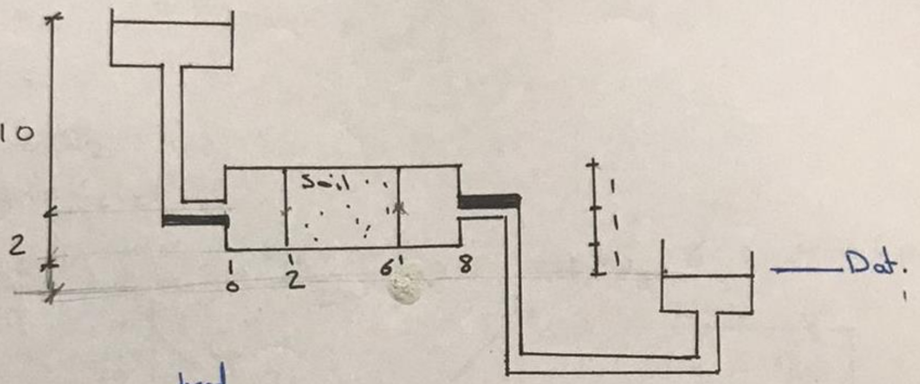
$V_s = \frac{V}{n} = \frac{1.333}{0.6} = 2.666$ ft/min

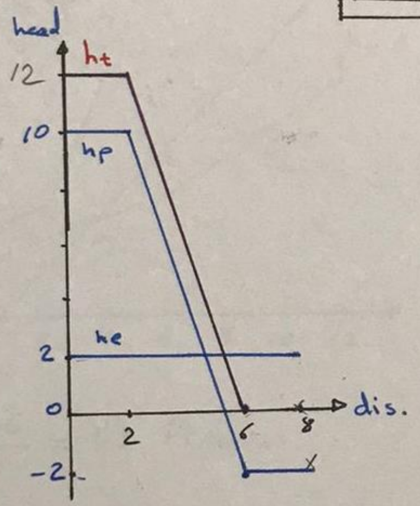
Point	he	hp	ht
A	12	0	12
B	8	4	12
C	2	18	20
D	0	20	20

②

C. Horizontal Flow:

let $K = 4$ ft/min, $n = 0.6$

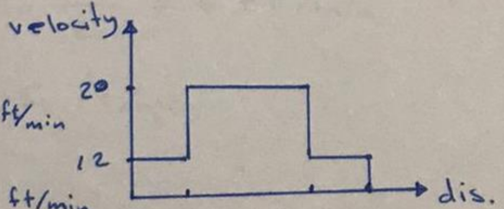




$V = K \cdot i$

$= 4 * \frac{12-0}{4} = 12$ ft/min

$V_s = \frac{V}{n} = \frac{12}{0.6} = 20$ ft/min



D. Flow in Two Soils:

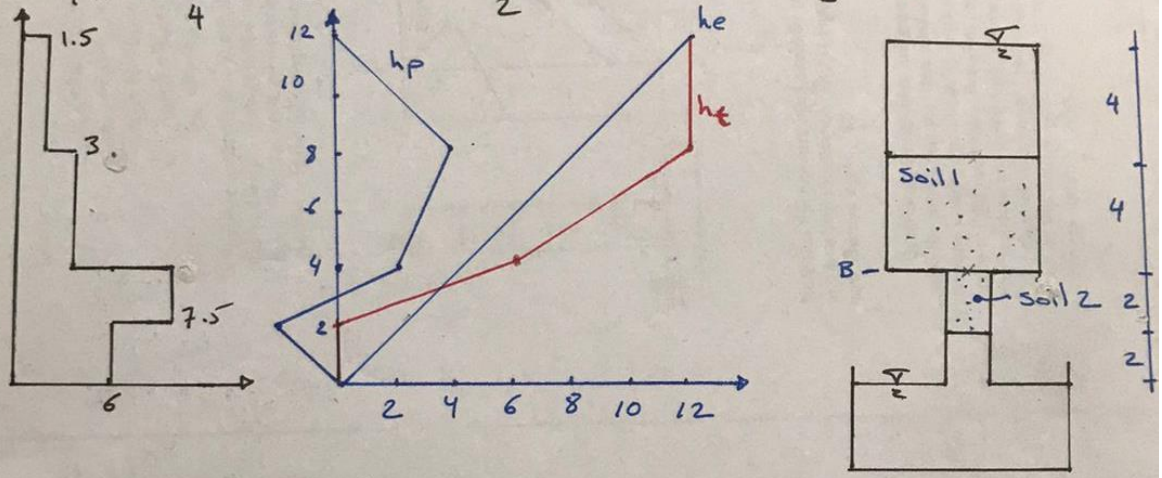
③

	Soil 1	Soil 2
A	2 ft^2	0.5 ft^2
K	1 ft/min	2 ft/min
n	0.5	0.8

$$Q_1 = Q_2$$

$$K_1 i_1 A_1 = K_2 i_2 A_2$$

$$1 * \frac{12 - h_{t})_B}{4} * 2 = 2 * \frac{h_{t})_B - 0}{2} * 0.5 \Rightarrow h_{t})_B = 6, \; h_e = 4 \Rightarrow h_p = 2$$

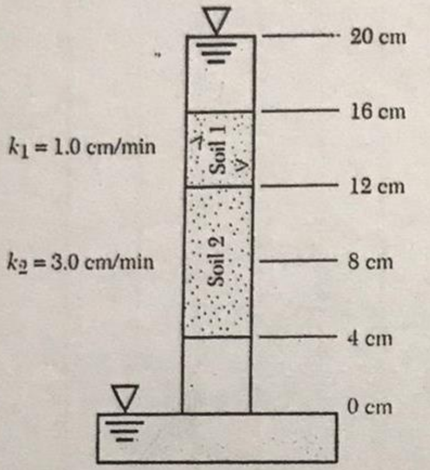


$$V_1 = K_1 \cdot i_1 = 1 * \frac{12-6}{4} = 1.5 \text{ ft/min}$$

$$V_{s1} = \frac{1.5}{0.5} = 3 \text{ ft/min}$$

$$V_2 = K_2 i_2 = 2 * \frac{6-0}{2} = 6 \text{ ft/min}$$

$$V_{s2} = \frac{6}{0.8} = 7.5 \text{ ft/min}$$



$q_1 = q_2$

$1 * A * \frac{\Delta h_1}{4} = 3 * A * \frac{\Delta h_2}{8}$

$\Rightarrow \Delta h_1 = \frac{3}{2} \Delta h_2$

$\Delta H = \Delta h_1 + \Delta h_2$

$\Delta H = 20 - 0 = 20$

$20 = \frac{3}{2} \Delta h_2 + \Delta h_2$

$\Rightarrow \Delta h_2 = 8$

$\Rightarrow \Delta h_1 = 12$

Solution

Although there is no best way to handle problems of this type, it is always a good idea to follow some specific methodology. In this case, follow the steps outlined below:

Step 1: Establish a datum (reference) at elevation 0 cm, then determine the total head difference. In this case

$$\Delta H = 20 - 0 = 20 \text{ cm}$$

Step 2: Determine the total head loss in layer 1 and layer 2 using results from the preceding example.

$$\alpha = \frac{k_2}{k_1}\frac{L_1}{L_2} = \frac{3.0}{1.0}\left(\frac{4}{8}\right) = 1.5$$

$$\Delta h_1 = \frac{\alpha \Delta H}{1+\alpha} = \frac{1.5(20)}{1+1.5} = 12 \text{ cm} \qquad \Delta h_2 = \frac{\Delta H}{1+\alpha} = \frac{(20)}{1+1.5} = 8 \text{ cm}$$

Step 3: Determine the elevation head versus height.

Step 4: Determine the total head at elevation 20 cm, 16 cm, 12 cm, 4 cm, and 0 cm. This is because at these elevations the total head is known. Thus

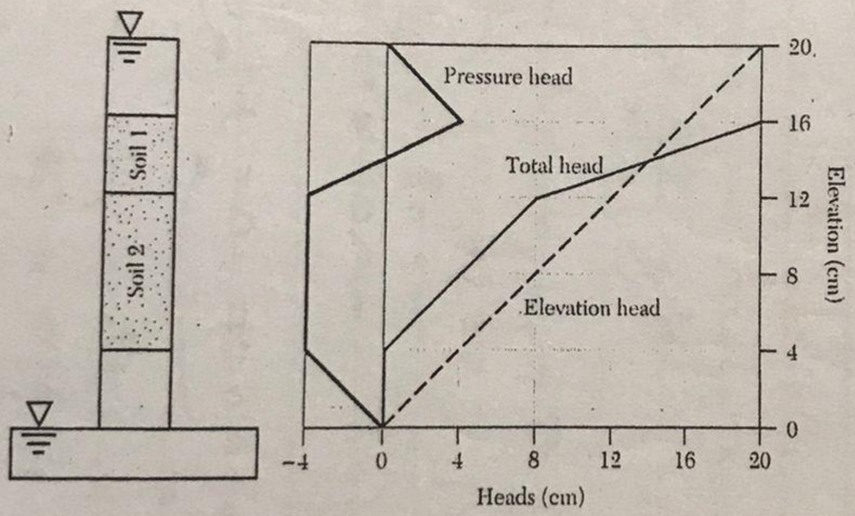
$$h_{20} = h_{e20} + h_{p20} = 20 + 0 = 20 \text{ cm}$$
$$h_{16} = h_{e16} + h_{p16} = 16 + 4 = 20 \text{ cm}$$
$$h_{12} = h_{16} - \Delta h_1 = 20 - 12 = 8 \text{ cm}$$
$$h_4 = h_{12} - \Delta h_2 = 8 - 8 = 0 \text{ cm}$$
$$h_0 = h_{e0} + h_{p0} = 0 + 0 = 0 \text{ cm}$$

Note that head loss occurs only when water flows through soil.

Step 5: Plot the total and elevation heads, then determine the pressure head as follows:

$$h_{p20} = h_{20} - h_{e20} = 20 - 20 = 0 \text{ cm}$$
$$h_{p16} = h_{16} - h_{e16} = 20 - 16 = 4 \text{ cm}$$
$$h_{p12} = h_{12} - h_{e12} = 8 - 12 = -4 \text{ cm}$$
$$h_{p4} = h_4 - h_{e4} = 0 - 4 = -4 \text{ cm}$$
$$h_{p0} = h_0 - h_{e0} = 0 - 0 = 0 \text{ cm}$$

The pore water pressure at any elevation can be calculated by multiplying the unit weight of water by the pressure head at that elevation. That is, $u = h_p \gamma_w$. The total head, elevation head, and pressure head versus elevation are shown in the following figure:



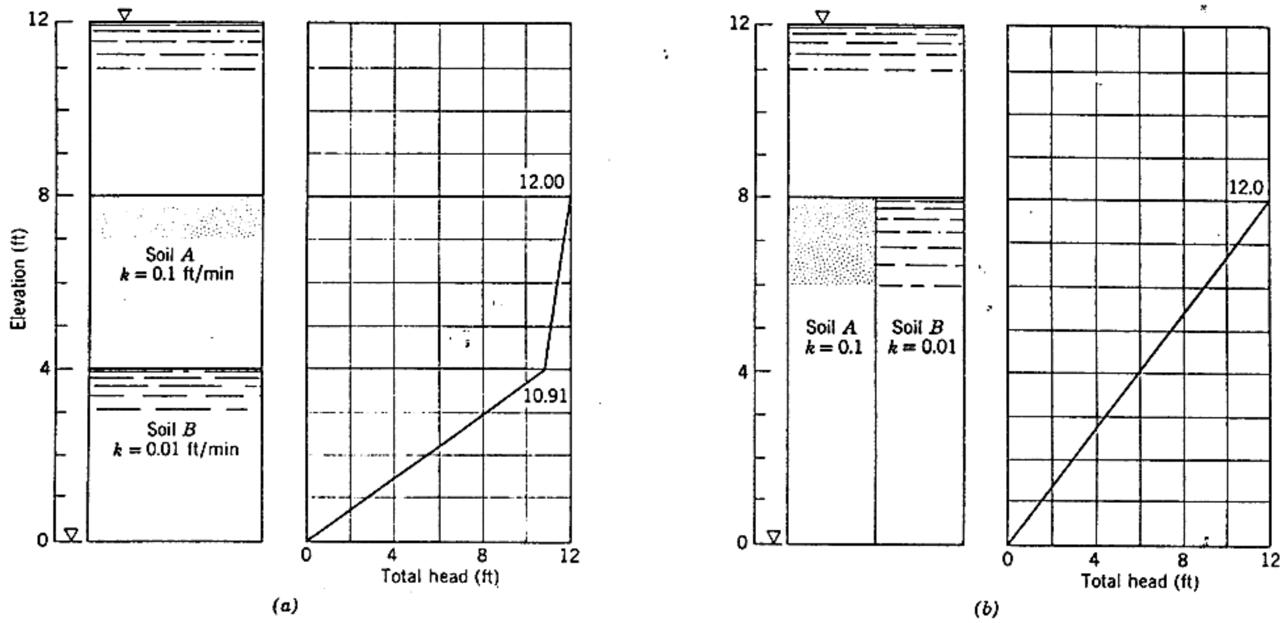
Note that the pressure head could be negative. This phenomenon is of theoretical and practical importance. This is especially true when dealing with capillary rise and effective stress. ■

Geotechnical engineers are especially interested in the pressure head because the pore water pressure needed to study soil behavior depends on it. The pressure head at a point can be measured directly or can be computed using principles of fluid mechanics. For most practical problems, the soil in question is nonhomogeneous and anisotropic. Consequently, field tests are normally required to determine actual pressure head values. The pressure head or water pressure at a point in the field is determined using a **piezometer**, a word literally meaning "pressure meter."

4

Example 5.10

Given the two soil-filled tubes in Fig. E18.5, find the quantity of flow and total head vs. elevation.



Soil A:

$$\frac{Q}{L} = 0.1 \text{ ft/min} \times \frac{1.09}{4} \times 4 \text{ ft} = 0.109 \text{ (ft}^3\text{/min)/ft}$$

Soil B:

$$\frac{Q}{L} = 0.01 \text{ ft/min} \times \frac{10.9}{4} \times 4 \text{ ft} = 0.109 \text{ (ft}^3\text{/min)/ft}$$

Soil A:

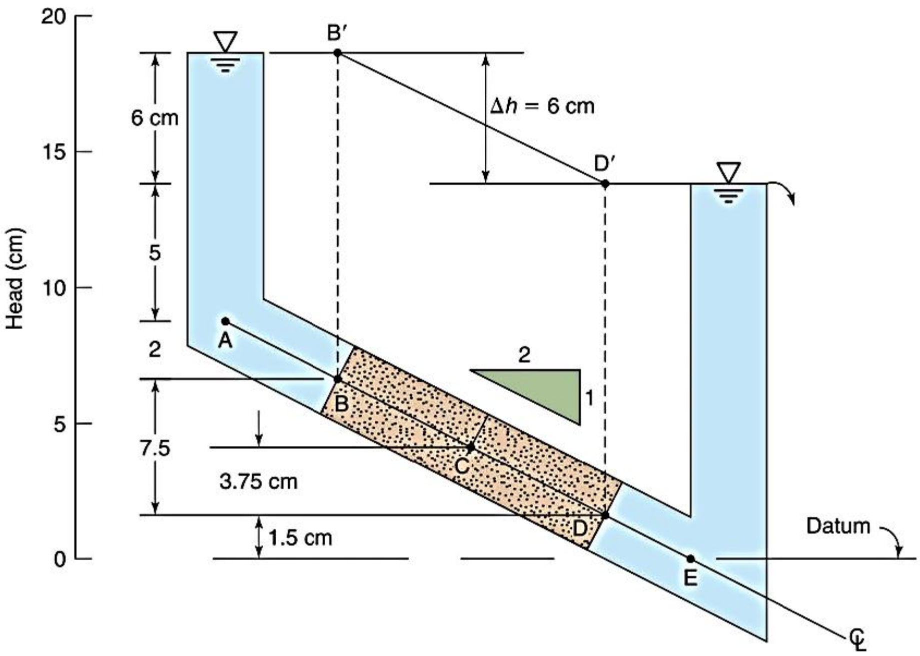
$$\frac{Q}{L} = 0.1 \text{ ft/min} \times \frac{12}{8} \times 2 = 0.30 \text{ (ft}^3\text{/min)/ft}$$

Soil B:

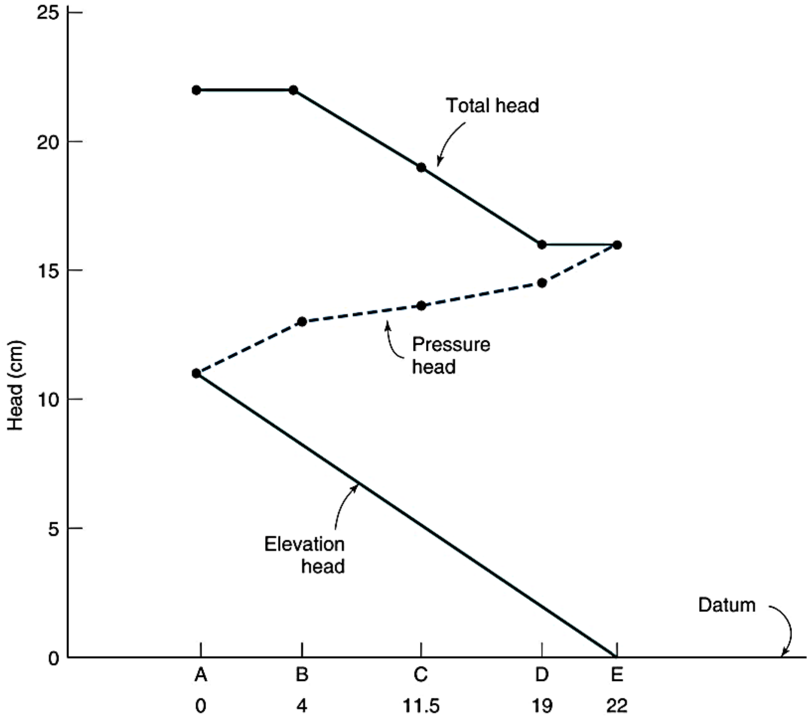
$$\frac{Q}{L} = 0.01 \text{ ft/min} \times \frac{12}{8} \times 2 = 0.03 \text{ (ft}^3\text{/min)/ft}$$

Example 5.11

For the following figure determine the pressure, elevation, and total head at sufficient points to be able to plot them versus horizontal distance.

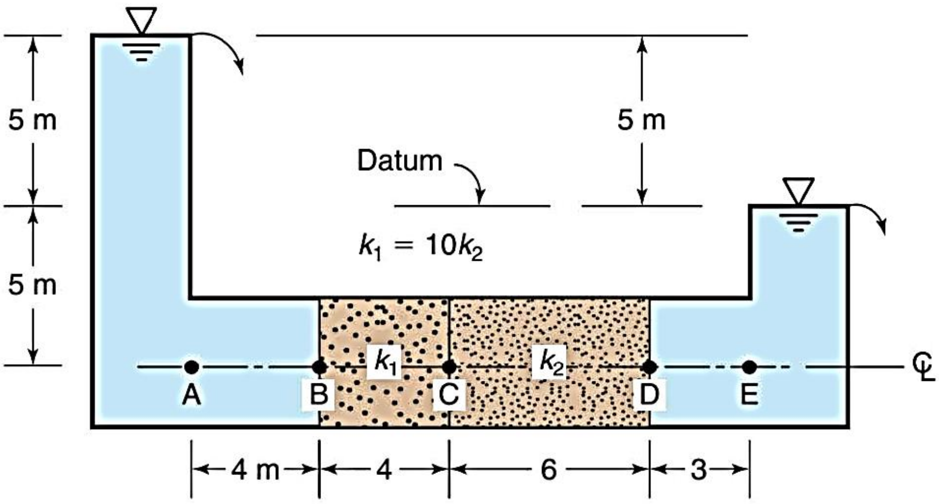


Point	Pressure Head (cm)	Elevation Head (cm)	Total Head (cm)	Head Loss (cm)
A	11	11	22	0
B	13	9	22	0
C	13.75	5.25	19	3
D	14.5	1.5	16	6
E	16	0	16	6



Example 5.11

For the following figure determine the pressure, elevation, and total head at sufficient points to be able to plot them versus horizontal distance.



Solution:

$$q_1 = k_1 i_1 A_1 = q_2 = k_2 i_2 A_2$$

Since the areas are the same, $q_{1,2} = k_1 i_1 = k_2 i_2$ with $k_1 = 10k_2$ and $i = \Delta h/l$. Substituting,

$$q_{1,2} = 10k_2 \frac{\Delta h_1}{L_1} = k_2 \frac{\Delta h_2}{L_2}$$

Also, the total head loss, $\Delta h = \Delta h_1 + \Delta h_2$. So, $\Delta h_1 = \Delta h - \Delta h_2$, and we obtain

$$q_{1,2} = 10k_2 \frac{(\Delta h - \Delta h_2)}{L_1} = k_2 \frac{\Delta h_2}{L_2}$$

Rearranging and multiplying out,

$$L_2 10k_2 \Delta h - L_2 10k_2 \Delta h_2 = k_2 \Delta h_2 L_1$$

Rearranging and canceling out the k_2's,

$$10L_2 \Delta h = \Delta h_2 (L_1 + 10L_2)$$

Solving for Δh_2,

$$\Delta h_2 = \frac{10L_2 \Delta h}{L_1 + 10L_2}$$

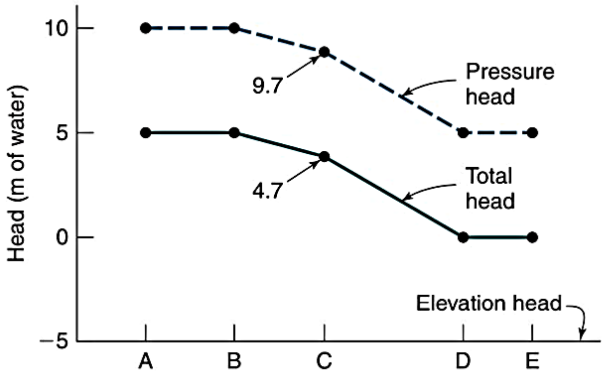
$$= \frac{10 \times 6 \text{ m} \times 5 \text{ m}}{(4 \text{ m} + 10 \times 6 \text{ m})} = \frac{300 \text{ m}^2}{64 \text{ m}}$$

$$= 4.69 \text{ m}$$

$$\therefore \Delta h_1 = \Delta h - \Delta h_2 = 5 - 4.69 = 0.31 \text{ m}$$

Point	Pressure Head (m)	Elevation Head (m)	Total Head (m)	Head Loss (m)
A	10	−5	5	0
B	10	−5	5	0
C	9.7	−5	4.7	0.31
D	5	−5	0	5
E	5	−5	0	5

Because the permeability of soil 2 is so much less than that of soil 1, most of the head is lost in soil 2.



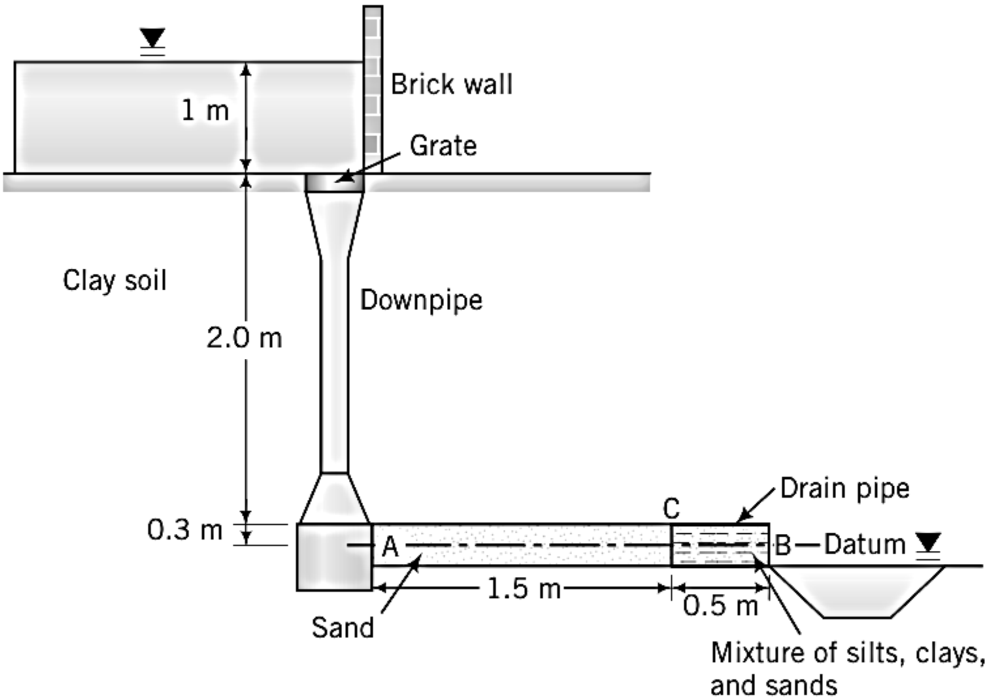
Example 5.12

A drainage pipe (shown in the following figure) became completely blocked during a storm by a plug of sand 1.5 m long, followed by another plug of a mixture of clays, silts, and sands 0.5 m long. When the storm was over, the water level above ground was 1 m. The hydraulic conductivity of the sand is 2 times that of the mixture of clays, silts, and sands.

(a) Plot the variation of pressure, elevation, and total head over the length of the pipe.

(b) Calculate the porewater pressure at (1) the center of the sand plug and (2) the center of the mixture of clays, silts, and sands.

(c) Find the average hydraulic gradients in the sand and in the mixture of clays, silts, and sands.



Solution:

Step 1: Select a datum.

Select the exit at B along the centerline of the drainage pipe as the datum.

Step 2: Determine heads at A and B.

$$(h_z)_A = 0 \text{ m}, \quad (h_p)_A = 0.3 + 2 + 1 = 3.3 \text{ m}, \quad H_A = 0 + 3.3 = 3.3 \text{ m}$$

$$(h_z)_B = 0 \text{ m}, \quad (h_p)_B = 0 \text{ m}, \quad H_B = 0 \text{ m}$$

Step 3: Determine the head loss in each plug.

Head loss between A and B = $|H_B - H_A|$ = 3.3 m (decrease in head taken as positive). Let ΔH_1, L_1, k_1, and q_1 be the head loss, length, hydraulic conductivity, and flow in the sand; let ΔH_2, L_2, k_2, and q_2 be the head loss, length, hydraulic conductivity, and flow in the mixture of clays, silts, and sands. Now,

$$q_1 = \mathrm{A}k_1\frac{\Delta H_1}{L_1} = \mathrm{A} \times 2k_2\frac{\Delta H_1}{L_1}$$

$$q_2 = \mathrm{A}k_2\frac{\Delta H_2}{L_2} = \mathrm{A} \times k_2\frac{\Delta H_2}{L_2}$$

From the continuity equation, $q_1 = q_2$.

$$\therefore\ \mathrm{A} \times 2k_2\frac{\Delta H_1}{L_1} = \mathrm{A}k_2\frac{\Delta H_2}{L_2}$$

Solving, we get

$$\frac{\Delta H_1}{\Delta H_2} = \frac{L_1}{2L_2} = \frac{1.5}{2 \times 0.5} = 1.5$$

$$\Delta H_1 = 1.5\Delta H_2 \qquad (1)$$

However, we know that

$$\Delta H_1 + \Delta H_2 = \Delta H = 3.3\ \mathrm{m} \qquad (2)$$

Solving for ΔH_1 and ΔH_2 from Equations (1) and (2), we obtain

$$\Delta H_1 = 1.98\ \mathrm{m} \quad \text{and} \quad \Delta H_2 = 3.3 - 1.98 = 1.32\ \mathrm{m}$$

Step 4: Calculate heads at the junction of the two plugs.

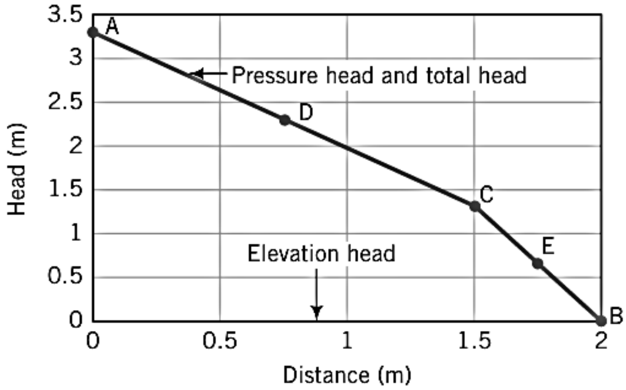
$$\text{Total head at C} = H_\mathrm{C} = H_\mathrm{A} - \Delta H_1 = 3.3 - 1.98 = 1.32\ \mathrm{m}$$

$$(h_z)_\mathrm{C} = 0$$

$$(h_p)_\mathrm{C} = H_\mathrm{C} - (h_z)_\mathrm{C} = 1.32\ \mathrm{m}$$

Step 5: Plot distribution of heads.

See Figure E6.3b.



Step 6: Calculate porewater pressures.

Let D be the center of the sand.

$$(h_p)_D = \frac{(h_p)_A + (h_p)_C}{2} = \frac{3.3 + 1.32}{2} = 2.31 \text{ m}$$

$$u_D = 2.31 \times \gamma_w = 2.31 \times 9.8 = 22.6 \text{ kPa}$$

Let E be the center of the mixture of clays, silts, and sands.

$$(h_p)_E = \frac{(h_p)_C + (h_p)_B}{2} = \frac{1.32 + 0}{2} = 0.66 \text{ m}$$

$$u_E = 0.66 \times 9.8 = 6.5 \text{ kPa}$$

Step 7: Find the average hydraulic gradients.

$$i_1 = \frac{\Delta H_1}{L_1} = \frac{1.98}{1.5} = 1.32$$

$$i_2 = \frac{\Delta H_2}{L_2} = \frac{1.32}{0.5} = 2.64$$

5.8 Two-Dimensional Flow of Water Through Soils

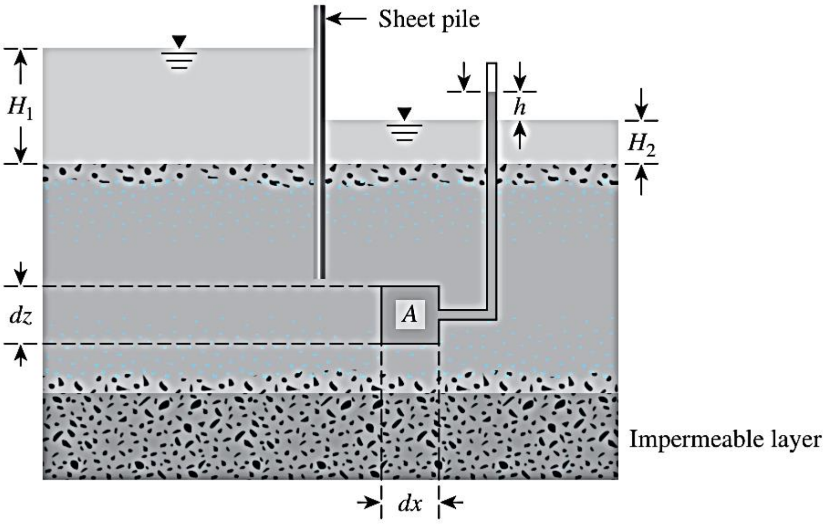
The two-dimensional flow of water through soils is described by Laplace's equation given as

$$k_x \frac{\partial^2 H}{\partial x^2} + k_z \frac{\partial^2 H}{\partial z^2} = 0$$

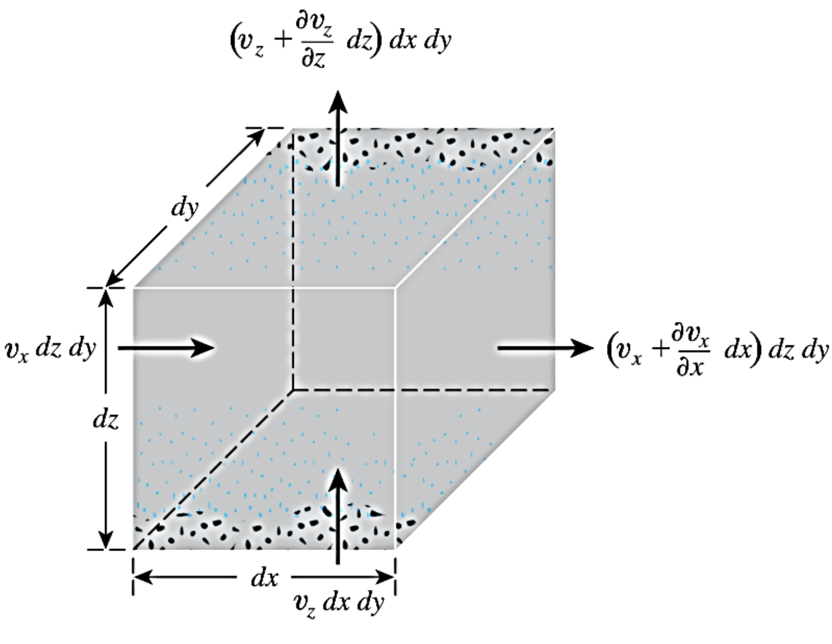
where H is the total head and kx and kz are the hydraulic conductivities in the x and z directions.

Laplace equation

The flow in porous media occurs in 3-D. Laplace equation is the combination of the continuity equation and the Darcy's law.



(a)



(b)

(a) Single-row sheet piles driven into permeable layer; (b) flow at A

Flow in are: $q = vA$

$$v_x dydz, \quad v_y dxdz, \quad v_z dxdy$$

Flow out are:

$$\left(v_x + \frac{\partial v_x}{\partial x}dx\right)dydz, \quad \left(v_y + \frac{\partial v_y}{\partial y}dy\right)dxdz, \quad \left(v_z + \frac{\partial v_z}{\partial z}dz\right)dxdy$$

Flow in = Flow out

$$\left(v_x + \frac{\partial v_x}{\partial x}dx\right)dydz + \left(v_y + \frac{\partial v_y}{\partial y}dy\right)dxdz + \left(v_z + \frac{\partial v_z}{\partial z}dz\right)dxdy - \left[v_x dydz + v_y dxdz + v_z dxdy\right] = 0$$

The equation of continuity is:

$$\frac{\partial v_x}{\partial x}dxdydz + \frac{\partial v_y}{\partial y}dydxdz + \frac{\partial v_z}{\partial z}dzdxdy = 0$$

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

Darcy's Law:

$$v_x = -k_x\frac{\partial h}{\partial x}, \quad v_y = -k_y\frac{\partial h}{\partial y}, \quad v_z = -k_z\frac{\partial h}{\partial z}$$

Apply Darcy's law to the equation of continuity:

$$k_x\frac{\partial^2 h}{\partial x^2} + k_y\frac{\partial^2 h}{\partial y^2} + k_z\frac{\partial^2 h}{\partial z^2} = 0$$

For isotropic soil:

$$k = k_x = k_y = k_z$$

Laplace Equation is:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

Flow nets

In an isotropic porous medium:

$$k = k_x = k_y = k_z$$

Laplace Equation

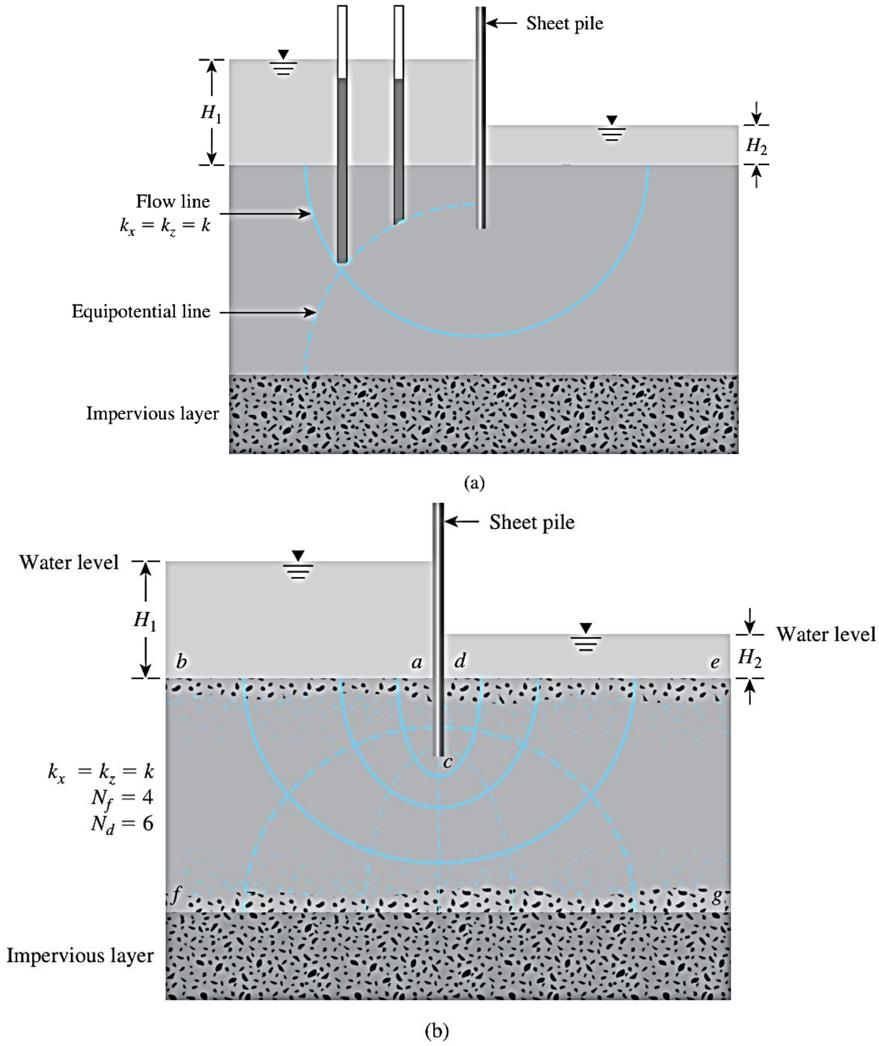
$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

Resents two orthogonal families of curves:

1-**Flow line**: the line along which a water particle will travel from upstream to the downstream side in the permeable soil medium;

2-**Equipotential line**: the line along which the potential (pressure) head at all points is equal;

Flow net: is the combination of flow lines and equipotential lines, which is the graphical solution of Laplace equation.



(a) Definition of flow lines and equipotential lines; (b) completed flow net

Drawing a flow net takes several trials. While constructing the flow net, keep the boundary conditions in mind. For the flow net shown in Figure, the following four boundary conditions apply:

Condition 1: The upstream and downstream surfaces of the permeable layer (lines ab and de) are equipotential lines.

Condition 2: Because ab and de are equipotential lines, all the flow lines intersect them at right angles.

Condition 3: The boundary of the impervious layer (line fg and the surface of the impervious sheet pile, line acd are flow lines).

Condition 4: The equipotential lines intersect acd and fg at right angles.

5.9 Seepage calculation from a flow net

In a flow net, the strip between any two adjacent flow lines is called a flow channel.

The drop in the piezometric level between any two adjacent equipotential lines is the same and is called the potential drop.

The rate of seepage through the flow channel per unit length can be calculated as follows.

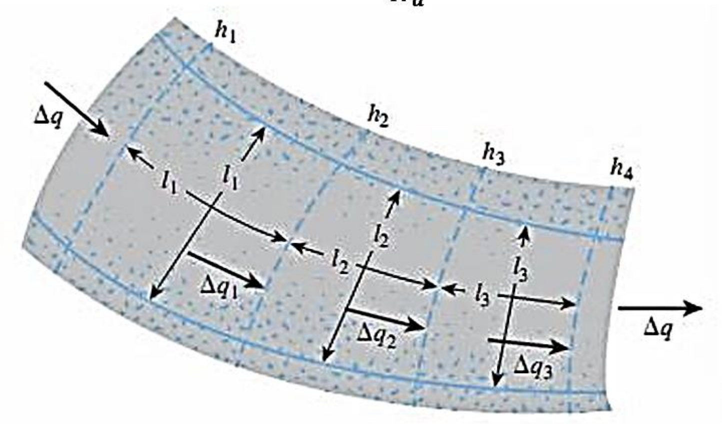
Because there is no flow across the flow lines,

$$\Delta q_1 = \Delta q_2 = \Delta q_3 = \cdots = \Delta q$$

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right) l_1 = k\left(\frac{h_2 - h_3}{l_2}\right) l_2 = k\left(\frac{h_3 - h_4}{l_3}\right) l_3 = \cdots$$

$$\Delta h = h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \frac{H}{N_d}$$

$$\Delta q = k\frac{H}{N_d}$$



Seepage through a flow channel with square elements.

where H = head difference between the upstream and downstream sides; and N_d = number of potential drops.

$$q = kH\frac{N_f}{N_d}$$

N_f = number of flow channels in a flow net.

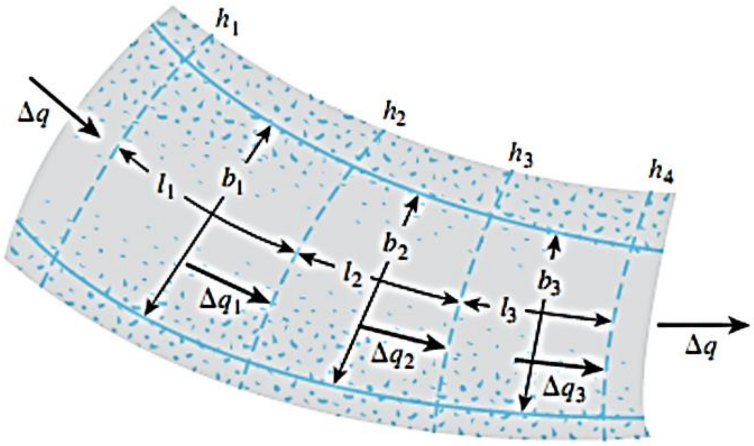
One can draw a **rectangular mesh** for flow channel, as shown in Figure, with constant ratios of width-to-length for all the rectangular elements in the flow net. In this case, the rate of flow is:

$$\Delta q = k\left(\frac{h_1 - h_2}{l_1}\right) b_1 = k\left(\frac{h_2 - h_3}{l_2}\right) b_2 = k\left(\frac{h_3 - h_4}{l_3}\right) b_3 = \cdots$$

$$b_1/l_1 = b_2/l_2 = b_3/l_3 = \cdots = n$$

$$\Delta q = kH\frac{n}{N_d}$$

$$q = kH\frac{N_f}{N_d}\,n$$

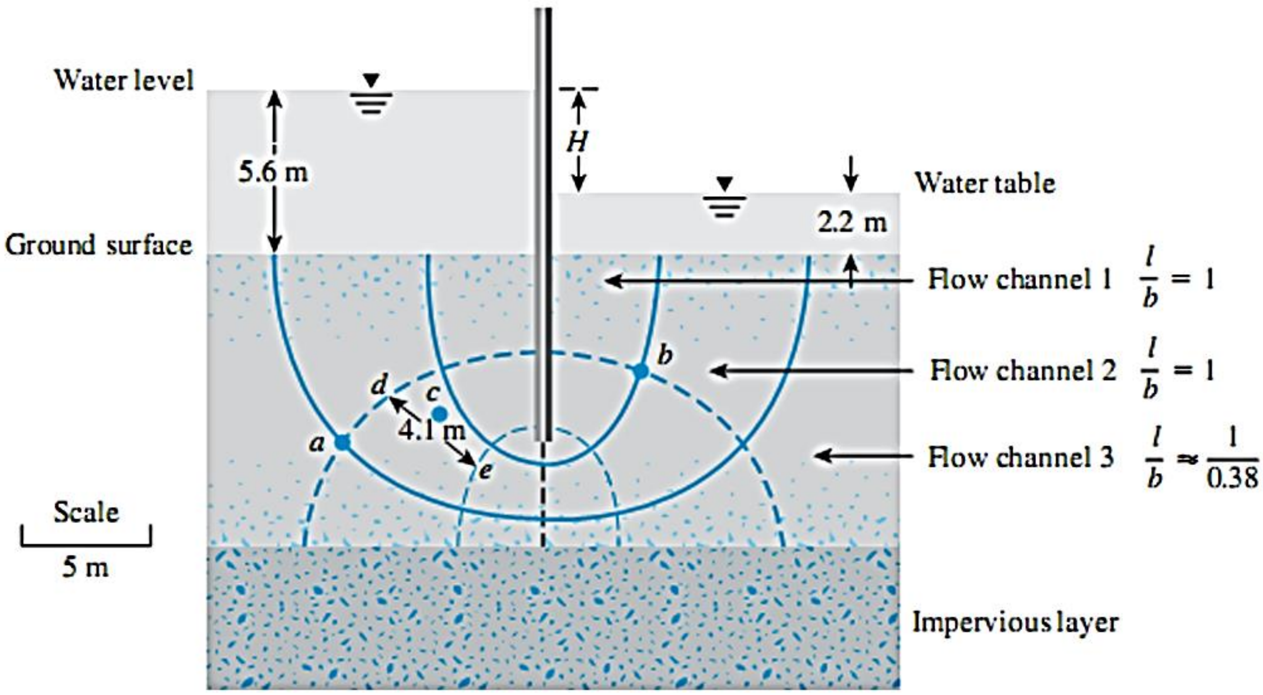


Seepage through a flow channel with rectangular elements.

Example 5.13

A flow net for flow around a single row of sheet piles in a permeable soil layer is shown in Figure. Given that $k_x = k_z = k = 5x10^{-3}$ cm/sec, determine:

a) How high (above the ground surface) the water will rise if piezometers are placed at points a and b.
b) The rate of seepage through the permeable layer per unit length.
c) The approximate average hydraulic gradient at c.



Solution:

Part a

From Figure 8.7, we have $N_d = 6$, $H_1 = 5.6$ m, and $H_2 = 2.2$ m. So the head loss of each potential drop is

$$\Delta H = \frac{H_1 - H_2}{N_d} = \frac{5.6 - 2.2}{6} = 0.567 \text{ m}$$

At point *a*, we have gone through one potential drop. So the water in the piezometer will rise to an elevation of

$$(5.6 - 0.567) = \mathbf{5.033 \text{ m above the ground surface}}$$

At point *b*, we have five potential drops. So the water in the piezometer will rise to an elevation of

$$[5.6 - (5)(0.567)] = \mathbf{2.765 \text{ m above the ground surface}}$$

Part b

From Eq. (8.25),

$$q = 2.38\frac{k(H_1 - H_2)}{N_d} = \frac{(2.38)(5 \times 10^{-5} \text{ m/sec})(5.6 - 2.2)}{6}$$

$$= \mathbf{6.74 \times 10^{-5} m^3/sec/m}$$

Part c

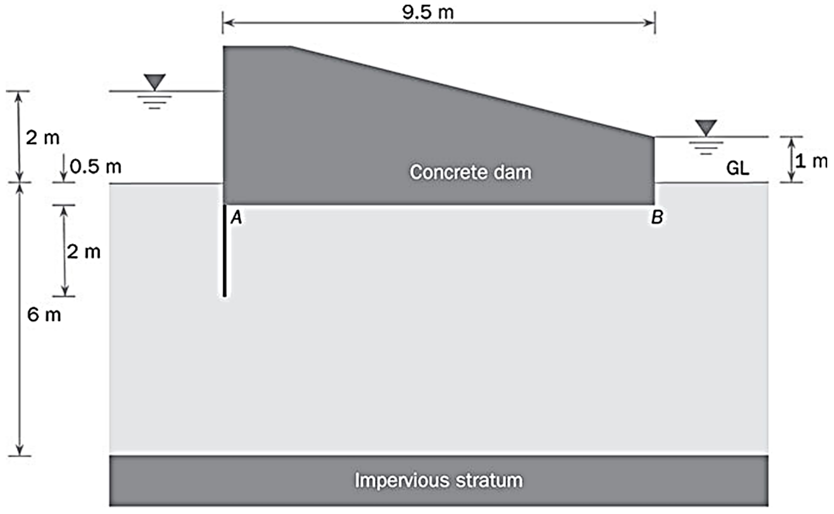
The average hydraulic gradient at *c* can be given as

$$i = \frac{\text{head loss}}{\text{average length of flow between } d \text{ and } e} = \frac{\Delta H}{\Delta L} = \frac{0.567 \text{ m}}{4.1 \text{ m}} = \mathbf{0.138}$$

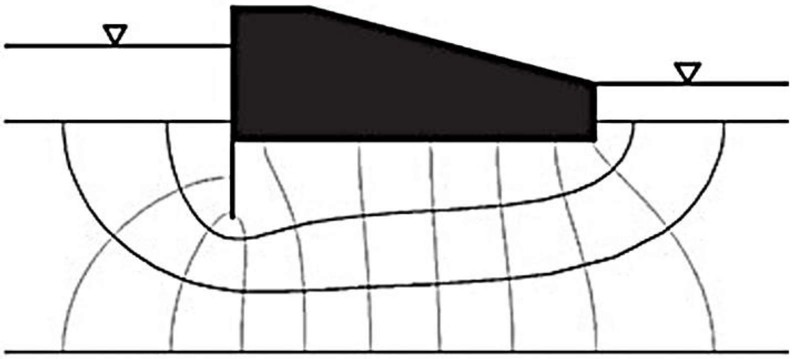
(*Note:* The average length of flow has been scaled.)

Example 5.14

A concrete dam with a sheet pile cut-off wall at the upstream end is shown in the following figure. The total head loss from upstream to downstream is 1.0 m. Hydraulic conductivity of the underlying soil is 3.2×10^{-3} cm/s. Determine the flow rate per unit width of the dam per day.



Solution:



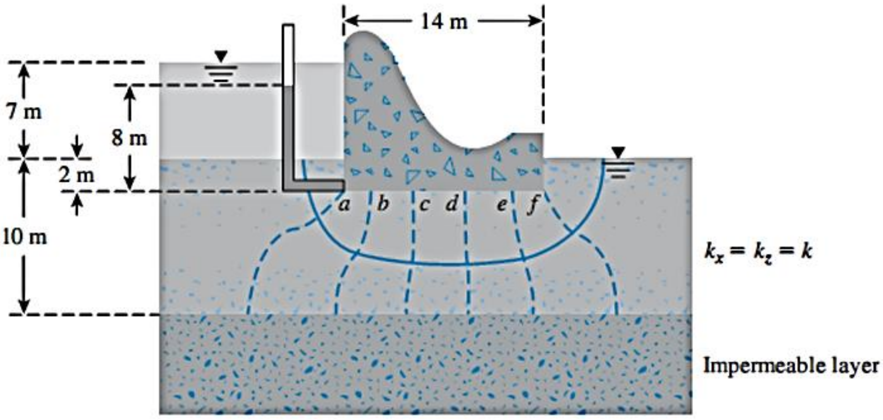
$$N_f = 3, N_d = 10$$

$$Q = kh_L \frac{N_f}{N_d} = (3.2 \times 10^{-5})(1.0)\left(\frac{3}{10}\right) = 0.96 \times 10^{-5} \text{ m}^3/\text{s per m width}$$

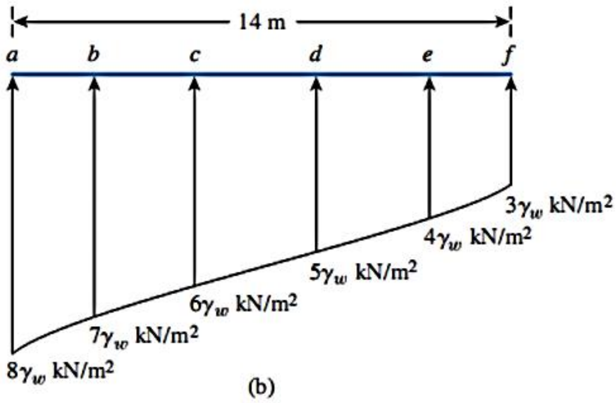
$$= (0.96 \times 10^{-5})(24 \times 3600) = \mathbf{0.83\ m^3\ per\ day\ per\ m\ width}$$

5.10 Uplift pressure under hydraulic structures

Flow nets can be used to determine the uplift pressure at the base of a hydraulic structure. The necessary flow net also has been drawn (assuming that kx = kz = k).



(a)



(b)

(a) A weir; (b) uplift force under a hydraulic structure

There are seven equipotential drops (Nd) in the flow net, and the difference in the water levels between the upstream and downstream sides is H 7 m. The head loss for each potential drop is H/7 = 7/7 =1 m. The uplift pressure at

$$(\text{left corner of the base}) = (\text{Pressure head at } a) \times (\gamma_w)$$
$$= [(7+2) - 1]\gamma_w = 8\gamma_w$$

Similarly, the uplift pressure at

$$b = [9 - (2)(1)]\gamma_w = 7\gamma_w$$

and at

$$f = [9 - (6)(1)]\gamma_w = 3\gamma_w$$

- The uplift force per unit length measured along the axis of the weir can be calculated by finding the area of the pressure diagram.

Example 5.14

In the concrete dam shown in Example 5.13, determine the pore water pressures at A and B and hence the uplift on the dam.

Solution Let us take the downstream water level as the datum.

The total head loss from upstream to downstream, h_L = 1.0 m

The total head loss between any two adjacent equipotential lines = 1.0/10 = 0.1 m

The total head at upstream and downstream ground levels are 1 and 0 m, respectively. There are about 3.5 equipotential drops from the upstream to point *A*, and hence the total head at *A* is 0.65 m. The elevation head at *A* is −1.5 m. Therefore, the pressure head at *A* is 2.15 m.

The pore water pressure at *A* = 2.15 × 9.81 = **21.1 kN/m²**

There is 0.9 equipotential drop from the downstream to point *B* (or 9.1 drops from upstream) where the total head is 0.9 × 0.1 = 0.09 m. The elevation head at *B* is −1.5 m. Therefore, the pressure head at *B* is 1.59 m.

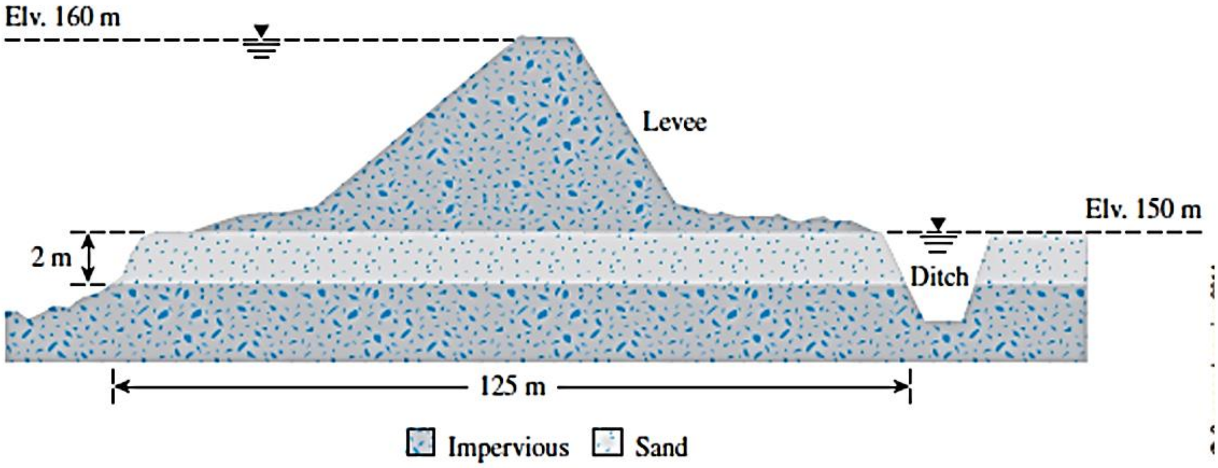
The pore water pressure at *B* = 1.59 × 9.81 = **15.6 kN/m²**

Assuming linear variation of pore water pressure between the points *A* and *B*, the uplift force on the bottom of the concrete dam can be computed as

$$U = \frac{(21.1 + 15.6)(9.5)}{2} = \mathbf{174.3\,kN\ per\,m\,width}$$

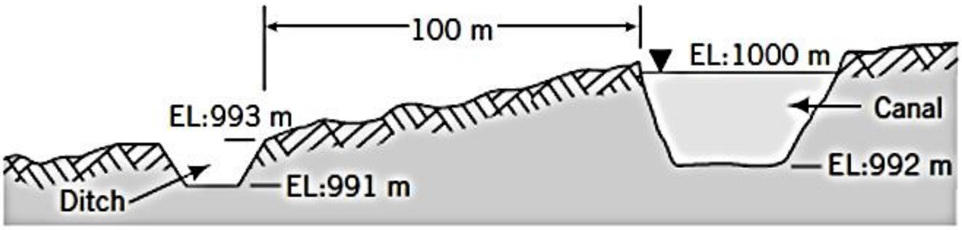
Problem 5.1

The cross section of a levee that is 500 m long and is underlain by a 2-m-thick permeable sand layer is shown in figure below. It was observed that the quantity of water flowing through the sand layer into the collection ditch is 250 m^3/day. What is the hydraulic conductivity of the sand layer?



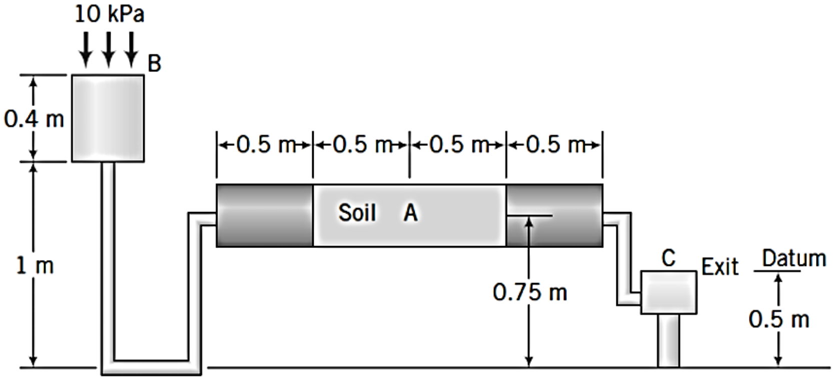
Problem 5.2

A ditch is required for a utility line near an ephemeral canal, which at the time of excavation was filled with water, as shown in Figure below. The average vertical and horizontal hydraulic conductivities are 1×10^{-5} cm/s and 2×10^{-4} cm/s, respectively. Assuming a 1-m length of ditch, determine the flow rate of water into it.



Problem 5.3

Determine the pressure head, elevation head, and total head at A, B, and C for the arrangement shown in Figure below. Take the water level at exit as datum.



Problem 5.4

Consider the setup shown in Figure below (similar to Example 7.13) in which three different soil layers, each 200 mm in length, are located inside a cylindrical tube of diameter 150 mm. A constant-head difference of 470 mm is maintained across the soil sample. The porosities and hydraulic conductivities of the three soils in the direction of the flow are given in the following Table:

Soil	n	k (cm/sec)
I	0.5	5×10^{-3}
II	0.6	4.2×10^{-2}
III	0.33	3.9×10^{-4}

Perform the following tasks:

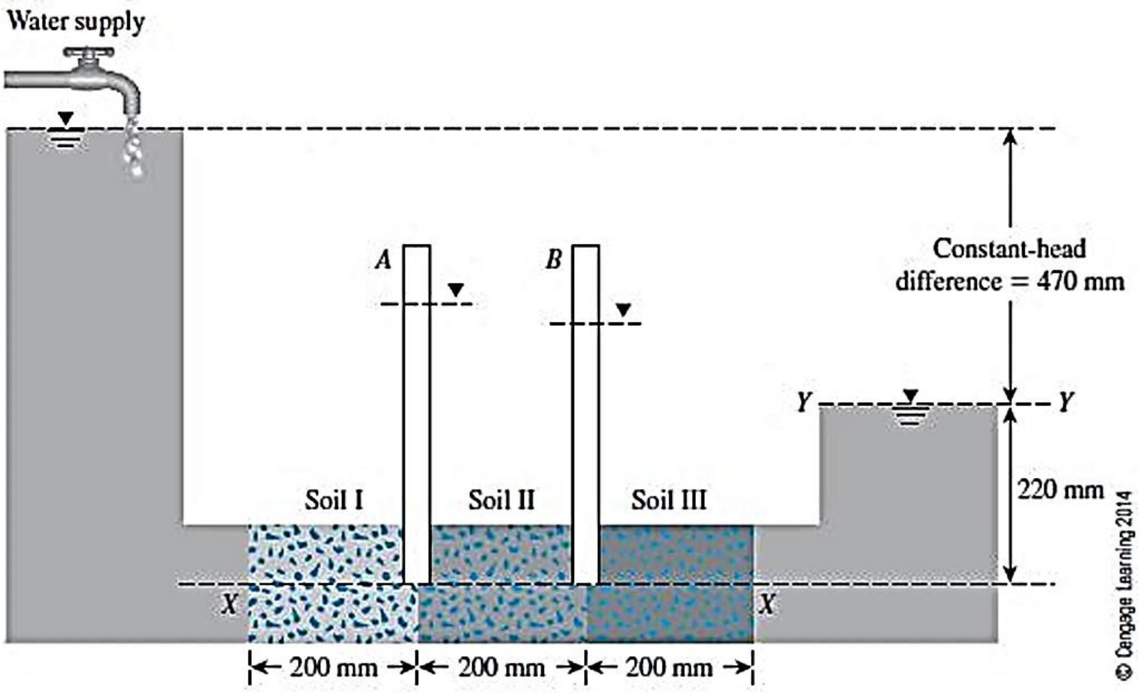
a) Determine the quantity of water flowing through the sample per hour.

b) Denoting the downstream water level (Y-Y) to be the datum, determine the elevation head (Z), pressure head (u/γ_w), and the total head (h) at the entrance and exit of each soil layer.

c) Plot the variation of the elevation head, pressure head and the total head with the horizontal distance along the sample axis (X-X).

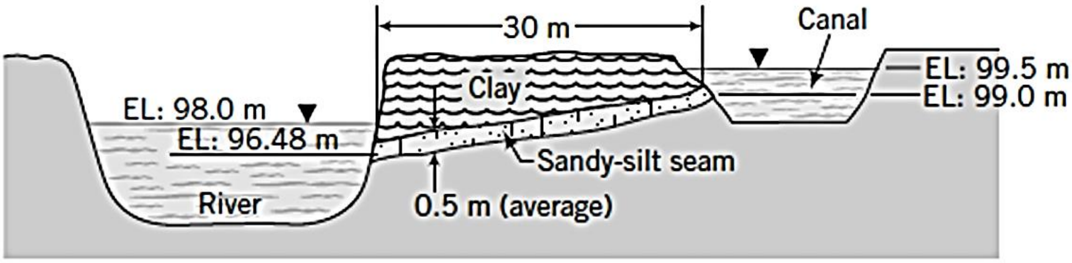
d) Plot the variations of discharge velocity and the seepage velocity along the sample axis.

e) What will be the height of the vertical columns of water inside piezometers A and B installed on the sample axis?



Problem 5.5

A canal is dug parallel to a river, as shown in Figure below. A sandy-silt seam of average thickness 0.5 m cuts across the otherwise impermeable clay. The average vertical and horizontal hydraulic conductivities are 1.5×10^{-5} cm/s and 15×10^{-5} cm/s, respectively. Assuming a 1-m length of canal, determine the flow rate of water from the canal to the river.



Problem 5.6

Draw a flow net for the weir shown in Figure below. Calculate the rate of seepage under the weir.